

# Table of Notation

Notation	Section	Definition
$D$	1.1	The unit disc in the plane
$\Omega$	1.1	A domain
$dV$	1.1	The volume element
$H^\infty(\Omega)$	1.1	Bounded holomorphic functions on $\Omega$
$A(\Omega)$	1.1	Functions holomorphic on $\Omega$ , continuous on the closure
$A^2(\Omega)$	1.1	Square integrable holomorphic functions on $\Omega$ (the Bergman space)
$C^k$	1.1	The $k$ -times continuously differentiable functions
$\ \cdot\ _{A^2(\Omega)}$	1.1	The Bergman norm
$K(z, \zeta) = K_\Omega(z, \zeta)$	1.1	The Bergman kernel
$\{\phi_j\}$	1.1	A complete orthonormal basis
$P$	1.1	The Bergman projection
$J_{\mathbb{C}}(f)$	1.1	The complex Jacobian matrix of $f$
$J_{\mathbb{R}}(f)$	1.1	The real Jacobian matrix of $f$
$g_{i,j}(z)$	1.1	The Bergman metric
$ \xi _{B,z}$	1.1	The Bergman metric
$\ell(\gamma)$	1.1	The Bergman length of $\gamma$
$d_\Omega(z, w)$	1.1	The Bergman distance of $z$ to $w$
$\Gamma(z)$	1.1	Euler's gamma function
$\Gamma(\zeta, z)$	1.1	Fundamental solution for the Laplacian
$dA$	1.1	The area element
$G(\zeta, z)$	1.1	The Green's function
$S(z, \zeta)$	1.2	The Szegő kernel
$\eta(z)$	1.2	The Leray form
$\bar{\partial}$	1.2	The Cauchy–Riemann operator
$S(z, \zeta)$	1.2	The Szegő projection
$\mathcal{P}(z, \zeta)$	1.2	The Poisson–Szegő kernel
$\delta_z$	1.2	The Dirac delta mass
$N$	1.2	The $\bar{\partial}$ -Neumann operator

Notation	Section	Definition
$H^2(\Omega)$	1.2	The square-integrable Hardy space
$\mathcal{H}$	1.3	A Hilbert space with reproducing kernel
$K(x, y)$	1.3	A reproducing kernel
$\mathbf{h}^2(D)$	1.6	The square-integrable harmonic functions
$\mathbb{Z}^+$	1.7	The nonnegative integers
$A$	1.8	An annulus
$d\sigma$	1.9	Area measure on the boundary of the ball
$W^s$	1.11	The Sobolev space
$\mathcal{U}_\epsilon^k(\Omega_0)$	2.1	Domains neighboring $\Omega_0$
$\sim$	2.1	Is biholomorphic to
$\text{Aut}(\Omega)$	2.1	The automorphism group of $\Omega$
$\bar{\partial}^*$	2.1	The adjoint of $\bar{\partial}$
Condition $R$	2.1	A regularity condition for the Bergman projection
$\bigwedge^{0,j}$	2.1	Differential forms
$WH^j(\Omega)$	2.1	$W^j(\Omega) \cap$ (holomorphic functions)
$WH^\infty(\Omega)$	2.1	$W^\infty \cap$ (holomorphic functions)
$W_0^j(\Omega)$	2.1	The $W^j$ closure of $C_c^\infty(\Omega)$
$\partial/\partial\nu_P$	2.1	The outward normal derivative at $P$
$b_j^i$	3.1	Bergman representative coordinates
$\bar{\partial}_b$	3.3	The boundary Cauchy–Riemann operator
$\mathcal{B}(z, \zeta)$	3.3	The Poisson–Bergman kernel
$\Lambda f(w, x)$	3.3	The Berezin transform
$\beta_2(z, r)$	3.3	Nonisotropic ball
$\mathcal{M}f(z)$	3.3	Maximal operator
$\mathcal{A}_\alpha(z)$	3.3	Admissible approach region
$\mathcal{L}$	3.5	Laplace–Beltrami operator
$\rho(z, \zeta)$	3.6	A nonisotropic metric on $B$
$\mathcal{P}_k$	4.1	All homogeneous polynomials of degree $k$
$\mathcal{A}_k$	4.1	Kernel of the Laplacian in $\mathcal{P}_k$
$\mathcal{B}_k$	4.1	Image of the Laplacian in $\mathcal{P}_k$
$\mathcal{H}_k$	4.1	The spherical harmonics
$Z_{x'}^{(k)}$	4.2	Zonal harmonic
