Appendix A
Description of Test Problems

Test Problems with Bound Constraints

- **Test Problem 1 [49]**
  - Number of variables: $n = 2$
  - Objective function:

$$f(\mathbf{x}) = -4x_1x_2 \sin(4\pi x_2)$$

  - Feasible region: $\mathbb{D} = [0, 1]^2$
  - Accuracy: $\varepsilon = 0.355$
  - Global minimum: $f^* = -2.51997258$
  - Global minimizer: $\mathbf{x}^* = (1.00000000, 0.63492204)$
  - Derivatives:

$$f_1'(\mathbf{x}) = -4x_2 \sin 4\pi x_2$$
$$f_2'(\mathbf{x}) = -4x_1 \sin 4\pi x_2 - 16\pi x_1 x_2 \cos 4\pi x_2$$

- **Test Problem 2 [49]**
  - Number of variables: $n = 2$
  - Objective function:

$$f(\mathbf{x}) = -\sin(2x_1 + 1) - 2\sin(3x_2 + 2)$$

  - Feasible region: $\mathbb{D} = [0, 1]^2$
  - Accuracy: $\varepsilon = 0.0446$
  - Global minimum: $f^* = -2.81859485$
  - Global minimizer: $\mathbf{x}^* = (0.28539815, 0.00000000)$
  - Derivatives:

R. Paulavičius and J. Žilinskas, *Simplicial Global Optimization*,
SpringerBriefs in Optimization, DOI 10.1007/978-1-4614-9093-7,

$$f_1'(\mathbf{x}) = -2\cos(2x_1 + 1)$$
$$f_2'(\mathbf{x}) = -6\cos(3x_2 + 2)$$

- **Test Problem 3 (Branin) [23, 49]**
  - Number of variables: $n = 2$
  - Objective function:

$$f(\mathbf{x}) = \left(x_2 - \frac{5.1}{4\pi^2}x_1^2 + \frac{5}{\pi}x_1 - 6\right)^2 + 10\left(1 - \frac{1}{8\pi}\right)\cos(x_1) + 10$$

  - Feasible region: $\mathbb{D} = [-5, 10] \times [0, 15]$
  - Accuracy: $\varepsilon = 11.9$
  - Global minimum: $f^* = 0.397887$
  - Global minimizers: $\mathbf{x}^* =$ (âĹŠπ, 12.275), $(\pi, 2.275)$, $(9.42478, 2.475)$
  - Derivatives:

$$f_1'(\mathbf{x}) = 2\left(\frac{5}{\pi} - \frac{2.55}{\pi^2}x_1\right)\left(-\frac{1.275}{\pi^2}x_1^2 + \frac{5}{\pi}x_1 + x_2 - 6\right) + 10(\sin x_1)\left(\frac{1}{8\pi} - 1\right)$$

$$f_2'(\mathbf{x}) = -\frac{2.55}{\pi^2}x_1^2 + \frac{10}{\pi}x_1 + 2x_2 - 12$$

- **Test Problem 4 [49]**
  - Number of variables: $n = 2$
  - Objective function:

$$f(\mathbf{x}) = \max\left(\sqrt{3}x_1 + x_2, -2x_2, x_2 - \sqrt{3}x_1\right)$$

  - Feasible region: $\mathbb{D} = [-1, 1]^2$
  - Accuracy: $\varepsilon = 0.0141$
  - Global minimum: $f^* = 0.00000000$
  - Global minimizer: $\mathbf{x}^* = (0.00000000, 0.00000000)$

- **Test Problem 5 [49]**
  - Number of variables: $n = 2$
  - Objective function:

$$f(\mathbf{x}) = -e^{-x_1^2}\sin x_1 + |x_2|$$

- Feasible region: $\mathbb{D} = [0, 10]^2$
- Accuracy: $\varepsilon = 0.1$
- Global minimum: $f^* = -0.39665295$
- Global minimizer: $\mathbf{x}^* = (0.65327118, 0.00000000)$
- Derivatives:

$$f_1'(\mathbf{x}) = -e^{-x_1^2} \cos x_1 + 2x_1 e^{-x_1^2} \sin x_1$$
$$f_2'(\mathbf{x}) = \text{signum}\,(x_2)$$

- **Test Problem 6 [49]**
  - Number of variables: $n = 2$
  - Objective function:

$$f(\mathbf{x}) = 2x_1^2 - 1.05x_1^4 + x_2^2 - x_1 x_2 + \frac{1}{6}x_2^2$$

  - Feasible region: $\mathbb{D} = [-2, 4]^2$
  - Accuracy: $\varepsilon = 44.9$
  - Global minimum: $f^* = -239.696629$
  - Global minimizer: $\mathbf{x}^* = (4.00000000, -1.11976261)$
  - Derivatives:

$$f_1'(\mathbf{x}) = -4.2x_1^3 + 4x_1 - x_2$$
$$f_2'(\mathbf{x}) = -x_1 + \frac{7}{3}x_2$$

- **Test Problem 7 [49]**
  - Number of variables: $n = 2$
  - Objective function:

$$f(\mathbf{x}) = \sum_{i=1}^{n-1} \left(100\left(x_{i+1} - x_i^2\right)^2 + (x_i - 1)^2\right)$$

  - Feasible region: $\mathbb{D} = [-3, 3] \times [-1.5, 4.5]$
  - Accuracy: $\varepsilon = 542.0$
  - Global minimum: $f^* = 0$
  - Global minimizer: $\mathbf{x}^* = (1, 1)$
  - Derivatives:

$$f_1'(\mathbf{x}) = -400x_1\left(x_2 - x_1^2\right) + 2x_1 - 2$$
$$f_2'(\mathbf{x}) = -200x_1^2 + 200x_2$$

- **Test Problem 8 [49]**
  - Number of variables: $n = 2$
  - Objective function:

$$f(\mathbf{x}) = (x_1 - 2x_2 - 7)^2 + (2x_1 + x_2 - 5)^2$$

  - Feasible region: $\mathbb{D} = [-2.5, 3.5] \times [-1.5, 4.5]$
  - Accuracy: $\varepsilon = 3.66$
  - Global minimum: $f^* = 0.450000$
  - Global minimizer: $\mathbf{x}^* = (3.40000000, -1.49999999)$
  - Derivatives:

$$f_1'(\mathbf{x}) = -34 + 10x_1$$
$$f_2'(\mathbf{x}) = 10x_2 + 18$$

- **Test Problem 9 (Goldstein-Price) [23, 49]**
  - Number of variables: $n = 2$
  - Objective function:

$$\begin{aligned} f(\mathbf{x}) = &\left(1 + (x_1 + x_2 + 1)^2 \left(19 - 14x_1 + 3x_1^2 - 14x_2 + 6x_1x_2 + 3x_2^2\right)\right) \\ &\times \left(30 + (2x_1 - 3x_2)^2 \left(18 - 32x_1 + 12x_1^2 + 48x_2 - 36x_1x_2 + 27x_2^2\right)\right) \end{aligned}$$

