

Bibliography

Anderson, T.W. and H. Rubin (1949), Estimation of the Parameters of a Single Equation in a Complete System of Stochastic Equations, *Annals of Mathematical Statistics*, pp. 46–63.

Anderson, T.W. and H. Rubin (1950), The Asymptotic Properties of Estimates of Parameters of in a Complete System of Stochastic Equations, *Annals of Mathematical Statistics*, pp. 570–582.

Balestra, P., & Nerlove, M. (1966). Pooling cross section time series data in the estimation of a dynamic model: The demand for natural gas. *Econometrica*, 34, 585–612.

Bellman, R. G. (1960). *Introduction to matrix analysis*. New York: McGraw-Hill.

Billingsley, P. (1968). *Convergence of probability measures*. New York: Wiley.

Billingsley, P. (1995). *Probability and measure* (3rd ed.). New York: Wiley.

Brockwell, P. J., & Davis, R. A. (1991). *Time series: Theory and methods* (2nd ed.). New York: Springer-Verlag.

Chow, Y. S., & Teicher, H. (1988). *Probability theory* (2nd ed.). New York: Springer-Verlag.

Dhrymes, P. J. (1969). Alternative asymptotic tests of significance and related aspects of 2SLS and 3SLS estimated parameters. *Review of Economic Studies*, 36, 213–226.

Dhrymes, P. J. (1970). *Econometrics: Statistical foundations and applications*. New York: Harper and Row; also (1974). New York: Springer-Verlag.

Dhrymes, P. J. (1973). Restricted and Unrestricted Reduced Forms: Asymptotic Distributions and Relative Efficiencies, *Econometrica*, vol. 41, pp. 119–134.

- Dhrymes, P. J. (1978). *Introductory economics*. New York: Springer-Verlag.
- Dhrymes, P.J. (1982) *Distributed Lags: Problems of Estimation and Formulation (corrected edition)*. Amsterdam: North Holland
- Dhrymes, P. J. (1989). *Topics in advanced econometrics: Probability foundations*. New York: Springer-Verlag.
- Dhrymes, P. J. (1994). *Topics in advanced econometrics: Volume II linear and nonlinear simultaneous equations*. New York: Springer-Verlag.
- Hadley, G. (1961). *Linear algebra*. Reading: Addison-Wesley.
- Kendall, M. G., & Stuart, A. (1963). *The advanced theory of statistics*. London: Charles Griffin.
- Kendall M. G., Stuart, A., & Ord, J. K. (1987). *Kendall's advanced theory of statistics*. New York: Oxford University Press.
- Kolassa, J. E. (1997). *Series approximation methods in statistics* (2nd ed.). New York: Springer-Verlag.
- Sims, C.A. (1980). Macroeconomics and Reality, *Econometrica*, vol. 48, pp.1-48.
- Shiryayev, A. N. (1984). *Probability*. New York: Springer-Verlag.
- Stout, W. F. (1974). *Almost sure convergence*. New York: Academic.
- Theil, H. (1953). *Estimation and Simultaneous Correlation in Complete Equation Systems, mimeograph*. The Hague: Central Plan Bureau.
- Theil, H. (1958). *Economic Forecasts and Policy*, Amsterdam: North Holland.

Index

A

Adjoint of matrix, 38
Aitken estimator, 321–323, 333, 358,
361

Algebra

of lag operators, 299
 σ - (see σ -algebra)

ARCH. *See* Auto, regressive
conditional
heteroskedasticity (ARCH)

Asymptotic

distributions, 326, 327, 334 (see
also Distribution, limiting)
expansions, 393–409

Auto

correlation function, 186, 302,
305
covariance function, 186, 187,
300, 301
covariance generating function
(AGF), 385–387
regressive conditional
heteroskedasticity (ARCH),
192, 194
regressive moving average
sequence (ARMA), 190–192
regressive sequence (AR),
188–190

B

Balanced panel, 335
Best linear predictor, 388–389,
391–392
Borel field, 394, 395
Block diagonal matrices, 43, 77, 133
Block triangular matrices, 43, 48
Block partitioned matrices, 42

C

Causal, 190, 191, 196, 298–300, 302,
304, 306, 308, 345, 347,
389
Causal and invertible, 389
Central limit theorem
of Liapounov, 262
of Lindeberg, 294, 296
of martingale difference, 296–297
Characteristic equation
of matrix, 179, 183
of vector difference equation, 353
Characteristic roots
of definite matrices, 83, 84,
88–90, 322
of idempotent matrices, 79
of matrix, 64, 76, 78, 83, 84,
88–90, 93, 94, 179, 183, 346
of a matrix in the metric of
another, 60
of orthogonal matrices, 68, 94
of semidefinite matrices, 83, 88,
94
of symmetric matrices, 71, 78
Characteristic function
of a matrix, 297
of a random variable, 298, 399,
403
Characteristic vectors
of a matrix, 80, 93, 101, 319
of a symmetric matrix, 71, 72,
74, 79, 101, 324, 362
Chi-square distribution, 316, 319
Choleski decomposition, 83. *See also*
Triangular decomposition
c-inverse, 96. *See also* Conditional
inverse of matrix