$P(x, t')$	4.2	The Poisson kernel
$P_k^\lambda(t)$	4.2	Gegenbauer polynomial
$\mathcal{H}^{p,q}$	4.3	Harmonic polynomials of bidegree $(p, q)$
$Q(f, g)$	4.4	Cauchy–Riemann inner product
$[L, M]$	5.3	First-order commutator
$\nu(\phi)$	5.3	The order of vanishing of $\phi$
$\mathcal{H}$	5.7	Hausdorff distance on domains
$A^2(M)$	5.9	The Bergman space on a manifold
$ds^2$	5.9	The Bergman metric on a manifold
$\mathcal{W}$	6.1	The Diederich–Fornæss worm domain
$\eta$	6.1	Function used to construct the worm
$\mathcal{A}$	6.2	Singular annulus in the worm
$B_\Omega^K(q, r)$	7.2	Kobayashi distance ball
$S_\Omega(p; \xi)$	7.3	Holomorphic sectional curvature

# Bibliography

- [ADA] R. Adams, *Sobolev Spaces*, Academic Press, 1975.
- [AFR] P. Ahern, M. Flores, and W. Rudin, An invariant volume-mean-value property, *Jour. Functional Analysis* 11(1993), 380–397.
- [AHL] L. Ahlfors, *Complex Analysis*, 3<sup>rd</sup> ed., McGraw-Hill, New York, 1979.
- [ARO] N. Aronszajn, Theory of reproducing kernels, *Trans. Am. Math. Soc.* 68(1950), 337–404.
- [BEG] T. N. Bailey, M. G. Eastwood, and C. R. Graham, Invariant theory for conformal and CR geometry, *Annals of Math.* 139(1994), 491–552.
- [BAR1] D. Barrett, Irregularity of the Bergman projection on a smooth bounded domain in  $\mathbb{C}^2$ , *Annals of Math.* 119(1984), 431–436.
- [BAR2] D. Barrett, The behavior of the Bergman projection on the Diederich–Fornæss worm, *Acta Math.*, 168(1992), 1–10.
- [BAR3] D. Barrett, Regularity of the Bergman projection and local geometry of domains, *Duke Math. Jour.* 53(1986), 333–343.
- [BAR4] D. Barrett, Behavior of the Bergman projection on the Diederich–Fornæss worm, *Acta Math.* 168(1992), 1–10.
- [BEDF] E. Bedford and P. Federbush, Pluriharmonic boundary values, *Tohoku Math. Jour.* 26(1974), 505–511.
- [BEF1] E. Bedford and J. E. Fornæss, A construction of peak functions on weakly pseudoconvex domains, *Ann. Math.* 107(1978), 555–568.
- [BEF2] E. Bedford and J. E. Fornæss, Counterexamples to regularity for the complex Monge–Ampère equation, *Invent. Math.* 50 (1978/79), 129–134.
- [BEL1] S. Bell, Biholomorphic mappings and the  $\bar{\partial}$  problem, *Ann. Math.*, 114(1981), 103–113.
- [BEL2] S. Bell, Local boundary behavior of proper holomorphic mappings, *Proc. Sympos. Pure Math*, vol. 41, American Math. Soc., Providence R.I., 1984, 1–7.
- [BEL3] S. Bell, Differentiability of the Bergman kernel and pseudo-local estimates, *Math. Z.* 192(1986), 467–472.
- [BEB] S. Bell and H. Boas, Regularity of the Bergman projection in weakly pseudoconvex domains, *Math. Annalen* 257(1981), 23–30.
- [BEC] S. Bell and D. Catlin, Proper holomorphic mappings extend smoothly to the boundary, *Bull. Amer. Math. Soc.* (N.S.) 7(1982), 269–272.
- [BEK] S. Bell and S. G. Krantz, Smoothness to the boundary of conformal maps, *Rocky Mt. Jour. Math.* 17(1987), 23–40.
- [BELL] S. Bell and E. Ligocka, A simplification and extension of Fefferman’s theorem on biholomorphic mappings, *Invent. Math.* 57(1980), 283–289.