  - Feasible region: $\mathbb{D} = [-2, 2]^2$
  - Accuracy: $\varepsilon = 62900$
  - Global minimum: $f^* = 3.$
  - Global minimizer: $\mathbf{x}^* = (0, -1)$
  - Derivatives:

$$\begin{aligned} f_1'(\mathbf{x}) = \; & 1152x_1^7 - 2016x_1^6x_2 - 5376x_1^6 - 3888x_1^5x_2^2 + 8064x_1^5x_2 \\ & + 5712x_1^5 + 6120x_1^4x_2^3 + 12960x_1^4x_2^2 - 840x_1^4x_2 + 6720x_1^4 \\ & + 5220x_1^3x_2^4 - 16320x_1^3x_2^3 - 21480x_1^3x_2^2 - 30720x_1^3x_2 - 9816x_1^3 \\ & - 5508x_1^2x_2^5 - 10440x_1^2x_2^4 + 3720x_1^2x_2^3 + 29520x_1^2x_2^2 + 17352x_1^2x_2 \\ & - 3216x_1^2 - 2916x_1x_2^6 + 7344x_1x_2^5 + 17460x_1x_2^4 + 10080x_1x_2^3 \\ & + 15552x_1x_2^2 + 14688x_1x_2 + 2520x_1 + 972x_2^7 + 1944x_2^6 \\ & - 1188x_2^5 - 11880x_2^4 - 23616x_2^3 - 19296x_2^2 - 4680x_2 + 720 \\ f_2'(\mathbf{x}) = \; & -288x_1^7 - 1296x_1^6x_2 + 1344x_1^6 + 3672x_1^5x_2^2 + 5184x_1^5x_2 - 168x_1^5 \\ & + 5220x_1^4x_2^3 - 12240x_1^4x_2^2 - 10740x_1^4x_2 - 7680x_1^4 - 9180x_1^3x_2^4 \end{aligned}$$

$$
\begin{aligned}
&- 13920x_1^3x_2^3 + 3720x_1^3x_2^2 + 19680x_1^3x_2 + 5784x_1^3 - 8748x_1^2x_2^5 \\
&+ 18360x_1^2x_2^4 + 34920x_1^2x_2^3 + 15120x_1^2x_2^2 + 15552x_1^2x_2 \\
&+ 7344x_1^2 + 6804x_1x_2^6 + 11664x_1x_2^5 - 5940x_1x_2^4 - 47520x_1x_2^3 \\
&- 70848x_1x_2^2 - 38592x_1x_2 - 4680x_1 + 5832x_2^7 - 4536x_2^6 \\
&- 26568x_2^5 + 9720x_2^4 + 57384x_2^3 + 36864x_2^2 + 6120x_2 + 720
\end{aligned}
$$

- **Test Problem 10 [49]**
  - Number of variables: $n = 2$
  - Objective function:

$$f(\mathbf{x}) = \sin(x_1 + x_2) + (x_1 - x_2)^2 - 1.5x_1 + 2.5x_2 + 1$$

  - Feasible region: $\mathbb{D} = [-1.5, 4] \times [-3, 3]$
  - Accuracy: $\varepsilon = 0.691$
  - Global minimum: $f^* = -1.91322295$
  - Global minimizer: $\mathbf{x}^* = (-0.54719386, -1.54720091)$
  - Derivatives:

$$
\begin{aligned}
f_1'(\mathbf{x}) &= -2x_2 + 2x_1 + \cos(x_1 - x_2) - 1.5 \\
f_2'(\mathbf{x}) &= -2x_1 + 2x_2 + \cos(x_1 - x_2) + 2.5
\end{aligned}
$$

- **Test Problem 11 [49]**
  - Number of variables: $n = 2$
  - Objective function:

$$f(\mathbf{x}) = (x_1 - 2)^2 + (x_2 - 1)^2 + \frac{0.04}{-\frac{x_1^2}{4} - x_2^2 + 1} + \frac{(x_1 - 2x_2 + 1)^2}{0.2}$$

  - Feasible region: $\mathbb{D} = [1, 2]^2$
  - Accuracy: $\varepsilon = 0.335$
  - Global minimum: $f^* = 0.16904267$
  - Global minimizer: $\mathbf{x}^* = (1.79541003, 1.37786415)$
  - Derivatives:

$$
\begin{aligned}
f_1'(\mathbf{x}) &= -20x_2 + 12x_1 + 0.02\frac{x_1}{\left(\frac{1}{4}x_1^2 + x_2^2 - 1\right)^2} + 6 \\
f_2'(\mathbf{x}) &= -20x_1 + 42x_2 + 0.08\frac{x_2}{\left(\frac{1}{4}x_1^2 + x_2^2 - 1\right)^2} - 22
\end{aligned}
$$

- **Test Problem 12 [49]**
  - Number of variables: $n = 2$
  - Objective function:

$$f(\mathbf{x}) = 0.1\left(12 + x_1^2 + \frac{\left(1 + x_2^2\right)}{x_1^2} + \frac{x_1^2 x_2^2 + 100}{x_1^4 x_2^4}\right)$$

  - Feasible region: $\mathbb{D} = [1, 3]^2$
  - Accuracy: $\varepsilon = 0.804$
  - Global minimum: $f^* = 1.74$
  - Global minimizer: $\mathbf{x}^* = (1.74333181, 2.02987349)$
  - Derivatives:

$$f_1'(\mathbf{x}) = -\frac{0.2}{x_1^3}\left(x_2^2 + 1\right) + \frac{0.2}{x_1^3 x_2^2} - \frac{0.4}{x_1^5 x_2^4}\left(x_1^2 x_2^2 + 100\right) + 0.1$$

$$f_2'(\mathbf{x}) = -\frac{0.4}{x_1^4 x_2^5}\left(x_1^2 x_2^2 + 100\right) + \frac{0.2}{x_1^2 x_2^3} + \frac{0.2}{x_1^2} x_2$$

- **Test Problem 13 [49]**
  - Number of variables: $n = 2$
  - Objective function:

$$f(\mathbf{x}) = \frac{1}{2}\left(x_1^2 + x_2^2\right) - \cos\left(10 \ln\left(2x_1\right)\right)\cos\left(10 \ln\left(3x_2\right)\right) + 1$$

