APPENDIX

The spectral theory of random Schrödinger operators still gives rise to a lot of papers and some questions left without solution in the preceding chapters have received partial answers during the redaction of this book.

1 The one dimensional case

F. Ledrappier has given a simpler proof of proposition II.3.3 avoiding utilization of the Thouless formula. This result now appears as a direct application of a general fact in ergodic theory (F. Ledrappier "Positivity of the exponent for stationnary sequences of matrices" preprint).

S. Kotani has pointed out that in corollary III.1.2, it is possible to handle the Helmotz case assuming now that there exists two consecutive indexes $n = n_o, n_o+1$ such that :

(i) b_n is bounded

(ii) for each $\lambda \in A$, the law of $a_n-\lambda b_n$ has a bounded density $\phi_{n,\lambda}$ such that $\sup_{\lambda\in A} \|\phi_{n,\lambda}\| < +\infty$.

The proof is the same as in III.1.2 and Corollary III.1.3 is obtained under the same hypothesis. (S. Kotani, private communication).

In [61], B. Simon and M. Taylor prove that for the classical independant Schrödinger operator, for any $\alpha > 0$ there exists a Bernouilli distribution of the potential such that the distribution function $K(\lambda)$ of the distribution of states is not Hölder of order α. This shows that the result (ii) of I.6.9 cannot be improved in general.

2. The strip

In their paper "Anderson localization for one and quasi one dimensional systems" (preprint) F. Delyon, Y. Levy, B. Souillard precise the theorem IV.3.9, proving that the eigenspaces of $H(\omega)$ are $\mathbb{P}$ almost surely one-dimensional. (See also the discussion following proposition IV.2.5).

3. The multidimensional case

At the beginning of 1985 the first proofs of the Anderson's conjecture (see the introduction) appear almost simultaneously in three preprints :

- J. Frölich, F. Martinelli, E. Scopolla and T. Spencer
  "Anderson localization for large disorder or low energy"
- F. Delyon, Y. Levy, B. Souillard
  "Anderson localization for multidimensional systems at large disorder or low energy"
- B. Simon, M. Taylor, T. Wolff
  "Some rigorous results for the Anderson model"

In these three papers, the localization property is proved in any dimension for the classical independent Schrödinger operator, assuming the disorder "large enough" and with a bounded density. The Kotani's criterion and Frölich and Spencer estimates (replacing the positivity of Lyapunov exponents in this context) are the essential tools.

REFERENCES

[1] ANDERSON P.W. (1958). Absence of diffusion in certain random lattices. Physical Review 109, 1492-1505.

[2] ATKINSON F.V. (1964). Discrete and continuous boundary problems. Mathematics in science and engineering Vol 8, Academic Press, New-York.

[3] AVRON J., SIMON B. (1983). Almost periodic Schrödinger operators II. The integrated density of states. Duke. Math. J. 50, 369-391.

[4] BEREZANSKII Ju. M. (1968). Expansions in eigenfunctions of self adjoint operators. Transl. of Math. Mono. 17, A.M.S. Providence.

[5] BORLAND R.E. (1963). Proc. R. Soc. London A 274, 529.

[6] BRUNEL A., REVUZ D. (1974). Quelques applications probabilistes de la quasi-compacité. Ann. Inst. Henri Poincaré, Vol X, 3, 301-337.

[7] CARMONA R. (1982). Exponential localization in one dimensional disordered systems. Duke Math. J. 49, 191-213.

[8] CARMONA R. (1983). One dimensional Schrödinger operators with random or deterministic potentials : new spectral types. J. Functionnal Anal 51, 229-258.

[9] CARMONA R. (1985). Ecole d'été de probabilités de Saint-Flour. To appear in Springer Lecture Notes Series.

[10] CASHER A., LEBOWITZ J.L. (1971). J. Math. Phys. N.Y. 12 1701.

[11] CRAIG W., SIMON B. (1983). Subharmonicity of the Lyapunov index. Duke Math. J. 50, 551-559.

[12] CRAIG W., SIMON B. (1983). Log Hölder continuity of the integrated density of states for stochastic Jacobi matrices. Comm. Math. Phys. 90, 207-218.