- Cofactors
 of matrix, 161, 165, 178
 complementary, 38
 expansion by, 35, 39
- Column space of matrix, 19, 20
- Convergence
 a.c. (almost certainly), 288
 a.s. (almost surely), 288
 complete, 289, 292
 in distribution, 289, 290
 in mean square (or quadratic mean), 189, 313
 in mean of order p , 288–289
 in probability, 226, 288, 290, 305, 313
 weak, 292
- Complex
 number, 1–5, 7, 173
 absolute value of a , 2
 argument of a , 3, 4
 modulus of a , 4
 vector, 4
- c -inverse, 96–107
- Conditional inverse of matrix, 96
- Cross section, 186, 335–337
 repeated, 336
- Cumulant(s), 401–408
- D**
- Degrees of freedom, 316, 319, 368
- Dependent variable, 171, 180, 188, 189, 195, 309, 310, 336, 339
- Definite matrices
 characteristic roots of, 83
 rank of, 17
 simultaneous decomposition of, 322
 square root of, 87, 94
 trace of, 30
- Determinant
 of definite matrix, 30
 of inverse of matrix, 39
 of matrix, 34, 38, 39, 57
 of partitioned matrix, 36, 42
 of transpose of matrix, 178
- Diagonal matrices, block, 68, 75, 77, 104, 133, 135
- Diagonalizable matrices, 79
- Difference equations
 general solution of, 172
 particular solutions of, 172
 homogeneous part of, 171, 172
 non-stochastic, 171, 176
 stability of, 172
 stochastic, 171, 172, 375
- Differentiation of
 determinants of matrices, 161
 inverse of matrices, 169
 quadratic forms, 153, 161
 symmetric matrices, 71
 trace of matrices, 157
- Dimension of
 matrix, 121–123, 152
 vector space, 8
- Distribution, limiting, 298, 307, 326, 328, 339, 340, 344, 356, 359–361, 366–368, 379, 380, 384, 389–393. *See also* Asymptotic
- E**
- Echelon matrices, 25, 26, 28
- Elementary matrices, 24, 26, 140
- Elementary transformation, 23, 25
- Endogenous variables, 181, 351–354, 356, 360, 369, 375, 378. *See also* Jointly dependent variables
 lagged, 181, 352, 360
- Ergodicity, 285–308. *See also* Mixing
- Ergodic, 301–304
 theorem, 302

- Estimator
 maximum likelihood, 370, 375
 minimum contrast, 372
 systemwide, 358–362
 three stage least squares (3SLS),
 356, 358–361, 364,
 378–380
 two stage least squares (2SLS),
 357, 358, 361, 365, 366,
 369, 378
- Exogenous variable, 181, 183, 184,
 191, 351, 352, 360, 378,
 381, 387
- Explanatory variable, 95, 183, 189,
 309, 310, 313, 326, 332,
 336, 337, 340, 356, 363,
 389
- F**
F distribution, 289
- Filter
 linear time invariant, 187
 time invariant, 187
- Function
 measurable, 195, 207–216,
 221–232, 234, 252–255, 257,
 261, 273, 275, 291, 301,
 302, 395
 characteristic, 232, 252, 297, 398,
 399, 401, 403
 of a random variable, 398
 moment generating, 314, 399,
 401, 408
 of a random variable, 399
 of a normal random variable,
 399
- G**
 Gauss-Markov theorem, 312, 321,
 323, 340, 342
 Gaussian time series, 300
- General linear model (GLM), 95,
 188, 298, 309–334, 336,
 338–341, 344, 349, 356, 358,
 361, 384, 388, 389
- General Linear Structural
 Econometric Model
 (GLSEM), 171–196, 298,
 351–392
- Generalized inverse. *See also*
 g-inverse
 of diagonal matrices, 101
 of idempotent matrices, 100, 102
 of symmetric matrices, 101, 102
 properties of, 100
 g-inverse, 96–107, 110, 114, 117, 118,
 137, 138. *See also*
 Generalized inverse
- Gram-Schmidt orthogonalization, 70,
 71, 331
- H**
 Hermite form of matrix
 canonical, 27
 in definition of conditional
 inverse, 96
- I**
 Idempotent matrices
 characteristic roots of, 79
 trace of, 79
- Identification in GLSEM, 183
- Independent variable, 339
- Inverse of a matrix
 characteristic roots of, 62
 computation of, 39–40
 conditional (*see* c-inverse)
 determinant of, 39
 generalized, 99–107, 110, 137,
 138
 least squares, 98
- Invertible matrices. *See* Nonsingular
 matrices