  - Feasible region: $\mathbb{D} = [0.01, 1]^2$
  - Accuracy: $\varepsilon = 6.92$
  - Global minimum: $f^* = -0.0001$
  - Global minimizer: $\mathbf{x}^* = (0.01152703, 0.01440453)$
  - Derivatives:

$$f_1'(\mathbf{x}) = x_1 + \frac{10}{x_1}\cos\left(10 \ln 3x_2\right)\sin\left(10 \ln 2x_1\right)$$

$$f_2'(\mathbf{x}) = x_2 + \frac{10}{x_2}\cos\left(10 \ln 2x_1\right)\sin\left(10 \ln 3x_2\right)$$

- **Test Problem 14 [49]**
  - Number of variables: $n = 3$
  - Objective function:

$$f(\mathbf{x}) = 100\left(x_3 - \left(\frac{x_1 + x_2}{2}\right)^2\right)^2 + (1 - x_1)^2 + (1 - x_2)^2$$

– Feasible region: $\mathbb{D} = [0, 1]^3$
– Accuracy: $\varepsilon = 2.12$
– Global minimum: $f^* = 0$
– Global minimizer: $\mathbf{x}^* = (1, 1, 1)$
– Derivatives:

$$f_1'(\mathbf{x}) = -200\left(x_3 - \left(\frac{1}{2}x_1 + \frac{1}{2}x_2\right)^2\right)\left(\frac{1}{2}x_1 + \frac{1}{2}x_2\right) + 2x_1 - 2$$

$$f_2'(\mathbf{x}) = -200\left(x_3 - \left(\frac{1}{2}x_1 + \frac{1}{2}x_2\right)^2\right)\left(\frac{1}{2}x_1 + \frac{1}{2}x_2\right) + 2x_2 - 2$$

$$f_3'(\mathbf{x}) = -200\left(\frac{1}{2}x_1 + \frac{1}{2}x_2\right)^2 + 200x_3$$

- **Test Problem 15 (Hartman-3) [23, 49]**
  – Number of variables: $n = 3$
  – Objective function:

$$f(\mathbf{x}) = -\sum_{i=1}^{4} c_i \exp\left(-\sum_{j=1}^{3} \alpha_{ij}\left(x_j - p_{ij}\right)^2\right)$$

– Feasible region: $\mathbb{D} = [0, 1]^3$
– Accuracy: $\varepsilon = 0.369$
– Global minimum: $f^* = -3.8627814$
– Global minimizer: $\mathbf{x}^* = (0.114540, 0.555784, 0.852538)$
– Derivatives:

$$\begin{aligned}
f_1'(\mathbf{x}) = {} & \alpha_{11}c_1 \exp\left(-\alpha_{11}(x_1 - p_{11})^2 - \alpha_{12}(x_2 - p_{12})^2 - \alpha_{13}(x_3 - p_{13})^2\right) \\
& \times (2x_1 - 2p_{11}) + \alpha_{21}c_2 \\
& \times \exp\left(-\alpha_{21}(x_1 - p_{21})^2 - \alpha_{22}(x_2 - p_{22})^2 - \alpha_{23}(x_3 - p_{23})^2\right) \\
& \times (2x_1 - 2p_{21}) + \alpha_{31}c_3 \\
& \times \exp\left(-\alpha_{31}(x_1 - p_{31})^2 - \alpha_{32}(x_2 - p_{32})^2 - \alpha_{33}(x_3 - p_{33})^2\right) \\
& \times (2x_1 - 2p_{31}) + \alpha_{41}c_4 \\
& \times \exp\left(-\alpha_{41}(x_1 - p_{41})^2 - \alpha_{42}(x_2 - p_{42})^2 - \alpha_{43}(x_3 - p_{43})^2\right) \\
& \times (2x_1 - 2p_{41})
\end{aligned}$$

$$
\begin{aligned}
f_2'(\mathbf{x}) = {} & \alpha_{12} c_1 \exp\left(-\alpha_{11}(x_1 - p_{11})^2 - \alpha_{12}(x_2 - p_{12})^2 - \alpha_{13}(x_3 - p_{13})^2\right) \\
& \times (2x_2 - 2p_{12}) + \alpha_{22} c_2 \\
& \times \exp\left(-\alpha_{21}(x_1 - p_{21})^2 - \alpha_{22}(x_2 - p_{22})^2 - \alpha_{23}(x_3 - p_{23})^2\right) \\
& \times (2x_2 - 2p_{22}) + \alpha_{32} c_3 \\
& \times \exp\left(-\alpha_{31}(x_1 - p_{31})^2 - \alpha_{32}(x_2 - p_{32})^2 - \alpha_{33}(x_3 - p_{33})^2\right) \\
& \times (2x_2 - 2p_{32}) + \alpha_{42} c_4 \\
& \times \exp\left(-\alpha_{41}(x_1 - p_{41})^2 - \alpha_{42}(x_2 - p_{42})^2 - \alpha_{43}(x_3 - p_{43})^2\right) \\
& \times (2x_2 - 2p_{42}) \\
f_3'(\mathbf{x}) = {} & \alpha_{13} c_1 \exp\left(-\alpha_{11}(x_1 - p_{11})^2 - \alpha_{12}(x_2 - p_{12})^2 - \alpha_{13}(x_3 - p_{13})^2\right) \\
& \times (2x_3 - 2p_{13}) + \alpha_{23} c_2 \\
& \times \exp\left(-\alpha_{21}(x_1 - p_{21})^2 - \alpha_{22}(x_2 - p_{22})^2 - \alpha_{23}(x_3 - p_{23})^2\right) \\
& \times (2x_3 - 2p_{23}) + \alpha_{33} c_3 \\
& \times \exp\left(-\alpha_{31}(x_1 - p_{31})^2 - \alpha_{32}(x_2 - p_{32})^2 - \alpha_{33}(x_3 - p_{33})^2\right) \\
& \times (2x_3 - 2p_{33}) + \alpha_{43} c_4 \\
& \times \exp\left(-\alpha_{41}(x_1 - p_{41})^2 - \alpha_{42}(x_2 - p_{42})^2 - \alpha_{43}(x_3 - p_{43})^2\right) \\
& \times (2x_3 - 2p_{43})
\end{aligned}
$$

where

i	c_i	α_{i1}	α_{i2}	α_{i3}	p_{i1}	p_{i2}	p_{i3}
1	1.0	3.0	10	30	0.3689	0.1170	0.2673
2	1.2	0.1	10	35	0.4699	0.4387	0.7470
3	3.0	3.0	10	30	0.1091	0.8732	0.5547
4	3.2	1.0	10	35	0.0382	0.5743	0.8828

- **Test Problem 16 [49]**
  - Number of variables: $n = 3$
  - Objective function:

$$
f(\mathbf{x}) = \frac{1}{3}(x_1^2 + x_2^2 + x_3^2) - \cos(10\ln(2x_1))\cos(10\ln(3x_2)) \times \cos(10\ln(4x_3)) + 1
$$

