SUGGESTIONS FOR FURTHER READINGS

We provide some recent references on limit theorems for random matrices and related topics. We make no claim for completeness but the quoted papers often contain a large bibliography.

(1) Lyapunov exponent for stationary sequences

The main properties of Lyapunov exponents in the stationary case can be found in Ledrappier [46] . Guivarc'h [32] (see also Royer [62] , Virtser [69],[70]) gives a criterion ensuring that two given exponents are distinct for markovian products. See Ledrappier [48] for an assumption implying that the exponents are not all equal, in the general stationary setting.

(2) Boundary theory

After the fundamental work of Furstenberg (see [20], [22], [23]) the set of bounded harmonic functions was determined

- for absolutely continuous distributions on connected groups by Raugi [59] (see also Guivarc'h [31]),

- for distributions on discrete groups of matrices by Ledrappier [47] .

(3) Limit theorems

(3.1) One can find a proof, under our usual irreducibility assumptions, of

- the functional central limit theorem,
- the law of iterated logarithm,
- the renewal theorem,

- the local limit theorem,

for the sequence $\text{Log} \|S_n x\|$ in Le Page [49], [50] . Recurrence properties are studied in Guivarc'h [33] (see also Bougerol [9]) .

The central limit theorem for S_n written in the polar and the Iwasawa decomposition is proved in Raugi [59] . References to earlier proofs and applications can be found in Tutubalin [68].

(3.2) Properties of the solutions of the difference equation on $\mathbb{R}^d$

$$X_{n+1} = Y_n X_n + B_n$$

(where B_n is in $\mathbb{R}^d$ and Y_n in $Gl(d,\mathbb{R})$) are studied in Kesten [41] and in Le Page [50] . Stationary solutions are given in [8] . For $d = 1$, Grincevicius [34] proves a central limit theorem.

(3.3) Without irreducibility assumptions, the central limit theorem is not yet fully understood. The latest reference is Raugi [60].

(4) Positive matrices

The reader will find in Cohen [14] a nice account of the applications of products of positive random matrices to demography and an extensive bibliography. Kesten and Spitzer [42] study the convergence in distribution of such products.

(5) Linear stochastic differential equation

A goog introduction to this subject is the survey of Arnold and Kliemann [1] . A nice application is given in Pardoux and Pignol [58].

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