[13] DELYON F., KUNZ H., SOUILLARD B. (1983). One dimensional wave equations in random media. J. Phys. A 16, 25-42.

[14] DELYON F., LEVY Y., SOUILLARD B. (1985). Anderson localization for multi-dimensional systems at large disorder or large energy. To appear in Comm. Math. Phys.

[15] DELYON F., SOUILLARD B. (1983). The rotation number for finite difference operators and its properties. Comm. Math. Phys. 89, 415-427.

[16] DERRIDA B., HILHORST H.J., (1983). Singular behaviour of certain infinite products of random 2×2 matrices. J. Phys. A. Math. Gen. 16, 2641-2654.

[17] DERRIENNIC Y., GUIVARCH Y. (1974). Théorème de renouvellement pour les groupes non moyennables. Com. Rend. Acad. Sc. PARIS 277, 613-615.

[18] FIGOTIN A.L., PASTUR L.A. (1984). An exactly solvable model of a multidimensional incommensurate structure. Comm. Math. Phys. 95, 401-425.

[19] FURSTENBERG H. (1963). Non commuting random products. Trans. Amer. Soc. 108, 377-428.

[20] FURSTENBERG H., KESTEN H. (1960). Products of random matrices. Ann. Math. Stat. 31, 457-469.

[21] FURSTENBERG H., KIFER Y. (1983). Random matrix products and measures on projective spaces. Israël J. of Math. 46, 12-32.

[22] FURSTENBERG H., TZKONI I. (1971). Spherical functions and integral geometry. Israël J. of Math 10, 327-338.

[23] GOLSHEID I. Ja. (1980). The structure of the spectrum of the Schrödinger random difference operator. Soviet Math Dokl 22, 670-675.

[24] GOLSHEID I. Ja. MOLCANOV S.A., PASTUR L.A. (1977). A pure point spectrum of the one dimensional Schrödinger operator. Funct. Anal. Appl. 11, 1-10.

[25] HENNION H. (1984). Loi des grands nombres et perturbations pour des produits réductibles de matrices aléatoires indépendantes. Wahrs. Verw. Gebiete 67, 265-278.

[26] HERMAN M. (1981). Ecole Polytechnique, preprint.

[27] HERMAN M. (1983). Une méthode pour minorer les exposants de Lyapounov et quelques exemples montrant le caractère local d'un théorème d'Arnold et de Moser sur le tore de dimension 2. Comment. Math. Helvetici 58 , 453-502.

[28] HUA L.K. (1963). Harmonic analysis of several complex variables in the classical domains. A.M.S. Providence.

[29] ISHII K. (1973). Localization of eigenstates and transport phenomena in the one dimensional disordered system. Suppl. Prog. Theor. Phys. 53, 77-138.

[30] JHONSTON R., KUNZ H. (1983). The conductance of a disordered wire. J.Phys. C : Solid State Phys. 16, 3895-3912.

[31] KATO T. (1966). Perturbation theory for linear operators. Springer Verlag, New-York.

[32] KATZNELSON Y. (1976). An introduction to harmonic analysis. New-York : Dover.

[33] KIRSH W., KOTANI S., SIMON B. (1985). Absence of absolutely continuous spectrum for some one dimensional random but deterministic Schrödinger operator. To appear in Ann. Inst. Henri Poincaré A.

[34] KIRSH W., MARTINELLI F. (1982). On the density of states of Schrödinger operators with a random potential. J. Phys. A 15, 2139-2156.

[35] KOTANI S. (1983). Lyapunov indices determine absolute continuous spectra of stationary one dimensional Schrödinger operators. Proc. Kyoto Stoch. Conf.

[36] KOTANI S. (1984). Lyapunov exponents and spectra for one dimensional random Schrödinger operators. Proceedings of the A.M.S. meeting on "Random matrices". Brunswick.

[37] KUNZ H., SOUILLARD B. (1980). Sur le spectre des opérateurs aux différences finies aléatoires. Comm. Math. Phys. 78, 201-246.