J

Jointly dependent variables, 363, 378.
See also Endogenous variables

K

Kronecker product of matrices,
 51–54, 67–68, 122, 148
 Kullback information, 372, 373

L

Lag operator, 171–196
 Lagrange multiplier, 50, 365–368
 Laplace expansion, 35, 36, 43
 Linearly independent set of vectors,
 6, 9, 66, 111
 Linear system of equations
 consistent, 109
 inconsistent
 least squares approximate
 solution (LS solution), 113
 minimum norm least squares
 approximate solution (MNLS
 solution), 115
 Law of Large Numbers (LLN)
 strong, 292, 371
 weak, 292

M

Martingale
 difference, 296–297, 306, 308
 sub, 296
 super, 296
 Matrix(-ces)
 addition, 15, 41, 42, 51
 conformable, 315, 346
 definite, 81, 82, 86–90, 94, 310,
 318, 323, 356, 376
 determinant of, 34, 38, 39, 57
 expansion by cofactors, 35, 39

diagonal, 14, 18, 43, 55, 58, 65,
 74, 76, 77, 79, 84, 89, 93,
 101, 104, 133, 135, 324
 block, 52, 134
 diagonalizable, 79, 177
 echelon, 25, 26, 28
 elimination, 131, 135–137, 139
 Hermite form of, 27
 identity, 14, 23, 24, 88, 96, 140,
 141, 146, 354
 idempotent, 15, 79, 80, 96, 311,
 319, 342
 inverse of, 17–23, 25, 39, 117,
 169–170
 linearly restricted, 124–138
 Jacobian, 149
 multiplication, 15, 16, 41, 52
 nilpotent, 134
 index of nilpotency, 134
 nonsingular, 18, 22, 23, 27, 28,
 45, 46, 48, 52, 67, 87–90,
 96, 97, 104, 138, 322, 355,
 358, 374, 380
 null, 15, 56, 87, 100
 orthogonal, 68–75, 77, 78, 84, 89,
 93, 94, 101, 126, 140, 146,
 147, 319, 324, 342, 376
 partitioned, 36, 40–53, 317, 330
 permutation, 139–147, 347
 rank factorization of, 29, 92
 restoration, 131, 135–137, 139,
 163, 164
 selection, 131, 133, 163, 319, 354,
 363, 374, 391
 semidefinite, 90, 261, 390
 similar, 55
 simultaneous decomposition of,
 90, 322
 singular value decomposition of,
 92

- square, 14, 15, 17–19, 27, 30–39, 42, 44, 52, 54, 55, 58–61, 63–65, 68, 79–81, 96, 97, 108, 126, 140, 147, 148, 158, 161, 346, 373
- square root of, 87, 94
- symmetric, 49, 71–78, 83–85, 92, 96, 99, 101, 107, 126, 129–132, 136, 155, 163, 311, 324, 333, 362
- simultaneous diagonalization of, 75
- Toeplitz, 139
- Toeplitz, 126–128, 135, 136, 139, 307, 391
- trace, 157
- transpose of, 178
- triangular, 14, 43, 48, 82, 83, 111, 132–134
- block, 43, 44, 48
- lower, 14, 82, 83, 132–135, 139
- upper, 14, 83, 111, 133, 134
- Vandermonde, 56
- vectorization of, 10, 119–148
- Minor of matrix, 36
- complementary, 36
- Mixing, 303, 304. *See also* Ergodicity
- Moment generating function (MGF), 314, 384, 399–409
- Moving average (MA), 178, 187, 188, 190–192, 298, 308, 384, 386, 389, 390
- of infinite extent, (MA(∞)), 191
- Mutually independent, 188, 194, 280, 282, 287, 316–318, 406
- N**
- Negative definite matrices, 300
- Negative semidefinite matrices, 261. *See also* Semidefinite matrices
- Nonsingular matrices, 23, 45, 46, 52, 67, 104
- Nullity of matrix, 79
- Null matrices, 15, 56, 87, 100
- Null space of matrix, 20, 79
- O**
- Operator
- identity, 131, 173
- lag, 171–196, 298, 299
- polynomials, 173, 177, 178
- vec, 124, 125, 139–148, 376
- Orders of magnitude (o , O), 402–403
- in probability (o_p , O_p), 402
- Order of matrix, 101
- Ordinary least squares (OLS), 305, 306, 312–323, 325, 326, 329–334, 348, 349, 354–359, 361, 376
- inconsistency of, in GLSEM, 357
- Orthogonal matrices, 68–71, 77, 94, 126, 146
- Orthogonal vectors, 70, 71
- Orthonormal vectors, 70, 74
- P**
- Particular solution of difference equations, 172
- Partitioned matrices
- determinant of, 36
- inverse of, 44
- Positive definite matrix, 86–88, 90, 310, 318, 356, 376. *See also* Definite matrices
- Positive semidefinite matrices, 50, 90, 114, 187, 261, 323, 390
- Pooled samples, 336
- Predetermined variable, 181, 351–354, 363, 381, 383