- Feasible region: $\mathbb{D} = [0.01, 1]^3$
- Accuracy: $\varepsilon = 8.333$
- Global minimum: $f^* = 0$
- Global minimizer: $\mathbf{x}^* = (0.040555, 0.069074, 0.052778)$
- Derivatives:

$$f_1'(\mathbf{x}) = \frac{2}{3}x_1 + \frac{10}{x_1}\cos(10\ln 3x_2)\cos(10\ln 4x_3)\sin(10\ln 2x_1)$$
$$f_2'(\mathbf{x}) = \frac{2}{3}x_2 + \frac{10}{x_2}\cos(10\ln 2x_1)\cos(10\ln 4x_3)\sin(10\ln 3x_2)$$
$$f_3'(\mathbf{x}) = \frac{2}{3}x_3 + \frac{10}{x_3}\cos(10\ln 2x_1)\cos(10\ln 3x_2)\sin(10\ln 4x_3)$$

- **Test Problem 17 [49]**
  - Number of variables: $n = 3$
  - Objective function:

$$f(\mathbf{x}) = -\sin x_1 \sin x_1x_2 \sin x_1x_2x_3$$

  - Feasible region: $\mathbb{D} = [0, 4]^3$
  - Accuracy: $\varepsilon = 0.672$
  - Global minimum: $f^* = -0.9$
  - Global minimizer: $\mathbf{x}^* = (1.572016, 2.998628, 2.998628)$
  - Derivatives:

$$\begin{aligned} f_1'(\mathbf{x}) &= -\sin x_1x_2 \cos x_1 \sin x_1x_2x_3 - x_2 \cos x_1x_2 \sin x_1 \sin x_1x_2x_3 \\ &\quad - x_2x_3 \sin x_1x_2 \sin x_1 \cos x_1x_2x_3 \\ f_2'(\mathbf{x}) &= -x_1 \cos x_1x_2 \sin x_1 \sin x_1x_2x_3 - x_1x_3 \sin x_1x_2 \sin x_1 \cos x_1x_2x_3 \\ f_3'(\mathbf{x}) &= -x_1x_2 \sin x_1x_2 \sin x_1 \cos x_1x_2x_3 \end{aligned}$$

- **Test Problem 18 [49]**
  - Number of variables: $n = 3$
  - Objective function:

$$f(\mathbf{x}) = -\left(x_1^2 - 2x_2^2 + x_3^2\right)\sin x_1 \sin x_2 \sin x_3$$

  - Feasible region: $\mathbb{D} = [-1, 1]^3$
  - Accuracy: $\varepsilon = 0.0506$
  - Global minimum: $f^* = -0.51637406$
  - Global minimizer: $\mathbf{x}^* = (-1.000000, -0.555968, -1.000000)$
  - Derivatives:

$$f_1'(\mathbf{x}) = -(\cos x_1 \sin x_2 \sin x_3)\left(x_1^2 - 2x_2^2 + x_3^2\right) - 2x_1 \sin x_1 \sin x_2 \sin x_3$$
$$f_2'(\mathbf{x}) = -(\cos x_2 \sin x_1 \sin x_3)\left(x_1^2 - 2x_2^2 + x_3^2\right) + 4x_2 \sin x_1 \sin x_2 \sin x_3$$
$$f_3'(\mathbf{x}) = -(\cos x_3 \sin x_1 \sin x_2)\left(x_1^2 - 2x_2^2 + x_3^2\right) - 2x_3 \sin x_1 \sin x_2 \sin x_3$$

- **Test Problem 19 [49]**
  - Number of variables: $n = 3$
  - Objective function:

$$f(\mathbf{x}) = -(x_1 - 1)(x_1 + 2)(x_2 + 1)(x_2 - 2)\,x_3^2$$

  - Feasible region: $\mathbb{D} = [-2, 2]^3$
  - Accuracy: $\varepsilon = 4.51$
  - Global minimum: $f^* = 35.99999997$
  - Global minimizer: $\mathbf{x}^* = (-0.500000, -2.000000, -2.000000)$
  - Derivatives:

$$f_1'(\mathbf{x}) = -x_3^2(x_1 - 1)(x_2 + 1)(x_2 - 2) - x_3^2(x_1 + 2)(x_2 + 1)(x_2 - 2)$$
$$f_2'(\mathbf{x}) = -x_3^2(x_1 - 1)(x_1 + 2)(x_2 + 1) - x_3^2(x_1 - 1)(x_1 + 2)(x_2 - 2)$$
$$f_3'(\mathbf{x}) = -2x_3(x_1 - 1)(x_1 + 2)(x_2 + 1)(x_2 - 2)$$

- **Test Problem 20 (Rosenbrock) [82]**
  - Number of variables: $n = 3$
  - Objective function:

$$f(\mathbf{x}) = \sum_{i=1}^{n-1}\left(100\left(x_{i+1} - x_i^2\right)^2 + (x_i - 1)^2\right)$$

  - Feasible region: $\mathbb{D} = [-3, 3]^3$
  - Accuracy: $\varepsilon = 2500$
  - Global minimum: $f^* = 0$
  - Global minimizer: $\mathbf{x}^* = (1, 1, 1)$
  - Derivatives:

$$f_1'(\mathbf{x}) = -400\left(x_2 - x_1^2\right)x_1 + 2(x_1 - 1)$$
$$f_2'(\mathbf{x}) = -400\left(x_{i+1} - x_i^2\right)x_i + 2(x_{i+1} - 1) + 200\left(x_i - x_{i-1}^2\right)$$
$$f_3'(\mathbf{x}) = 200\left(x_n - x_{n-1}^2\right)$$

- **Test Problem 21 (Levy 15) [60]**
  - Number of variables: $n = 4$

– Objective function:

$$f(\mathbf{x}) = \sin^2 3\pi x_1 + \sum_{i=1}^{n-1} (x_i - 1)^2 \left(1 + \sin^2 3\pi x_{i+1}\right)$$
$$+ (x_n - 1)^2 \times \left(1 + \sin^2 2\pi x_n\right)$$

– Feasible region: $\mathbb{D} = [-10, 10]^4$
– Accuracy: $\varepsilon = L_2$
– Global minimum: $f^* = 0$
– Global minimizer: $\mathbf{x}^* = (1, 1, 1, 1)$
– Derivatives:

$$f_1'(\mathbf{x}) = 6 \sin(3\pi x_1) \cos(3\pi x_1)\, \pi + 2(x_1 - 1)\left(1 + \sin^2(3\pi x_2)\right)$$
$$f_i'(\mathbf{x}) = 6(x_{i-1} - 1)^2 \sin(3\pi x_i) \cos(3\pi x_i)\, \pi + 2(x_i - 1) \times \left(1 + \sin^2(3\pi x_{i+1})\right)$$
$$f_n'(\mathbf{x}) = 6(x_{n-1} - 1)^2 \sin(3\pi x_n) \cos(3\pi x_n)\, \pi + 1 + \sin^2(2\pi x_n) + 4(x_n - 1) \sin(2\pi x_n) \cos(2\pi x_n)$$