- [BERE] F. A. Berezin, Quantization in complex symmetric spaces, *Math. USSR Izvestia* 9(1975), 341–379.
- [BER1] S. Bergman, Über die Entwicklung der harmonischen Funktionen der Ebene und des Raumes nach Orthogonal funktionen, *Math. Annalen* 86(1922), 238–271.
- [BER2] S. Bergman, *The Kernel Function and Conformal Mapping*, Am. Math. Soc., Providence, RI, 1970.
- [BES] S. Bergman and M. Schiffer, *Kernel Functions and Elliptic Differential Equations in Mathematical Physics*, Academic Press, New York, 1953.
- [BEC] B. Berndtsson, P. Charpentier, A Sobolev mapping property of the Bergman kernel, *Math. Z.* **235** (2000), 1–10.
- [BERS] L. Bers, *Introduction to Several Complex Variables*, New York Univ. Press, New York, 1964.
- [BLK] B. E. Blank and S. G. Krantz, *Calculus*, Key Press, Emeryville, CA, 2006.
- [BLP] Z. Blocki and P. Pflug, Hyperconvexity and Bergman completeness, *Nagoya Math. J.* 151(1998), 221–225.
- [BLG] T. Bloom and I. Graham, A geometric characterization of points of type  $m$  on real submanifolds of  $\mathbb{C}^n$ , *J. Diff. Geom.* 12(1977), 171–182.
- [BOA1] H. Boas, Counterexample to the Lu Qi-Keng conjecture, *Proc. Am. Math. Soc.* 97(1986), 374–375.
- [BOA2] H. Boas, The Lu Qi-Keng conjecture fails generically, *Proc. Amer. Math. Soc.* 124(1996), 2021–2027.
- [BOS1] H. Boas and E. Straube, Sobolev estimates for the  $\bar{\partial}$ -Neumann operator on domains in  $\mathbb{C}^n$  admitting a defining function that is plurisubharmonic on the boundary, *Math. Z.* **206** (1991), 81–88.
- [BOS2] H. Boas and E. Straube, Equivalence of regularity for the Bergman projection and the  $\bar{\partial}$ -Neumann operator, *Manuscripta Math.* 67(1990), 25–33.
- [BKP] H. Boas, S. G. Krantz, and M. M. Peloso, unpublished.
- [BOC1] S. Bochner, Orthogonal systems of analytic functions, *Math. Z.* 14(1922), 180–207.
- [BOU] L. Boutet de Monvel, Le noyau de Bergman en dimension 2, *Séminaire sur les Équations aux Dérivées Partielles* 1987–1988, Exp. no. XXII, École Polytechnique Palaiseau, 1988, p. 13.
- [BOS] L. Boutet de Monvel and J. Sjöstrand, Sur la singularité des noyaux de Bergman et Szegő, *Soc. Mat. de France Asterisque* 34–35(1976), 123–164.
- [BRE] H. J. Bremermann, Holomorphic continuation of the kernel function and the Bergman metric in several complex variables, *Lectures on Functions of a Complex Variable*, Michigan, 1955, 349–383.
- [BUN] L. Bungart, Holomorphic functions with values in locally convex spaces and applications to integral formulas, *Trans. Am. Math. Soc.* 111(1964), 317–344.
- [BSW] D. Burns, S. Shnider, R. O. Wells, On deformations of strictly pseudoconvex domains, *Invent. Math.* 46(1978), 237–253.
- [CKNS] L. Caffarelli, J. J. Kohn, L. Nirenberg, and J. Spruck, The Dirichlet problem for nonlinear second-order elliptic equations. II. Complex Monge–Ampère, and uniformly elliptic, equations, *Comm. Pure Appl. Math.* 38(1985), 209–252.
- [CAR] L. Carleson, *Selected Problems on Exceptional Sets*, Van Nostrand, Princeton, NJ, 1967.
- [CCP] G. Carrier, M. Crook, and C. Pearson, *Functions of a Complex Variable*, McGraw-Hill, New York, 1966.
- [CAT1] D. Catlin, Necessary conditions for subellipticity of the  $\bar{\partial}$ -Neumann problem, *Ann. Math.* 117(1983), 147–172.
