

# Index

- $\mathcal{F}$ -measurable function, 257
- $\sigma$ -algebra, 254
  - generated by random variable, 258
  - generated by stochastic process, 270
- adapted processes, 273, 402
- almost surely continuous stochastic process, 264
- alpha, 192
- Amortization, 53
- Annuity, 46
  - application to bond valuation, 66
  - application to equity in a house, 61
  - application to sinking funds, 62
  - application to stock valuation, 63
  - applications to saving, borrowing, and spending, 59
  - future value, 49
  - present value, 50
  - with varying payments and interest rates, 56
- APR, 33
- APY, 33
- arbitrage, 84, 334
- asset, 329
  - financial, 329
- at-the-money, 364
- average value-at-risk, 182
- Banker's Rule, 16
- basis points, 5
- bearish, 363
- beta, 158, 160
  - for portfolios, 162, 192
  - linear factor beta, 195
- bid-ask spread, 9
- bid/ask price, 8
- binomial pricing
  - comparison with BSM pricing, 422
  - of European calls, 411–422
  - of underliers, 209–246
- binomial trees
  - Cox-Ross-Rubinstein tree, 218
  - general, 209–218
  - Jarrow-Rudd tree, 251
  - probability measure, 214
  - recombining property, 211
- Black-Scholes-Merton model, *see* BSM model
- bonds, 66
  - bond prices vs. interest rates, 70
  - bond prices vs. YTM, 72
  - bond valuation formula, 69
  - callable, 66
  - convertible, 66
  - coupon payment, 68
  - coupon rate, 68
  - current yield, 68
  - issue date, 67
  - maturity date, 67
  - maturity, par, or face value, 67
  - par bonds, 67
  - premium bonds, 67
  - yield to maturity (YTM), 68
  - zero-coupon bond, 68
- book value, 196
- book-to-market ratio, 197
- boundary conditions, 391
  - for European calls, 392
  - for European puts, 396

- Brownian motion, 282–289
  - with drift, 289
  - with drift and scaling, 284, 314
  - with starting point, 284
- Brownian path, 284
- BSM model, 384–391
  - unique risk-neutral probability measure, 461
  - versus market data, 440–448
- BSM p.d.e., 391
  - equivalent to heat equation, 393
  - existence and uniqueness of solutions, 391
  - solving for European calls, 392
- bullish, 363
  
- Cameron-Martin-Girsanov theorem, *see* Girsanov theorem
- Capital Allocation Line (CAL), 154
- Capital Asset Pricing Model, *see* CAPM
- Capital Market Line (CML), 153–157
  - tangent point—market portfolio, 156
- CAPM, 152, 158–165
  - beta versus linear factor beta, 195
  - for portfolios, 162
  - formula, 159
  - risk premium of a security, 158
  - risk premium of the market portfolio, 158
  - security price, 160
  - security risk decomposition, 164
- cash market, 337
- coherent risk measure, 184
- coincident indicators, 10
- commercial banking, 3
- commodity, 331
- commodity swap, *see* swaps
- compound interest, 21
  - continuous compounding, 31
    - formula, 27
  - fractional compounding, 28
  - fractional vs. simple compounding, 30
  - future value, 27
  - generalized compounding, 31
  - nonnegative integer number of periods, 22
  - nonnegative real number of periods, 24
  - present value, 27
- conditional expectation, 270–273
- conditional value-at-risk, 182
- contingent claim, 330
- continuous stochastic process, 264
- continuous-state processes, 262
- continuous-time processes, 262
  - converge
    - almost surely, 265
    - in distribution, 265
    - in mean square, 265
    - in probability, 265
  - correlation of Brownian motion, 295
  - covered call, *see* options
  - Cox-Ross-Rubinstein tree, *see* CRR tree
  - credit default swaps, *see* swaps
  - CRR tree, 218–246
    - continuous-time limit, 237–246
    - CRR equations, 226, 234
    - real world, 219–230
      - real-world uptick probability, 226, 234
      - risk-neutral uptick probability, 233, 234
      - risk-neutral world, 230–236
      - security price formula, 229, 236
- dealers, 8
- deep in-the-money, 364
- deep out-the-money, 364
- delivery market, 338
- delta, 390
  - discrete version, 415
    - European call and put deltas at expiration, 398
    - of a European call, 396
    - of a European put, 397
- delta hedging, 422–433
  - application, 427–433
  - theoretical framework, 422–427
- derivatives
  - characteristics of its valuation, 332
  - commodity, 331
  - defined, 330
  - financial, 331
  - purposes of, 331
- Descartes's Rule of Signs, 43
- diffusion coefficient, 315
- diffusion equation, *see* heat equation
- diffusion process, 289
- Dirichlet density, 139
- discounted price process, 406
- discrete-state processes, 262
- discrete-time processes, 262
- diversifiable risk, 143, 151
- diversification, *see* Markowitz portfolio theory
- diversified portfolio, *see* Markowitz portfolio theory
- dividend
  - continuously reinvested, 340