- **Test Problem 22 (Rosenbrock) [82]**
  – Number of variables: $n = 4$
  – Objective function:

$$f(\mathbf{x}) = \sum_{i=1}^{n-1} \left(100\left(x_{i+1} - x_i^2\right)^2 + (x_i - 1)^2\right)$$

  – Feasible region: $\mathbb{D} = [-4, 4]^4$
  – Accuracy: $\varepsilon = L_2$
  – Global minimum: $f^* = 0$
  – Global minimizer: $\mathbf{x}^* = (1, 1, 1, 1)$
  – Derivatives:

$$f_1'(\mathbf{x}) = -400\left(x_2 - x_1^2\right) x_1 + 2(x_1 - 1)$$
$$f_i'(\mathbf{x}) = -400\left(x_{i+1} - x_i^2\right) x_i + 2(x_{i+1} - 1) + 200\left(x_i - x_{i-1}^2\right)$$
$$f_n'(\mathbf{x}) = 200\left(x_n - x_{n-1}^2\right)$$

- **Test Problem 23 (Shekel-5) [23, 60]**
  – Number of variables: $n = 4$

– Objective function:

$$f(\mathbf{x}) = -\sum_{i=1}^{5} \frac{1}{(x-a_i)(x-a_i)^T + c_i}$$

– Feasible region: $\mathbb{D} = [0, 10]^4$
– Accuracy: $\varepsilon = L_2$
– Global minimum: $f^* = -10.1532$
– Global minimizer: $\mathbf{x}^* = (4.00004, 4.00013, 4.00004, 4.00013)$
– Derivatives:

$$f'_j(\mathbf{x}) = -\sum_{i=1}^{5} \frac{2\left(x_j - a_{i,j}\right)}{\left((x-a_i)(x-a_i)^T + c_i\right)^2}$$

where

i	a_{i1}	a_{i2}	a_{i3}	a_{i4}	c_i
1	4.0	4.0	4.0	4.0	0.1
2	1.0	1.0	1.0	1.0	0.2
3	8.0	8.0	8.0	8.0	0.2
4	6.0	6.0	6.0	6.0	0.4
5	3.0	7.0	3.0	7.0	0.4

- **Test Problem 24 (Shekel-7) [23, 60]**

– Number of variables: $n = 4$
– Objective function:

$$f(\mathbf{x}) = -\sum_{i=1}^{7} \frac{1}{(x-a_i)(x-a_i)^T + c_i}$$

– Feasible region: $\mathbb{D} = [0, 10]^4$
– Accuracy: $\varepsilon = L_2$
– Global minimum: $f^* = -10.4029$
– Global minimizer: $\mathbf{x}^* = (4.00057, 4.00069, 3.99949, 3.99961)$
– Derivatives:

$$f'_j(\mathbf{x}) = -\sum_{i=1}^{7} \frac{2\left(x_j - a_{i,j}\right)}{\left((x-a_i)(x-a_i)^T + c_i\right)^2}$$

where

i	a_{i1}	a_{i2}	a_{i3}	a_{i4}	c_i
1	4.0	4.0	4.0	4.0	0.1
2	1.0	1.0	1.0	1.0	0.2
3	8.0	8.0	8.0	8.0	0.2
4	6.0	6.0	6.0	6.0	0.4
5	3.0	7.0	3.0	7.0	0.4
6	2.0	9.0	2.0	9.0	0.6
7	5.0	5.0	3.0	3.0	0.6

- **Test Problem 25 (Shekel-10) [23, 60]**
  - Number of variables: $n = 4$
  - Objective function:

$$f(\mathbf{x}) = -\sum_{i=1}^{10} \frac{1}{(x - a_i)(x - a_i)^T + c_i}$$

  - Feasible region: $\mathbb{D} = [0, 10]^4$
  - Accuracy: $\varepsilon = L_2$
  - Global minimum: $f^* = -10.5364$
  - Global minimizer: $\mathbf{x}^* = (4.00075, 4.00059, 3.99966, 3.99951)$
  - Derivatives:

$$f_j'(\mathbf{x}) = -\sum_{i=1}^{10} \frac{2\left(x_j - a_{i,j}\right)}{\left((x - a_i)(x - a_i)^T + c_i\right)^2}$$

where

i	a_{i1}	a_{i2}	a_{i3}	a_{i4}	c_i
1	4.0	4.0	4.0	4.0	0.1
2	1.0	1.0	1.0	1.0	0.2
3	8.0	8.0	8.0	8.0	0.2
4	6.0	6.0	6.0	6.0	0.4
5	3.0	7.0	3.0	7.0	0.4
6	2.0	9.0	2.0	9.0	0.6
7	5.0	5.0	3.0	3.0	0.6
8	8.0	1.0	8.0	1.0	0.7
9	6.0	2.0	6.0	2.0	0.5
10	7.0	3.6	7.0	3.6	0.5

- **Test Problem 26 (Schwefel 1.2) [60]**
  - Number of variables: $n = 4$
  - Objective function:

$$f(\mathbf{x}) = \sum_{i=1}^{4} \left(\sum_{j=1}^{i} x_j \right)^2$$

  - Feasible region: $\mathbb{D} = [-5, 10]^4$
  - Accuracy: $\varepsilon = L_2$
  - Global minimum: $f^* = 0$
  - Global minimizer: $\mathbf{x}^* = (1, 1, 1, 1)$
  - Derivatives:

$$f_1'(\mathbf{x}) = 8x_1 + 6x_2 + 4x_3 + 2x_4$$
$$f_2'(\mathbf{x}) = 6x_1 + 6x_2 + 4x_3 + 2x_4$$
$$f_3'(\mathbf{x}) = 4x_1 + 4x_2 + 4x_3 + 2x_4$$
$$f_4'(\mathbf{x}) = 2x_1 + 2x_2 + 2x_3 + 2x_4$$

- **Test Problem 27 (Powel) [60]**
  - Number of variables: $n = 4$
  - Objective function:

$$f(\mathbf{x}) = (x_1 + 10x_2)^2 + 5\,(x_3 - x_4)^2 + (x_2 - 2x_3)^4 + 10\,(x_1 - x_4)^4$$