- [CAT2] D. Catlin, Subelliptic estimates for the  $\bar{\partial}$ -Neumann problem, *Ann. Math.* 126(1987), 131–192.
- [CAT3] D. Catlin, Boundary behavior of holomorphic functions on pseudoconvex domains, *J. Differential Geom.* 15(1980), 605–625.

- [CNS] D.-C. Chang, A. Nagel, and E. M. Stein, Estimates for the  $\bar{\partial}$ -Neumann problem in pseudoconvex domains of finite type in  $\mathbb{C}^2$ , *Acta Math.* 169(1992), 153–228.
- [CHE] S.-C. Chen, A counterexample to the differentiability of the Bergman kernel function, *Proc. AMS* 124(1996), 1807–1810.
- [CHF] B.-Y. Chen and S. Fu, Comparison of the Bergman and Szegő kernels, *Advances in Math.* 228(2011), 2366–2384.
- [CHS] S.-C. Chen and M. C. Shaw, *Partial Differential Equations in Several Complex Variables*, AMS/IP Studies in Advanced Mathematics, 19. American Mathematical Society, Providence, RI; International Press, Boston, MA, 2001.
- [CHENG] S.-Y. Cheng, Open problems, *Conference on Nonlinear Problems in Geometry Held in Katata*, September, 1979, Tohoku University, Dept. of Mathematics, Sendai, 1979, p. 2.
- [CHM] S. S. Chern and J. Moser, Real hypersurfaces in complex manifolds, *Acta Math.* 133(1974), 219–271.
- [CHR1] M. Christ, Global  $C^\infty$  irregularity of the  $\bar{\partial}$ -Neumann problem for worm domains, *J. Amer. Math. Soc.* 9(1996), 1171–1185.
- [CHR2] M. Christ, Remarks on global irregularity in the  $\bar{\partial}$ -Neumann problem, *Several complex variables* (Berkeley, CA, 1995–1996), 161–198, Math. Sci. Res. Inst. Publ. 37, Cambridge Univ. Press, Cambridge, 1999.
- [COL] E. Coddington and N. Levinson, *Theory of Ordinary Differential Equations*, McGraw-Hill, New York, 1955.
- [COW] R. R. Coifman and G. Weiss, *Analyse Harmonique Non-Commutative sur Certains Espaces Homogenes*, Springer Lecture Notes vol. 242, Springer Verlag, Berlin, 1971.
- [COH] R. Courant and D. Hilbert, *Methods of Mathematical Physics*, 2nd ed., Interscience, New York, 1966.
- [DAN1] J. P. D’Angelo, Real hypersurfaces, orders of contact, and applications, *Annals of Math.* 115(1982), 615–637.
- [DAN2] J. P. D’Angelo, Intersection theory and the  $\bar{\partial}$ -Neumann problem, *Proc. Symp. Pure Math.* 41(1984), 51–58.
- [DAN3] J. P. D’Angelo, Finite type conditions for real hypersurfaces in  $\mathbb{C}^n$ , in *Complex Analysis Seminar*, Springer Lecture Notes vol. 1268, Springer Verlag, 1987, 83–102.
- [DAN4] J. P. D’Angelo, *Several Complex Variables and the Geometry of Real Hypersurfaces*, CRC Press, Boca Raton, FL, 1993.
- [DIE1] K. Diederich, Das Randverhalten der Bergmanschen Kernfunktion und Metrik in streng pseudo-konvexen Gebieten, *Math. Ann.* 187(1970), 9–36.
- [DIE2] K. Diederich, Über die 1. and 2. Ableitungen der Bergmanschen Kernfunktion und ihr Randverhalten, *Math. Ann.* 203(1973), 129–170.
- [DIF1] K. Diederich and J. E. Fornæss, Pseudoconvex domains: An example with nontrivial Nebenhülle, *Math. Ann.* 225(1977), 275–292.
- [DIF2] K. Diederich and J. E. Fornæss, Pseudoconvex domains with real-analytic boundary, *Annals of Math.* 107(1978), 371–384.
- [DIF3] K. Diederich and J. E. Fornæss, Pseudoconvex domains: Bounded strictly plurisubharmonic exhaustion functions, *Invent. Math.* 39 (1977), 129–141.