- cum-dividend, 18, 31, 387
- ex-dividend, 18, 31
- ex-dividend date, 18
- yield, 19, 215
- dividend discount model, 64
- dividend yield, 281
- drift parameter, 386
- drift process, 302
  
- economic cycle, 10
- economic indicator, 10
- efficient frontier, *see* Markowitz portfolio theory
- equity, 330
- equity in a house, 61
- equivalent martingale measure, 314
- equivalent measures, 313
- European call price, 394
  - behavior relative to security price, 397
  - behavior relative to strike price, 397
- European put price, 395
  - behavior relative to security price, 397
  - behavior relative to strike price, 397
- exact time, 16
- exchanges, 7
- expected short fall, 183
- expected tail loss, 183
  
- Fama-French three-factor model, 196–199
- feasible portfolios, 125
- federal discount rate, 2
- federal funds rate, 2
- Federal Reserve, 2
- filtered probability space, 268
- filtrations, 268–270
- final condition, 391
  - for European calls, 392
  - for European puts, 396
- financial markets, 1
- First Fundamental Theorem of Asset Pricing, 408
- forward commitment, 330
  - delivery date, 330
- forwards, 337–345
  - contract size, 337
  - delivery price, 338
  - delivery, expiry, expiration, exercise, or maturity date, 337
  - forward or exercise price, 337
  - forward price formula, 341
  - forward value formula, 344
  - long forward, 337
  - relation to put-call parity, 369
  - short forward, 337
  - spot-forward parity formula, 342
  - underlier, 337
  - writer, 337
- fundamental factor model, 191
- futures, 345–348
  - evolution from forwards to futures, 345
  - futures contract, 346
  - futures price, 346, 347
  - futures value, 347
  - impact of daily settlement, 347
  - maintenance margin, 347
  - margin account, 346
  - margin requirement, 347
  - mark-to-market, 346
  
- geometric Brownian motion, 314–319
- Girsanov theorem, 235, 311, 405, 459
- global minimum-variance portfolio, *see* Markowitz portfolio theory
- gradient, 120
- Greeks
  - delta, 390, 396–398, 434
  - for a portfolio, 434
  - gamma, 434
  - theta, 434
  - vega, 446
- gross return, 19
  
- heat equation, 390, 393
- hedgers, 331
- Hessian, 120
- hurdle rate, 161
  
- i.i.d., 211
- idiosyncratic risk, 151
- in-the-money, 364
- incomplete market, 458
- index rates, 4
- innovation process, 279
- intensity, 449
- interest, 15
  - exact, 16
  - interest rate per period, 15
  - negative interest rate, 15
  - ordinary, 16
  - quoted rate, 15
  - total interest on a loan, 56
- intrinsic value, 370
- investment banking, 3