[38] LACROIX J. (1982). Problèmes probabilistes liés à l'étude des opérateurs aux différences aléatoires. Ann. Inst. Elie Cartan 7, Marches aléatoires et processus stochastiques sur les groupes de Lie, 80-95.

[39] LACROIX J. (1983). Singularité du spectre de l'opérateur de Schrödinger aléatoire dans un ruban ou un demi-ruban. Ann. Inst. H. Poincaré A 38, 385-399.

[40] LACROIX J. (1984). Localisation pour l'opérateur de Schrödinger aléatoire dans un ruban. Ann. Inst. Henri Poincaré A 40, 97-116.

[41] LACROIX J. (1984). The Schrödinger operator in a strip. Probability measures on groups VII, Proceedings Oberwolfach, Springer lecture notes series 1064, 280-297.

[42] LEDRAPPIER F. (1984). Ecole d'été de probabilités de Saint-Flour XII. Springer lecture notes séries 1097.

[43] LE PAGE E. (1984). Répartition d'état d'un opérateur de Schrödinger aléatoire. Probability measures on groups VII, Proceedings Oberwolfach, Springer lectures notes series 1064, 309-367.

[44] MATSUDA, ISHII K. (1970). Localization of normal modes and energy transport in the disordered harmonic chain. Supp. Prog. Théor. Phys. 45, 56-89.

[45] MOLCANOV S.A. (1978). The structure of eigenfunctions of one dimensional unordered structures. Math. U.S.S.R Izvestija 12, 69-101.

[46] MOTT N. TWOSE W.D. (1961). The theory of impurity conduction. Adv. in Physics 10, 107-155.

[47] NERI U. (1971). Singular Integrals VII. Springer Lectures Notes in mathematics series 200.

[48] O'CONNOR (1975). Disordered harmonic chain. Comm. Math. Phys. 45, 63-77.

[49] OSSELEDEC V. (1968). A multiplicative ergodic theorem. Lyapunov characteristic numbers for dynamical systems". Trans. Moscow Math. Soc. 19, 197-231.

[50] PASTUR L.A. (1980). Spectral properties of disordered systems in the one body approximation. Comm. Math. Phys. 75, 167-196.

[51] PICHARD J.L. (1984). Contribution à une théorie quantique des phénomènes de transport ... . Thesis, Université de Paris-Sud Orsay.

[52] ROYER G. (1982). Etude des opérateurs de Schrödinger à potentiel aléatoire en dimension un. Bull. Soc. Math. France 110, 27-48.

[53] RUELLE D. (1969). A remark on bound states in potential scattering theory. Nuovo Cimento A 61, 655.

[54] RUELLE D. (1978). Analyticity properties of the characteristic exponents of random matrix products. Adv. Math. 32, 68.

[55] SARNAK P. (1982). Spectral behavior of quasi periodic potentials. Comm. Math. Phys. 84, 377-401.

[56] SCHNEIDER W. (1984). Rigorous scaling laws for Dyson measures. To appear in Bibos symposium on stochastic processes, Springer lecture notes series.

[57] SIMON B. (1982). Almost periodic operators : a review. Adv. Appl. Math 3, 463-490.

[58] SIMON B. (1983). Kotani theory for one dimensional stochastic Jacobi matrices. Comm. Math. Phys. 89 227.

[59] SIMON B. (1985). Almost periodic Schrödinger operators IV. The Maryland model. To appear in Ann. Phys.

[60] SIMON B. (1983). Equality of the density of states in a wide class of tight - binding Lorentzian random models. Phys. Rev. B 27, 3859-3860.

[61] SIMON B. TAYLOR M. (1985). Harmonic analysis on SL(2,$\mathbb{R}$) and smoothness of the density of states in the one dimensional Anderson model. To appear in Comm. Math. Phys.

[62] THOULESS D. (1972). A relation between the density of states and range of localization for one dimensional random systems. J. Phys. C 5, 77-81.

[63] VERHEGGEN T. (1979). Transmission coefficient and heat conduction

of a harmonic chain with random masses ... Comm. Math. Phys. 68, 69-82.

[64] YOSHIOKA Y. (1973). On the singularity of the spectral measures of a semi-infinite random system. Proc. Japan Acad. 49, 665-668.

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