

## PROBLEM SESSION

**J. Berman :** Let  $F_S(n)$  denote the free join semilattice on  $n$  free generators. It is known that  $F_S(n)$  is isomorphic to the set of all nonvoid subsets of  $\{1, 2, \dots, n\}$  with the operation of union. By the result of Freese & Nation it is known that  $\text{Con}(F_S(n))$  satisfies no lattice identities.

**Problem:** Determine or estimate the size of  $\text{Con}(F_S(n))$ . The motivation for this question comes from the theory of relational databases, since there is a one-one-correspondence between congruences  $\theta$  on  $F_S(n)$  and closed sets of functional dependencies in a relational database with  $n$  attributes.

**R. Wille :** A lattice  $L$  is order-polynomially-complete iff every order preserving  $n$ -ary operation on  $L$  is a polynomial-function on  $L$ .

**Problem:** Is every order-polynomially-complete lattice finite?

**R. Freese :** It can be effectively decided whether a word  $\omega$  in a finitely presented lattice has a lower cover.

**Problem:** Is there an algorithm doing this in polynomial time?

**R. Wille :** **Problem:** For which partial lattices  $P$  is the free lattice  $\text{FL}(P)$  generated by  $P$  finite?

**Solutions:** Jezek & Slavik for semilattices.

Wille for partially ordered sets.

**G. Lallement:** **Theorem:** For any word  $w \in \{a, b\}^*$  the word problem for the semigroup  $\langle a, b; a = bwa \rangle$  is decidable. (Oganesyan 1987)

**Problem 1:** Is there a reasonable algorithm for this?

**Problem 2:** Is the word problem for  $\langle A; u = v \rangle$  decidable, where  $u, v \in A^*$ ?

(for further references, see Semigroups, Theory and Applications, Lecture Notes in Mathematics 1320, p.176-182, Springer-Verlag).

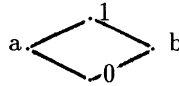
P. Higgins: Let  $A=(a_{ij})$  be a real matrix such that  $\forall i,j a_{i,j} > 0$  and  $\forall x \neq 0 xAx^T > 0$ .

**Problem:** Is  $A^{(k)} := (a_{ij}^k)$  positive definite as well?

True if  $k$  is a positive integer or if  $k \geq B$ , for some number  $B$  which depends on  $A$ . It is also true if the order of  $A$  is less than four.

This problem came from Dr. Alan Russel of the University of Melbourne.

I. Loureiro: De Morgan algebras are generated by the four-element lattice



with the negation  $\sim$ :

$x$	$\sim x$
0	1
a	a
b	b
1	0

Considering the unary operation  $\nabla$  on De Morgan algebras, that satisfies:

$$A_5) \sim x \vee \nabla x = 1$$

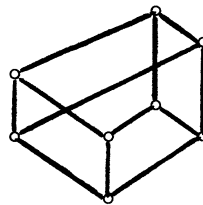
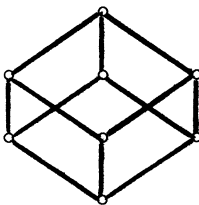
$$A_6) \sim x \wedge x = \sim x \wedge \nabla x,$$

we obtain a variety that coincides with congruence permutable De Morgan algebras (see papers from I. Loureiro and from M. Adams).

**Problem:** Is it possible to define a unary operation  $\nabla$  on MS-algebras (see papers from J. Varlet and from T. Blyth) such that we have a generalization of the last result?

R. Wille: **Problem:** How many “essentially different” diagrams of  $B_n$  exist with  $n$  slopes ?

**Example:** For  $B_3$  there exist exactly two:



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