

Index

Numbers in italics refer to entire sections.

- Abelian theorems, 171
- Absorbing barriers, 93
- Absorbing state, 105
- Accident proneness, 155
- Accidents, 15
- Age, 204
- Arrival stream, 225
- Asynchronous counting, 209, 213

- Backward differential equation, 192
- Backward recurrence time, 204
- Balking, *6.16*
 - geometric, 226
- Bayes' theorem, 2.2
- Before-and-after studies, 80
- Bernoulli distribution, 42
- Bernoulli trials, *2.4*
 - first passage times, 129
- Beta-binomial relation, 154
- Beta distribution, *4.6*
 - mixing, 156
- Beta function, *1.9*
- Binomial distribution, 9, 16
 - in Bernoulli trials, 65
 - distribution function, 153
 - mixed, 156
 - probability generating function, 34
- Binomial–Poisson limit, 67, 154
- Birth distribution, 120
- Birth process, divergent, *5.14*
- Birth-and-death process, 194
 - linear, 196, 224
 - nonequilibrium, 224

- Boltzmann's constant, 181
- Borel's distribution, 253, 257
- Borel's method, *6.11*
- Borel–Tanner distribution, 263
- Branching chain, *3.12*
- Branching process, as busy period, *6.12*
- Busy period, *6.11*
 - as a branching process, *6.12*
 - continuous, *6.13*
 - discrete, 261
 - generalized, *6.14*

- Car length, 213
- Catalan distribution, *1.12*, 255, 257
- Cauchy distribution, 144
- Cauchy–Schwartz inequality, 74
- Causal distribution, 19, 158; *see also*
 - Deterministic distribution
- Change of variable, *4.3*
 - probability generating function, 37
- Chapman–Kolmogorov equation, 102, 185
- Chi-square distribution, 181
- Closed set of states, 105
- Coin tossing, 1, 13
- Column vector, 96
- Compound event, 14
- Conditional sample space, 77
- Conditioning, 2.3
- Continuous busy period, *6.13*, 264
- Convolution, *2.6*
 - of density functions, 147
 - geometric distribution, 74
 - n*-fold, 75
 - Poisson distribution, 74

- Correlation coefficient, 69
- Counting distribution, 152, 5.11
- Counting process, 183
- Covariance, 69
- Cumulative probabilities, 25
 - generating function, 37
- Dead time, 139, 213
- Deficient distribution, 130
- Deficit, 204
- Delta function, 158
 - method, 168
- Density function, convolution, 147
- Dependence, 2.4
- Deterministic distribution, 19; *see also*
 - Causal distribution
 - balking, 272
- Deterministic process, 186
- Diagonal distributions, 2.8
 - binomial, 79
 - joint probability generating function, 78
- Discrete busy period, 261
- Displaced gaps, 5.13
- Displaced negative exponential distribution, 215
- Distribution: *see* Probability distribution
- Distribution function, 1.8, 144
 - binomial distribution, 153
 - of a bivariate distribution, 2.5
 - gamma distribution, 150
 - geometric distribution, 28
 - mixed distribution, 157
 - Poisson distribution, 150
 - rectangular distribution, 28
- Divergent birth process, 5.14
- Doubly stochastic matrix, 137
- Ehrenfest chain, 136, 3.11
- Elementary renewal theorem, 203
- Equilibrium, 94, 5.6
- Equilibrium renewal process, 207, 211, 213
- Ergodic chain, 3.8
- Erlang loss formula, 6.5
- Erlang process, 5.12, 186, 222
- Event, 1.5
- Excess life, 204
- Expected value, 19ff
- Exponential distribution, 143, 147
 - displaced, 215
 - memoryless property, 152
- Exponential probability generating function, 47
- Extinction probability, 121, 3.13, 3.15
- Extinction of surnames, 119
- Failure rate, 187
- False theta function, 272
- Family problem, 60
- Feedback, 285
- FIFO, 277
- Finite storage, 235
- First passage time, 3.16
- Fisher distribution, 9
 - probability generating function, 35
- Forward differential equation, 192
- Forward recurrence time, 204
- Functional equation, 79
- Gambler's ruin, 94, 113, 3.14, 3.15
- Gamma distribution, 143
 - distribution function, 150
- Gamma function, 1.9
- Gap, 152, 184, 5.10
 - density, 186
 - distribution, 204
- Generalized busy period, 6.14
- Generalized renewal process, 207
- Generating function, 126
 - of a bivariate distribution, 2.5
 - cumulative probabilities, 37
- Geometric distribution, 9, 125
 - balking, 271
 - convolution, 74
 - distribution function, 28
 - mean, 23
 - memoryless property, 86
 - mixed, 156
 - of Poisson variables, 123
 - probability generating function, 35
 - variance, 24
- Harmonic mean, 222
- Head-of-line priority, 273
- Imbedded Markov chain, 6.6
- Incomplete beta function, 4.6
- Incomplete gamma function, 143, 4.5
- Independence, 2.4
- Indicator, 1.5
- Infinite mean, 20

- Infinitesimal probabilities, 192
- Initial distribution, 92
- Initial vector, 3.4
- Integral equation of renewal theory, 202
- Inverse Laplace transform, 4.12, 212
- Irreducible chain, 105