- Probability
 measure, 209–216, 223, 231,
 242–250, 264, 266, 285, 286,
 394, 395
 Space, 211, 213, 221, 223, 243,
 250, 255, 263, 267, 269,
 270, 273, 274, 277, 278,
 280–283, 285–288, 290, 291,
 295, 296, 302, 303, 305,
 339, 394–396, 406
 Pseudo-inverse (pseudoinverse), 95,
 99, 100, 108–112, 117
- Q**
 Quadratic forms, 81, 153, 154, 156,
 161,
- R**
 Rank
 of definite matrix, 323
 of matrix, 101, 139, 319, 323,
 342, 366, 367
 of semidefinite matrix, 50, 323
 Rank factorization of matrix, 23–29,
 92, 99
 Reduced form system, 357
 Regressand, 188, 309
 Regressor, 189, 309
 orthogonal, 328–332
 Rematricization, 157
 Restricted subspace
 for diagonal matrices, 133
 for Toeplitz matrices, 139
 Row space of matrix, 19, 20, 112
- S**
 Scalar polynomials, 178, 179, 191
 Seemingly unrelated regressions
 (SUR), 332, 334, 336, 341,
 344
- Semidefinite matrices
 characteristic roots of, 83, 88,
 90
 determinant of, 324
 rank of, 86, 356
 square root of, 87, 94
 σ -algebra, 205–207, 211, 230,
 235–242, 247–249, 251–253,
 256, 264, 266, 273, 274, 278,
 279, 281, 283, 285, 286,
 296, 303, 305, 307, 360,
 394–396
- Similar matrices, 55
- Singular
 multivariate normal distribution,
 314, 315
 value decomposition, 92, 148
- Stability
 of GLSEM, 183
 with identities, 183
 of vector difference equations,
 180
- Stationary
 covariance (or weakly), 186, 300,
 345
 strictly, 186, 300, 340, 345, 348
- Stochastic
 basis, 296, 306
 filtration, 296
 processes, 185, 280–282, 335, 340
 sequences, 185–188, 190, 192,
 280, 296, 340, 384, 385,
 387, 389, 390
- Structural
 equations, 353–357, 363, 364,
 368, 369
 form, 353, 354, 362, 378, 381
 parameters, 354–357, 379
- Symmetric matrices
 characteristic roots of, 362

- characteristic vectors of, 72, 74, 362
 - simultaneous diagonalization of, 75
- T**
- t distribution, 316
 - Three stage least squares (3SLS), 356–362, 364, 365, 378–383
 - efficiency of, relative to 2SLS, 361
 - Trace
 - as sum of characteristic roots, 54
 - function of vectorized matrices, 122
 - of definite matrices, 35
 - of idempotent matrices, 79
 - of product of matrices, 119
 - Transpose of matrix, 178
 - determinant of, 178
 - Triangular decomposition, 82, 83, 377. *See also* Choleski decomposition
 - Triangular matrix, 14, 43, 48, 82, 83, 111, 132, 134
 - block, 43, 48
 - Two stage least squares (2SLS), 356–362, 364–366, 368–370, 378–381
- V**
- VAR model, 374–377. *See also* Vector(s), autoregressive model (VAR)
 - Vandermonde matrix, 56
 - Vector(s)
 - addition, 2
 - autoregressive model (VAR), 374–377
 - characteristic of a matrix, complex, 1–5
 - difference equation, 176
 - inner product of, 6
 - linearly dependent set of, 6, 74
 - linearly independent set of, 6, 9, 66, 111
 - multiplication by scalar, 2, 7, 8
 - orthogonal set, 70, 71
 - orthonormal set, 8, 69–71, 74
 - space
 - basis of, 8–9
 - dimension of, 8–9
 - subspace of a, 9–11
 - of lag operators, 173
 - Vectorized matrices, 122
- W**
- White noise, 128, 193, 298