  - Feasible region: $\mathbb{D} = [-4, 5]^4$
  - Accuracy: $\varepsilon = L_2$
  - Global minimum: $f^* = 0$
  - Global minimizer: $\mathbf{x}^* = (0, 0, 0, 0)$
  - Derivatives:

$$f_1'(\mathbf{x}) = 2x_1 + 20x_2 + 40\,(x_1 - x_4)^3$$
$$f_2'(\mathbf{x}) = 20x_1 + 200x_2 + 4\,(x_2 - 2x_3)^3$$
$$f_3'(\mathbf{x}) = -10x_4 + 10x_3 - 8\,(x_2 - 2x_3)^3$$
$$f_4'(\mathbf{x}) = -10x_3 + 10x_4 - 40\,(x_1 + x_4)^3$$

- **Test Problem 28 (Levy 9) [60]**
  - Number of variables: $n = 4$
  - Objective function:

$$f(\mathbf{x}) = \sin^2 3\pi y_1 + \sum_{i=1}^{n-1} (y_i - 1)^2 \left(1 + 10\sin^2 \pi y_{i+1}\right) + (y_n - 1)^2,$$
$$y_i = 1 + (x_i - 1)/4$$

- Feasible region: $\mathbb{D} = [-10, 10]^4$
- Accuracy: $\varepsilon = L_2$
- Global minimum: $f^* = 0$
- Global minimizer: $\mathbf{x}^* = (1, 1, 1, 1)$
- Derivatives:

$$f_1'(\mathbf{x}) = \left(10\sin^2 \pi \left(\frac{1}{4}x_2 + \frac{3}{4}\right) + 1\right)(2x_1 - 2) + \frac{3}{2}\pi \cos 3\pi \left(\frac{1}{4}x_1 + \frac{3}{4}\right) \sin 3\pi \left(\frac{1}{4}x_1 + \frac{3}{4}\right)$$
$$f_2'(\mathbf{x}) = \left(10\sin^2 \pi \left(\frac{1}{4}x_4 + \frac{3}{4}\right) + 1\right)\left(\frac{1}{8}x_3 - \frac{1}{8}\right) + 5\pi \left(\cos \pi \left(\frac{1}{4}x_3 + \frac{3}{4}\right) \sin \pi \left(\frac{1}{4}x_3 + \frac{3}{4}\right)\right)\left(\frac{1}{4}x_2 - \frac{1}{4}\right)^2$$
$$f_3'(\mathbf{x}) = \frac{1}{8} + 5\pi \left(\cos \pi \left(\frac{1}{4}x_4 + \frac{3}{4}\right) \sin \pi \left(\frac{1}{4}x_4 + \frac{3}{4}\right)\right)\left(\frac{1}{4}x_3 - \frac{1}{4}\right)^2 + \frac{1}{8}x_4$$

- **Test Problem 29 (Levy 16) [60]**
  - Number of variables: $n = 5$
  - Objective function:

$$f(\mathbf{x}) = \sin^2 3\pi x_1 + \sum_{i=1}^{n-1} (x_i - 1)^2 \left(1 + \sin^2 3\pi x_{i+1}\right) + (x_n - 1)^2 \left(1 + \sin^2 2\pi x_n\right)$$

  - Feasible region: $\mathbb{D} = [-5, 5]^5$
  - Accuracy: $\varepsilon = 1.5L_2$
  - Global minimum: $f^* = 0$
  - Global minimizer: $\mathbf{x}^* = (1, 1, 1, 1, 1)$
  - Derivatives:

$$f_1'(\mathbf{x}) = 6\sin(3\pi x_1)\cos(3\pi x_1)\pi + 2(x_1 - 1)\left(1 + \sin^2(3\pi x_2)\right)$$
$$f_i'(\mathbf{x}) = 6(x_{i-1} - 1)^2 \sin(3\pi x_i)\cos(3\pi x_i)\pi + 2(x_i - 1) \times \left(1 + \sin^2(3\pi x_{i+1})\right)$$

$$f_n'(\mathbf{x}) = 6\,(x_{n-1}-1)^2 \sin(3\pi x_n)\cos(3\pi x_n)\,\pi + 1 + \sin^2(2\pi x_n) \\ + 4\,(x_n-1)\sin(2\pi x_n)\cos(2\pi x_n)$$

- **Test Problem 30 (Rosenbrock) [82]**
  - Number of variables: $n = 5$
  - Objective function:

$$f(\mathbf{x}) = \sum_{i=1}^{n-1}\left(100\left(x_{i+1}-x_i^2\right)^2 + (x_i-1)^2\right)$$

  - Feasible region: $\mathbb{D} = [-5, 5]^5$
  - Accuracy: $\varepsilon = 1.5L_2$
  - Global minimum: $f^* = 0$
  - Global minimizer: $\mathbf{x}^* = (1, 1, 1, 1, 1)$
  - Derivatives:

$$f_1'(\mathbf{x}) = -400\left(x_2 - x_1^2\right)x_1 + 2\,(x_1-1)$$
$$f_i'(\mathbf{x}) = -400\left(x_{i+1} - x_i^2\right)x_i + 2\,(x_{i+1}-1) + 200\left(x_i - x_{i-1}^2\right)$$
$$f_n'(\mathbf{x}) = 200\left(x_n - x_{n-1}^2\right)$$

- **Test Problem 31 (Levy 10) [60]**
  - Number of variables: $n = 5$
  - Objective function:

$$f(\mathbf{x}) = \sin^2 3\pi y_1 + \sum_{i=1}^{n-1}(y_i-1)^2\left(1+10\sin^2\pi y_{i+1}\right) + (y_n-1)^2,$$
$$y_i = 1 + (x_i-1)/4$$

  - Feasible region: $\mathbb{D} = [-10, 10]^5$
  - Accuracy: $\varepsilon = 1.5L_2$
  - Global minimum: $f^* = 0$
  - Global minimizer: $\mathbf{x}^* = (1, 1, 1, 1, 1)$
  - Derivatives:

$$f_1'(\mathbf{x}) = \left(10\sin^2\pi\left(\frac{1}{4}x_2+\frac{3}{4}\right)+1\right)(2x_1-2) \\ + \frac{3}{2}\pi\cos 3\pi\left(\frac{1}{4}x_1+\frac{3}{4}\right)\sin 3\pi\left(\frac{1}{4}x_1+\frac{3}{4}\right)$$

$$f_2'(\mathbf{x}) = \left(10\sin^2\pi\left(\frac{1}{4}x_3+\frac{3}{4}\right)+1\right)\left(\frac{1}{8}x_2-\frac{1}{8}\right)$$
$$+5\pi\left(\cos\pi\left(\frac{1}{4}x_2+\frac{3}{4}\right)\sin\pi\left(\frac{1}{4}x_2+\frac{3}{4}\right)\right)(x_1-1)^2$$
$$f_i'(\mathbf{x}) = \left(10\sin^2\pi\left(\frac{1}{4}x_{i+1}+\frac{3}{4}\right)+1\right)\left(\frac{1}{8}x_i-\frac{1}{8}\right)$$
$$+5\pi\left(\cos\pi\left(\frac{1}{4}x_i+\frac{3}{4}\right)\sin\pi\left(\frac{1}{4}x_i+\frac{3}{4}\right)\right)\left(\frac{1}{4}x_{i-1}-\frac{1}{4}\right)$$
$$f_n'(\mathbf{x}) = -\frac{1}{8}+5\pi\left(\cos\pi\left(\frac{1}{4}x_n+\frac{3}{4}\right)\sin\pi\left(\frac{1}{4}x_n+\frac{3}{4}\right)\right)\left(\frac{1}{4}x_{n-1}-\frac{1}{4}\right)^2$$
$$+\frac{1}{8}x_n$$