- [DIF4] K. Diederich and J. E. Fornæss, Smooth extendability of proper holomorphic mappings, *Bull. Amer. Math. Soc. (N.S.)* 7(1982), 264–268.
- [EBI1] Ebin, D. G., On the space of Riemannian metrics, *Bull. Amer. Math. Soc.* 74(1968), 1001–1003.
- [EBI2] Ebin, D. G., The manifold of Riemannian metrics, 1970 Global Analysis (*Proc. Sympos. Pure Math.*, Vol. XV, Berkeley, Calif., 1968), pp. 11–40, Amer. Math. Soc., Providence, R.I.
- [ENG1] M. Engliš, Functions invariant under the Berezin transform, *J. Funct. Anal.* 121(1994), 233–254.

- [ENG2] M. Engliš, Asymptotics of the Berezin transform and quantization on planar domains, *Duke Math. J.* 79(1995), 57–76.
- [EPS] B. Epstein, *Orthogonal Families of Functions*, Macmillan, New York, 1965.
- [ERD] A. Erdelyi, et al, *Higher Transcendental Functions*, McGraw-Hill, New York, 1953.
- [FED] H. Federer, *Geometric Measure Theory*, Springer-Verlag, New York, 1969.
- [FEF1] C. Fefferman, The Bergman kernel and biholomorphic mappings of pseudoconvex domains, *Invent. Math.* 26(1974), 1–65.
- [FEF2] C. Fefferman, Parabolic invariant theory in complex analysis, *Adv. Math.* 31(1979), 131–262.
- [FOK] G. B. Folland and J. J. Kohn, *The Neumann Problem for the Cauchy-Riemann Complex*, Princeton University Press, Princeton, NJ, 1972.
- [FOL] G. B. Folland, Spherical harmonic expansion of the Poisson–Szegő kernel for the ball, *Proc. Am. Math. Soc.* 47(1975), 401–408.
- [FOM] J. E. Fornæss and J. McNeal, A construction of peak functions on some finite type domains. *Amer. J. Math.* 116(1994), no. 3, 737–755.
- [FOR] F. Forstneric, An elementary proof of Fefferman’s theorem, *Expositiones Math.*, 10(1992), 136–149.
- [FRI] B. Fridman, A universal exhausting domain, *Proc. Am. Math. Soc.* 98(1986), 267–270.
- [FUW] S. Fu and B. Wong, On strictly pseudoconvex domains with Kähler–Einstein Bergman metrics, *Math. Res. Letters* 4(1997), 697–703.
- [GAM] T. Gamelin, *Uniform Algebras*, Prentice-Hall, Englewood Cliffs, NJ, 1969.
- [GAS] T. Gamelin and N. Sibony, Subharmonicity for uniform algebras. *J. Funct. Anal.* 35 (1980), 64–108.
- [GAR] J. B. Garnett, *Analytic Capacity and Measure*, Lecture Notes in Math. 297, Springer, New York, 1972.
- [GARA] P. R. Garabedian, A Green’s function in the theory of functions of several complex variables, *Ann. of Math.* 55(1952), 19–33.
- [GAR] J. Garnett, *Bounded Analytic Functions*, Academic Press, New York, 1981.
- [GLE] A. Gleason, The abstract theorem of Cauchy–Weil, *Pac. J. Math.* 12(1962), 511–525.
- [GOL] Goluzin, *Geometric Theory of Functions of a Complex Variable*, American Mathematical Society, Providence, 1969.
- [GRA1] C. R. Graham, The Dirichlet problem for the Bergman Laplacian I, *Comm. Partial Diff. Eqs.* 8(1983), 433–476.
- [GRA2] C. R. Graham, The Dirichlet problem for the Bergman Laplacian II, *Comm. Partial Diff. Eqs.* 8(1983), 563–641.
- [GRA3] C. R. Graham, Scalar boundary invariants and the Bergman kernel, *Complex analysis, II* (College Park, Md., 1985–86), 108–135, Lecture Notes in Math. 1276, Springer, Berlin, 1987.
- [GRL] C. R. Graham and J. M. Lee, Smooth solutions of degenerate Laplacians on strictly pseudoconvex domains, *Duke Jour. Math.* 57(1988), 697–720.