- IRR, 41, 42
  - multiple, 45
  - relation to NPV, 44
- Itô diffusion, 302
- Itô integral, 299–302, 314
- Itô process, 302, 387
  - unique representation, 389
- Itô product rule, 292
- Itô's formula, 302–387
- Itô's lemma, 304
- jumps in security prices, 440
- kurtosis, 267, 442
  - excess, 443
  - in S&P 500 log returns, 446
- Lévy process, 456
  - exponential, 456
- lagging indicators, 10
- Lagrange Multiplier Theorem, 122
- latent factor, 191
- Law of One Price, 334, 389, 399, 413
  - consequence of no-arbitrage condition, 335
- leading indicators, 10
- leptokurtic, 443
- leverage ratio, 3
- LIBOR, 5, 350
- limit buy/sell order, 8
- Lindeberg Central Limit Theorem, 237
- Lindeberg condition, 239
- linear factor models, 185
- long-term rates, 5
- macroeconomic factor model, 191
- maintenance margin, *see* futures
- margin requirement, *see* futures
- mark-to-market accounting rule, 346
- market capitalization, 196
- market liquidity, 9
- market portfolio, 156, 157
- market risk, 151
- market sentiment, 363
- Markov process, 288
- Markov property, 244, 288
- Markowitz bullet, 126
- Markowitz portfolio theory
  - diversification, 138–143
  - diversified portfolio, 130
  - efficient frontier for  $N$  securities, 117–128
  - efficient frontier for two securities, 107–117
  - expected portfolio return rate, 94
  - global minimum-variance portfolio, 124, 128
  - model, 83
  - multivariate normality, 87
  - Mutual Fund Theorem, 130
  - one-period assumption, 88
  - optimal portfolios, 132
  - portfolio log return, 100–103
  - portfolio log return versus portfolio return rate, 103
  - portfolio risk, 96, 119
  - return rates, 85
  - securities' variances and covariances, 96–100
  - two-security portfolio analysis, 104–117
  - utility function, 131
  - weight vector for minimum-variance portfolio, 123
  - weights, 89
- martingale representation theorem, 407
- martingales, 275–278, 406
  - necessary condition of efficient market, 277
- maximum drawdown, 172
- Merton jump-diffusion model, *see* MJD model
- mesokurtic, 443
- method of least squares, 192
- MJD model, 448–465
  - assumptions, 456
  - European call pricing, 461–464
  - recovering BSM price, 464
  - solving the MJD s.d.e., 453–455
  - volatility smile, 465
- money market account, 386
- moneyiness, 364
- multivariate normality, 87
- mutual fund theorem, *see* Markowitz portfolio theory
- naked call, *see* options
- NPV, 38, 42
  - relation to IRR, 44
- numéraire, 386
- observable factor, 191
- opportunity cost, 16
- optimization problem, 120
- options, 353–376, 383–466
  - American, 354, 372–376
  - buyer, holder, or owner, 353
  - call option, 354
  - contract size, 353, 356
  - covered call, 357
  - European, 354, 359–372, 383–466

- exercise an option, 353
- expiration, exercise, or maturity date, 353, 356
- final or terminal payoff, 355
- how options work, 357–359
- moneyness, 364
- naked or uncovered call, 358
- premium, 353
- put option, 354
- seller or writer, 353
- strike or exercise price, 353
- styles, 354
- trading strategies, 365
- types, 354
- underlier, 353
- vanilla, 355
- order statistic, 176
- out-the-money, 364
- over-the-counter market (OTC), 7
  
- p-quantile, 174
- p.d.e., *see* partial differential equations
- partial differential equations, 390–395
  - connection with probability, 408
  - parabolic p.d.e., 391
- payoff diagram
  - forward, 339
  - terminal, 359–363
- perpetuity, 51
- physical market, 337
- platykurtic, 443
- Poisson process, 449
  - compound, 456
- portfolio
  - alpha, 192
  - beta, 162, 192
  - delta-gamma-neutral, 438–440
  - delta-neutral, 435–438
  - log return, 101
  - replicating, 388
  - replicating condition, 389
  - risk, 96, 119
  - risk measures, 151, 165–180
  - self-financing, 388, 412
  - self-financing condition, 389
  - trading strategy, 89, 388
  - weights, 89
- positive definite matrix, 86, 105, 119
- power set, 254
- price discovery, 331
- price-to-book ratio, 197
- primary market, 6, 67
- prime rate, 5
- principal, 15
- probability measure, 253, 255
- probability space, 253
- profit diagram
  - terminal, 359–363
- put-call parity
  - American options, 373
  - European options, 362, 368, 395
  - relation to forward, 369
  