- **Test Problem 32 (Levy 17) [60]**
  - Number of variables: $n = 6$
  - Objective function:

$$f(\mathbf{x}) = \sin^2 3\pi x_1 + \sum_{i=1}^{n-1}(x_i-1)^2\left(1+\sin^2 3\pi x_{i+1}\right)$$
$$+(x_n-1)^2\left(1+\sin^2 2\pi x_n\right)$$

  - Feasible region: $\mathbb{D} = [-5, 5]^6$
  - Accuracy: $\varepsilon = 4L_2$
  - Global minimum: $f^* = 0$
  - Global minimizer: $\mathbf{x}^* = (1, 1, 1, 1, 1, 1)$
  - Derivatives:

$$f_1'(\mathbf{x}) = 6\sin(3\pi x_1)\cos(3\pi x_1)\pi + 2(x_1-1)\left(1+\sin^2(3\pi x_2)\right)$$
$$f_i'(\mathbf{x}) = 6(x_{i-1}-1)^2\sin(3\pi x_i)\cos(3\pi x_i)\pi$$
$$+2(x_i-1)\times\left(1+\sin^2(3\pi x_{i+1})\right)$$
$$f_n'(\mathbf{x}) = 6(x_{n-1}-1)^2\sin(3\pi x_n)\cos(3\pi x_n)\pi + 1 + \sin^2(2\pi x_n)$$
$$+4(x_n-1)\sin(2\pi x_n)\cos(2\pi x_n)$$

- **Test Problem 33 (Rosenbrock) [82]**
  - Number of variables: $n = 6$
  - Objective function:

$$f(\mathbf{x}) = \sum_{i=1}^{n-1}\left(100\left(x_{i+1}-x_i^2\right)^2+(x_i-1)^2\right)$$

– Feasible region: $\mathbb{D} = [-6, 6]^6$
– Accuracy: $\varepsilon = 4L_2$
– Global minimum: $f^* = 0$
– Global minimizer: $\mathbf{x}^* = (1, 1, 1, 1, 1, 1)$
– Derivatives:

$$f_1'(\mathbf{x}) = -400\left(x_2 - x_1^2\right) x_1 + 2\,(x_1 - 1)$$
$$f_i'(\mathbf{x}) = -400\left(x_{i+1} - x_i^2\right) x_i + 2\,(x_{i+1} - 1) + 200\left(x_i - x_{i-1}^2\right)$$
$$f_n'(\mathbf{x}) = 200\left(x_n - x_{n-1}^2\right)$$

- **Test Problem Six-Hump Camelback [143]**

– Number of variables: $n = 2$
– Objective function:

$$f(\mathbf{x}) = \left(4 - 2.1x_1^2 + \frac{1}{3}x_1^4\right) x_1^2 + x_1x_2 + (-4 + 4x_2^2)x_2^2$$

– Feasible region: $\mathbb{D} = [-3, 3] \times [-2, 2]$
– Global minimum: $f^* = -1.031628$
– Global minimizers: $\mathbf{x}^* = (0.089842, -0.712656), (-0.089842, 0.712656)$

- **Test Problem Shubert [143]**

– Number of variables: $n = 2$
– Objective function:

$$f(\mathbf{x}) = \left(\sum_{i=1}^{5} i \cos\left((i+1)x_1 + i\right)\right) \times \left(\sum_{i=1}^{5} i \cos\left((i+1)x_2 + i\right)\right)$$

– Feasible region: $\mathbb{D} = [-10, 10]^2$
– Global minimum: $f^* = -186.7309$
– Global minimizers: 18 global minimizers

- **Test Problem Alolyan [46]**

– Number of variables: $n = 2$
– Objective function:

$$f(\mathbf{x}) = x_1x_2^2 + x_2x_1^2 - x_1^3 - x_2^3$$

– Feasible region: $\mathbb{D} = [-1, 1]^2$
– Global minimum: $f^* =$ âĹŠ1.18519
– Global minimizers: $\mathbf{x}^* = (-\frac{1}{3}, 1), (\frac{1}{3}, -1)$

- **Test Problem Easom [46]**
  - Number of variables: $n = 2$
  - Objective function:

$$f(\mathbf{x}) = -\cos x_1 \cos x_2 \exp(-(x_1 - \pi)^2 - (x_2 - \pi)^2)$$

  - Feasible region: $\mathbb{D} = [-100, -100]$
  - Global minimum: $f^* = -1$
  - Global minimizer: $\mathbf{x}^* = (\pi, \pi)$

- **Test Problem Rastrigin [46]**
  - Number of variables: $n = 2$
  - Objective function:

$$f(\mathbf{x}) = 20 + x_1^2 + x_2^2 - 10(\cos 2\pi x_1 + \cos 2\pi x_2)$$

  - Feasible region: $\mathbb{D} = [-5, 6]^2$
  - Global minimum: $f^* = 0$
  - Global minimizer: $\mathbf{x}^* = (0, 0)$

- **Test Problem Hartman-6 [23]**
  - Number of variables: $n = 6$
  - Objective function:

$$f(\mathbf{x}) = -\sum_{i=1}^{4} c_i \exp\left(-\sum_{j=1}^{6} \alpha_{ij} \left(x_j - p_{ij}\right)^2\right)$$

where

i	c_i	α_{i1}	α_{i2}	α_{i3}	α_{i4}	α_{i5}	α_{i6}
1	1.0	10.00	3.00	17.00	3.50	1.70	8.00
2	1.2	0.05	10.00	17.00	0.10	8.00	14.00
3	3.0	3.00	3.50	1.70	10.00	17.00	8.00
4	3.2	17.00	8.00	0.05	10.00	0.10	14.00
i		p_{i1}	p_{i2}	p_{i3}	p_{i4}	p_{i5}	p_{i6}
1		0.1312	0.1696	0.5569	0.0124	0.8283	0.5886
2		0.2329	0.4135	0.8307	0.3736	0.1004	0.9991
3		0.2348	0.1451	0.3522	0.2883	0.3047	0.6650
4		0.4047	0.8828	0.8732	0.5743	0.1091	0.0381