- [GRA] I. Graham, Boundary behavior of the Carathéodory and Kobayashi metrics on strongly pseudoconvex domains in  $\mathbb{C}^n$  with smooth boundary, *Trans. Am. Math. Soc.* 207(1975), 219–240.
- [GRL] H. Grauert and I. Lieb, Das Ramirezsche Integral und die Gleichung  $\bar{\partial}u = \alpha$  im Bereich der beschränkten Formen, *Rice University Studies* 56(1970), 29–50.
- [GKK] R. E. Greene, K.-T. Kim, and S. G. Krantz, *The Geometry of Complex Domains*, Birkhäuser Publishing, Boston, MA, 2011.
- [GRK1] R. E. Greene and S. G. Krantz, Stability properties of the Bergman kernel and curvature properties of bounded domains, *Recent Progress in Several Complex Variables*, Princeton University Press, Princeton, 1982.
- [GRK2] R. E. Greene and S. G. Krantz, Deformation of complex structures, estimates for the  $\bar{\partial}$  equation, and stability of the Bergman kernel, *Adv. Math.* 43(1982), 1–86.

- [GRK3] R. E. Greene and S. G. Krantz, The automorphism groups of strongly pseudoconvex domains, *Math. Annalen* 261(1982), 425–446.
- [GRK4] R. E. Greene and S. G. Krantz, The stability of the Bergman kernel and the geometry of the Bergman metric, *Bull. Am. Math. Soc.* 4(1981), 111–115.
- [GRK5] R. E. Greene and S. G. Krantz, Stability of the Carathéodory and Kobayashi metrics and applications to biholomorphic mappings, *Proc. Symp. in Pure Math.*, Vol. 41 (1984), 77–93.
- [GRK6] R. E. Greene and S. G. Krantz, Normal families and the semicontinuity of isometry and automorphism groups, *Math. Zeitschrift* 190(1985), 455–467.
- [GRK7] R. E. Greene and S. G. Krantz, Characterizations of certain weakly pseudo-convex domains with non-compact automorphism groups, in *Complex Analysis Seminar*, Springer Lecture Notes 1268(1987), 121–157.
- [GRK8] R. E. Greene and S. G. Krantz, Characterization of complex manifolds by the isotropy subgroups of their automorphism groups, *Indiana Univ. Math. J.* 34(1985), 865–879.
- [GRK9] R. E. Greene and S. G. Krantz, Biholomorphic self-maps of domains, *Complex Analysis II* (C. Berenstein, ed.), Springer Lecture Notes, vol. 1276, 1987, 136–207.
- [GRK10] R. E. Greene and S. G. Krantz, Techniques for Studying the automorphism Groups of Weakly Pseudoconvex Domains, *Several Complex Variables* (Stockholm, 1987/1988), 389–410, Math. Notes, 38, Princeton Univ. Press, Princeton, NJ, 1993.
- [GRK11] R. E. Greene and S. G. Krantz, Invariants of Bergman geometry and results concerning the automorphism groups of domains in  $\mathbb{C}^n$ , Proceedings of the 1989 Conference in Cetraro (D. Struppa, ed.), to appear.
- [GRK12] R. E. Greene and S. G. Krantz, *Function Theory of One Complex Variable*, 3<sup>rd</sup> ed., American Mathematical Society, Providence, RI, 2006.
- [HAP1] R. Harvey and J. Polking, Fundamental solutions in complex analysis. I. The Cauchy-Riemann operator, *Duke Math. J.* 46(1979), 253–300.
- [HAP2] R. Harvey and J. Polking, Fundamental solutions in complex analysis. II. The induced Cauchy-Riemann operator, *Duke Math. J.* 46(1979), 301–340.
- [HEL] S. Helgason, *Differential Geometry and Symmetric Spaces*, Academic Press, New York, 1962.
- [HEN] G. M. Henkin, Integral representations of functions holomorphic in strictly pseudoconvex domains and some applications, *Mat. Sb.* 78(120)(1969), 611–632.
- [HIL] E. Hille, *Analytic Function Theory*, 2<sup>nd</sup> ed., Ginn and Co., Boston, 1973.
- [HIR1] K. Hirachi, The second variation of the Bergman kernel of ellipsoids, *Osaka J. Math.* 30(1993), 457–473.