- QQ-plot, 445
- quadratic covariation, 291
- quadratic variation, 289, 290
- quantile function, 175
- quantiles, 445
- quoted interest rate, *see* interest
  
- Radon-Nikodym derivative, 313
- random variables, 257
  - convergence of, 265
  - independent, 259
  - independent of  $\sigma$ -algebra, 260
- random walk, 276, 280, 320
  - simple, 320
  - symmetric, 320
- reserve ratio, 2
- return
  - arithmetic mean return, 36
  - capital-gain return, 212
  - geometric mean return, 36
  - gross return, 211
  - log return, 212
  - required return rate, 16
- risk-averse investor, 87, 404
- risk-free rate, 16
  - proxy, 17
  - real, 16
- risk-neutral investor, 137, 404
- risk-neutral pricing
  - of European calls and puts, 410
  - of European-style derivatives, 404, 408
  - with binomial trees, 415–422
- risk-neutral probability measure, 309, 406
  - and no-arbitrage, 409
  - for Merton jump-diffusion model, 458
  - uniqueness for BSM model, 461
- risk-seeking investor, 136, 404
- rounding errors, 429

- s.d.e., *see* stochastic differential equation
- sample-continuous stochastic process, 264
- scale parameter, 315
- Second Fundamental Theorem of Asset Pricing, 409
- secondary market, 6, 67
- securities
  - basic behavior, 278–282
  - cum-dividend price, 386
  - debt securities, 330
  - definition, 329
  - derivative securities, 330
  - equity securities, 330
  - ex-dividend price, 386
- securities markets, 6
  - professional participants, 8
- Security Market Line (SML), 163
- semivariance, 171
- Sharpe ratio, 166–170, 244
  - as slope of CML, 167
  - in BSM model, 460
- short selling, 90
- short-term rates, 4
- simple interest, 20
  - formula, 21
  - future value, 21
  - present value, 21
  - return rate, 21
  - versus fractional compounding, 30
- sinking funds, 62
- size premium, 197
- skewness, 266, 442
  - in S&P 500 log returns, 445
- Sortino ratio, 170, 174
- speculators, 331
- spot market, 337
- spot price, 338
- spread, 366
  - bear, 366
  - bull, 366
  - butterfly, 367
  - calendar, 367
  - horizontal, 367
  - price, 366
  - time, 367
  - vertical, 366
- statistical factor model, 191
- stochastic differential equation
  - for cum-dividend security price, 387
  - for geometric Brownian motion, 386, 387, 451
  - for Merton jump diffusion, 453
- stochastic processes
  - basics, 260–265
  - Merton jump diffusion, 450–458
- stock valuation, 63
- straddle, 365
- strangle, 366
- sub-sigma algebra, 255
- swap contract, 349
- swaps, 348–353
  - commodity swaps, 350
  - credit default swap, 350
  - currency swap, 349
  - fixed leg, 349
  - floating leg, 349
  - interest rate swap buyer, 349
  - interest rate swap seller, 349
  - interest rate swaps, 349
  - mechanics of interest rate swaps, 351
  - notional principal, 349
  - plain vanilla swap, 349, 350
  - swap bank, 351
  - variance swap, 352
- systematic risk, 143, 151, 165
- tail VaR, 183
- time value, 370
- total variation, 289
- tower property, 403, 420
- trading costs, 9
- trading strategies with options, *see* options
- uncovered call, *see* options
- unobservable factor, 191
- unsystematic risk, 143, 165
- utility function, 131–137
  - concave, 134
  - convex, 136
  - marginal utility, 132
- value premium, 198
- value-at-risk, 178
- VaR, 177–180
- variance swap, *see* swaps
- volatility
  - implied, 446
  - MJD volatility smile, 465
  - parameter, 386
  - skews, 445
  - smiles, 447
  - surface, 447

- volatility parameter, 315
- volatility process, 302
- warrants, 398–400
- Weak Efficient Market Hypothesis, 244
- weights, 89, 118
- white noise, 280
  - Gaussian, 280
  - independent, 280
  - strict, 280
- yield curve, 6