  - Feasible region: $\mathbb{D} = [0, 1]^6$
  - Accuracy: $\varepsilon = 0.369$

– Global minimum: $f^* =$ âĹŠ3.32237
– Global minimizer:

$$\mathbf{x}^* = (0.20169, 0.15001, 0.47687, 0.27533, 0.31165, 0.65730)$$

Test Problems with Linear Constraints

- **Test Problem Horst 1 [55]**
  – Number of variables: $n = 2$
  – Objective function:

$$f(\mathbf{x}) = -x_1^2 - 4x_2^2 + 4x_1x_2 + 2x_1 + 4x_2$$

  – Constraints:

$$\begin{aligned} -4x_1 + 2x_2 &\le 1, \\ x_1 + x_2 &\le 4, \\ x_1 - 4x_2 &\le 1, \\ 0 \le x_1 &\le 3, \\ 0 \le x_2 &\le 2. \end{aligned}$$

  – Global minimum: $f^* = -1.0625$
  – Global minimizer: $\mathbf{x}^* = (0.75, 2.0)$

- **Test Problem Horst 2 [55]**
  – Number of variables: $n = 2$
  – Objective function:

$$f(\mathbf{x}) = -x_1^2 - x_2^{3/2}$$

  – Constraints:

$$\begin{aligned} x_1 + 2x_2 &\le 4, \\ x_1 - 2x_2 &\le 1, \\ -x_1 + x_2 &\le 1, \\ 0 \le x_1 &\le 2.5, \\ 0 \le x_2 &\le 2. \end{aligned}$$

- Global minimum: $f^* = -6.8995$
- Global minimizer: $\mathbf{x}^* = (2.5, 0.75)$

- **Test Problem Horst 3 [55]**
  - Number of variables: $n = 2$
  - Objective function:

$$f(\mathbf{x}) = -x_1^2 + \frac{4}{3}x_1 + \ln(1 + x_2) - \frac{4}{9}$$

  - Constraints:

$$\begin{aligned} -2x_1 + x_2 &\leq 1, \\ x_1 + x_2 &\leq \frac{3}{2}, \\ x_1 + \frac{1}{10}x_2 &\leq 1, \\ x_1, x_2 &\geq 0. \end{aligned}$$

  - Global minimum: $f^* = -\frac{4}{9}$
  - Global minimizer: $\mathbf{x}^* = (0.0, 0.0)$

- **Test Problem Horst 4 [55]**
  - Number of variables: $n = 3$
  - Objective function:

$$f(\mathbf{x}) = -|x_1 + \frac{1}{2}x_2 + \frac{2}{3}x_3|^{\frac{3}{2}}$$

  - Constraints:

$$\begin{aligned} x_1 + x_2 + 2x_3 &\leq 6, \\ x_1 + \frac{1}{2}x_2 &\leq 2, \\ -x_2 - 2x_3 &\leq -1, \\ -x_1 &\leq -\frac{1}{2}, \\ x_1, x_2, x_3 &\geq 0. \end{aligned}$$

  - Global minimum: $f^* = -6.0858$
  - Global minimizer: $\mathbf{x}^* = (2.0, 0.0, 2.0)$

- **Test Problem Horst 5 [55]**
  - Number of variables: $n = 3$
  - Objective function:

$$f(\mathbf{x}) = -|x_1 + \frac{1}{2}x_2 + \frac{2}{3}x_3|^{\frac{3}{2}} - x_1^2$$

  - Constraints:

$$\begin{aligned} x_1 + x_2 + x_3 &\leq 2, \\ x_1 + x_2 - \frac{1}{4}x_3 &\leq 1, \\ -2x_1 - 2x_2 + x_3 &\leq 1, \\ x_3 &\leq 3, \\ x_1, x_2, x_3 &\geq 0. \end{aligned}$$

  - Global minimum: $f^* = -3.722$
  - Global minimizer: $\mathbf{x}^* = (1.2, 0.0, 0.8)$

- **Test Problem Horst 6 [55]**
  - Number of variables: $n = 3$
  - Objective function:

$$f(\mathbf{x}) = \mathbf{x}^T \mathbf{Q} \mathbf{x} + \mathbf{q}^T \mathbf{x}$$

  - Constraints:

$$\begin{aligned} 0.488509x_1 + 0.063565x_2 + 0.945686x_3 &\leq 2.865062, \\ -0.578592x_1 - 0.324014x_2 - 0.501754x_3 &\leq -1.491608, \\ -0.719203x_1 + 0.099562x_2 + 0.445225x_3 &\leq 0.519588, \\ -0.346896x_1 + 0.637939x_2 - 0.257623x_3 &\leq 1.584087, \\ -0.202821x_1 + 0.647361x_2 + 0.920135x_3 &\leq 2.198036, \\ -0.983091x_1 - 0.886420x_2 - 0.802444x_3 &\leq -1.301853, \\ -0.305441x_1 - 0.180123x_2 - 0.515399x_3 &\leq -0.738290, \\ x_1, x_2, x_3 &\geq 0, \end{aligned}$$

  where

$$\mathbf{Q} = \begin{pmatrix} 0.992934 & -0.640117 & 0.337286 \\ -0.640117 & -0.814622 & 0.960807 \\ 0.337286 & 0.960807 & 0.500874 \end{pmatrix}, \; \mathbf{q} = \begin{pmatrix} -0.992372 \\ -0.046466 \\ 0.891766 \end{pmatrix}.$$

- Global minimum: $f^* = -31.5285$
- Global minimizer: $\mathbf{x}^* = (5.210677, 5.027908, 0.000000)$

- **Test Problem Horst 7 [55]**
  - Number of variables: $n = 3$
  - Objective function:

$$f(\mathbf{x}) = -\left(x_1 + \frac{1}{2}x_3 - 2\right)^2 - |x_1 + \frac{1}{2}x_2 + \frac{2}{3}x_3|^{\frac{3}{2}}$$

  - Constraints:

$$\begin{aligned} -x_1 - x_2 + \frac{1}{2}x_3 &\le 1, \\ x_1 + 2x_2 &\le 6, \\ 2x_1 + 4x_2 + 2x_3 &\ge 1, \\ x_3 &\le 3, \\ x_1, x_2, x_3 &\ge 0. \end{aligned}$$

  - Global minimum: $f^* = -44.859$
  - Global minimizer: $\mathbf{x}^* = (6.0, 0.0, 2.0)$

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