- [HIR2] K. Hirachi, Scalar pseudo-Hermitian invariants and the Szegő kernel on three-dimensional CR manifolds, *Complex Geometry* (Osaka, 1990), Lecture Notes Pure Appl. Math., v. 143, Marcel Dekker, New York, 1993, 67–76.
- [HIR3] K. Hirachi, Construction of boundary invariants and the logarithmic singularity in the Bergman kernel, *Annals of Math.* 151(2000), 151–190.
- [HIR] M. Hirsch, *Differential Topology*, Springer-Verlag, New York, 1976.
- [HOR1] L. Hörmander,  $L^2$  estimates and existence theorems for the  $\bar{\partial}$  operator, *Acta Math.* 113(1965), 89–152.
- [HOR2] L. Hörmander, *Linear Partial Differential Operators*, Springer-Verlag, New York, 1963.
- [HOR3] L. Hörmander, Pseudo-differential operators and non-elliptic boundary problems, *Ann. Math.* 83(1966), 129–209.
- [HOR5] L. Hörmander, “Fourier integral operators,” *The Analysis of Linear Partial Differential Operators IV*, Reprint of the 1994 ed., Springer, Berlin, Heidelberg, New York, 2009.
- [HUA] L. K. Hua, *Harmonic Analysis of Functions of Several Complex Variables in the Classical Domains*, American Mathematical Society, Providence, 1963.
- [ISK] A. Isaev and S. G. Krantz, Domains with non-compact automorphism group: A Survey, *Advances in Math.* 146 (1999), 1–38.

- [JAK] S. Jakobsson, Weighted Bergman kernels and biharmonic Green functions, Ph.D. thesis, Lunds Universitet, 2000, 134 pages.
- [KAT] Y. Katznelson, *Introduction to Harmonic Analysis*, John Wiley and Sons, New York, 1968.
- [KEL] O. Kellogg, *Foundations of Potential Theory*, Dover, New York, 1953.
- [KER1] N. Kerzman, Hölder and  $L^p$  estimates for solutions of  $\bar{\partial}u = f$  on strongly pseudoconvex domains, *Comm. Pure Appl. Math.* 24(1971), 301–380.
- [KER2] N. Kerzman, The Bergman kernel function. Differentiability at the boundary, *Math. Ann.* 195(1972), 149–158.
- [KER3] N. Kerzman, A Monge–Ampère equation in complex analysis. Several Complex Variables (Proc. Sympos. Pure Math., Vol. XXX, Part 1, Williams Coll., Williamstown, Mass., 1975), pp. 161–167. Amer. Math. Soc., Providence, R.I., 1977.
- [KIMYW] Y. W. Kim, Semicontinuity of compact group actions on compact differentiable manifolds, *Arch. Math.* 49(1987), 450–455.
- [KIS] C. Kiselman, A study of the Bergman projection in certain Hartogs domains, *Proc. Symposia Pure Math.*, vol. 52 (E. Bedford, J. D’Angelo, R. Greene, and S. Krantz eds.), American Mathematical Society, Providence, 1991.
- [KLE] P. Klembeck, Kähler metrics of negative curvature, the Bergman metric near the boundary and the Kobayashi metric on smooth bounded strictly pseudoconvex sets, *Indiana Univ. Math. J.* 27(1978), 275–282.
- [KOB1] S. Kobayashi, Geometry of bounded domains, *Trans. AMS* 92(1959), 267–290.
- [KOB2] S. Kobayashi, *Hyperbolic Manifolds and Holomorphic Mappings*, Dekker, New York, 1970.
- [KON] S. Kobayashi and K. Nomizu, *Foundations of Differential Geometry*, Vols. I and II, Interscience, New York, 1963, 1969.
- [KOH1] J. J. Kohn, Quantitative estimates for global regularity, *Analysis and geometry in several complex variables* (Katata, 1997), 97–128, Trends Math., Birkhäuser Boston, Boston, MA, 1999.
- [KOH2] J. J. Kohn, Boundary behavior of  $\bar{\partial}$  on weakly pseudoconvex manifolds of dimension two, *J. Diff. Geom.* 6(1972), 523–542.
- [KOR1] A. Koranyi, Harmonic functions on Hermitian hyperbolic space, *Trans. A. M. S.* 135(1969), 507–516.