- Jacobian, 147
- Joint density function, 144

- Kendall symbol, 225

- Lagrange series, 259
- Laplace inversion, 4.12
- Laplace transform, 144, 4.10, 4.11
 - of Erlang distribution, 211
 - notation, 209
 - of renewal density, 201, 211
- Laplace–Stieltjes transform, 165ff
- Lebesgue integration, 158
- Left continuity, 27, 164
- Length bias, 200
- Lifetime, 186
- LIFO, 6.18
- Linear birth process: *see* Yule process
- Linear birth-and-death process, 196, 224
- Loss formula, 6.5
- Lower incomplete gamma function, 150

- Markov infinitesimal matrix, 5.8
- Markov process, 184, 5.5
- Markov queue, 195
- Matrix, 3.3
- Mean, 144, 1.6, 1.7
 - recurrence time, 106
- Memoryless property
 - of exponential distribution, 152
 - of geometric distribution, 86
- Method of marks, 66, 5.7
 - in renewal theory, 202
- Mixed discrete and continuous, 147
- Mixed distributions, 155
 - distribution function, 157
- Mixed random variables, 140
- Mixing, parameter, 4.7
- Moment generating function, 46
- Moments
 - of a bivariate distribution, 2.5
 - of sums of random variables, 73

- Negative aging, 187
- Negative binomial distribution, 1.10, 155, 193
 - in Bernoulli trials, 66
 - mean, 33
 - probability generating function, 34
- Negative exponential distribution: *see* Exponential distribution
- Negative hypergeometric distribution, 156
- Nesting, 122
- Normalization to unity, 7
- Null chain, 107

- Ordered sample, 154
- Ordinary renewal process, 207, 212

- Palm input, 267
- Parameter mixing, 4.7
- Periodic chain, 3.7
- Petersburg paradox, 21, 87
- Pochhammer's symbol, 30
- Point process, 183
- Poisson distribution, 8, 12, 16
 - convolution, 74
 - diagonal distribution, 71
 - distribution function, 150
 - as Erlang loss formula, 236
 - geometric number, 123
 - mean, 22
 - mixing, 156
 - $s = \phi(s)$, 42
 - truncated, 236
 - variance, 23
- Poisson–gamma relation, 152, 188, 189, 209
- Poisson mixing, 267
- Poisson process, 186, 5.3, 192
 - method of marks, 196
- Pollaczek–Khintchine formula 6.7, 244
- Polya's urn model, 63
- Positive aging, 187
- Preemptive priority, 273
- Priority service, 6.17
- Probability density function, 4.2
- Probability distribution, 1.3
 - deficient, or improper, 130
 - see also names of particular distributions:*
 - Bernoulli, Beta, Binomial, Borel, Borel–Tanner, Catalan, Cauchy, Causal, Chi-square, Displaced negative exponential, Erlang, Exponential,

- Probability distribution (*Continued*)
 - Fisher, Gamma, Geometric, Negative binomial, Negative hypergeometric, Poisson, Rectangular, Truncated Poisson, Yule
- Probability of extinction, 121, 3.13
- Probability generating function, 1.11, 1.13
 - binomial distribution, 34
 - change of variable, 37
 - of an event, 47
 - geometric distribution, 35
 - Fisher distribution, 35
 - negative binomial distribution, 34
 - rectangular distribution, 34
- Probability vector, 96
- Pure birth process, 193

- Quality control, 17
- Queueing, 3.10
 - departure process, 233
 - equilibrium, 6.4
 - with feedback, 285
 - finite storage, 235
 - saturation, 6.3
 - table of notation, 228

- Random sum, 122, 4.13
- Random variable, 1.4
 - sums, 2.6
- Random walk, 99, 3.1, 3.9
- Range, 139
- Rectangular distribution, 8, 28, 34
- Recurrent state, 106
- Reducible chain, 3.6
- Reflecting barrier, 93
- Regularity, 3.5
- Renewal density function, 202
- Renewal function, 5.9
- Renewal process, 5.2
 - equilibrium, 207, 211, 213
 - general, 207
 - ordinary, 207, 212
 - thinning, 197
- Renewal theory, integral equation, 202
- Repeat preemptive priority, 273
- Residual lifetime, 204
- Residual service time, 247
- Resume preemptive priority, 273
- Reverse-order service, 6.18

- Riemann integration, 159
- Row vector, 96

- Sample point, 13
- Sample space, 2, 1.2
- Scalar, 96
- Scheduled arrivals, 226
- Schwartz function, 168
- Service process, 225
- Simple event, 14
- Sojourn time, 184, 198
- Spread distribution, 204
- Stages, 211
- Stationary probabilities, 94
- Stick problem, 60, 200, 203
- Stieltjes integration, 419
- Stochastic matrix, 96
- Stopped distribution, 176
- Stopping distribution, 176
- Sums of random variables, 2.6, 4.4, 4.13
 - moments, 73
- Survivor function, 186
- Synchronous counting, 209, 212, 214

- Tail of a distribution, 25, 144
- Tanner's combinatorial method, 261, 265
- Tauberian theorems, 171
- Thinning a renewal stream, 197
- Transient states, 106
- Transition, 91
 - matrix, 3.4
 - probabilities, 92
- Truncated Poisson distribution, 236
- Two-state process, 5.4, 192, 200

- Unit vector, 96
- Upper incomplete gamma function, 150

- Variance, 1.6, 1.7, 144
- Vector, 3.3
- Virtual queueing time, 6.9

- Waiting time, 141, 6.8

- Yule process, 193, 221
 - distribution, 193

- Zero aging, 184