- [KOR2] A. Koranyi, Boundary behavior of Poisson integrals on symmetric spaces, *Trans. A.M.S.* 140(1969), 393–409.
- [KRA1] S. G. Krantz, *Function Theory of Several Complex Variables*, 2<sup>nd</sup> ed., American Mathematical Society, Providence, RI, 2001.
- [KRA2] S. G. Krantz, On a construction of L. Hua for positive reproducing kernels, *Michigan Journal of Mathematics* 59(2010), 211–230.
- [KRA3] S. G. Krantz, Boundary decomposition of the Bergman kernel, *Rocky Mountain Journal of Math.*, to appear.
- [KRA4] S. G. Krantz, *Partial Differential Equations and Complex Analysis*, CRC Press, Boca Raton, FL, 1992.
- [KRA5] S. G. Krantz, *Cornerstones of Geometric Function Theory: Explorations in Complex Analysis*, Birkhäuser Publishing, Boston, 2006.
- [KRA6] S. G. Krantz, Invariant metrics and the boundary behavior of holomorphic functions on domains in  $\mathbb{C}^n$ , *Jour. Geometric. Anal.* 1(1991), 71–98.
- [KRA7] S. G. Krantz, Calculation and estimation of the Poisson kernel, *J. Math. Anal. Appl.* 302(2005)143–148.
- [KRA8] S. G. Krantz, A new proof and a generalization of Ramadanov’s theorem, *Complex Variables and Elliptic Eq.* 51(2006), 1125–1128.
- [KRA9] S. G. Krantz, *Complex Analysis: The Geometric Viewpoint*, 2nd ed., Mathematical Association of America, Washington, D.C., 2004.
- [KRA11] S. G. Krantz, Canonical kernels versus constructible kernels, preprint.



- [KRA12] S. G. Krantz, Lipschitz spaces, smoothness of functions, and approximation theory, *Expositiones Math.* 3(1983), 193–260.
- [KRA13] S. G. Krantz, Characterizations of smooth domains in  $\mathbb{C}$  by their biholomorphic self maps, *Am. Math. Monthly* 90(1983), 555–557.
- [KRA14] S. G. Krantz, *A Guide to Functional Analysis*, Mathematical Association of America, Washington, D.C., 2013, to appear.
- [KRA15] S. G. Krantz, A direct connection between the Bergman and Szegő projections, *Complex Analysis and Operator Theory*, to appear.
- [KRPA1] S. G. Krantz and H. R. Parks, *The Geometry of Domains in Space*, Birkhäuser Publishing, Boston, MA, 1996.
- [KRPA2] S. G. Krantz and H. R. Parks, *The Implicit Function Theorem*, Birkhäuser, Boston, 2002.
- [KRP1] S. G. Krantz and M. M. Peloso, The Bergman kernel and projection on non-smooth worm domains, *Houston J. Math.* 34 (2008), 9.3.-950.
- [KRP2] S. G. Krantz and M. M. Peloso, Analysis and geometry on worm domains, *J. Geom. Anal.* 18(2008), 478–510.
- [LEM1] L. Lempert, La métrique Kobayashi et las representation des domaines sur la boule, *Bull. Soc. Math. France* 109(1981), 427–474.
- [LI] S.-Y. Li, S.-Y. Li, Neumann problems for complex Monge–Ampère equations, *Indiana University J. of Math.* 43(1994), 1099–1122.
- [LIG1] E. Ligocka, The Sobolev spaces of harmonic functions, *Studia Math.* 54(1986). 79–87.
- [LIG2] E. Ligocka, Remarks on the Bergman kernel function of a worm domain, *Studia Mathematica* 130(1998), 109–113.
- [MIN] B.-L. Min, Domains with prescribed automorphism group, *J. Geom. Anal.* 19 (2009), 911–928.
- [NAR] R. Narasimhan, *Several Complex Variables*, University of Chicago Press, Chicago, 1971.
- [NWY] L. Nirenberg, S. Webster, and P. Yang, Local boundary regularity of holomorphic mappings. *Comm. Pure Appl. Math.* 33(1980), 305–338.
- [OHS] T. Ohsawa, A remark on the completeness of the Bergman metric, *Proc. Japan Acad. Ser. A Math. Sci.*, 57(1981), 238–240.
- [PAI] Painlevé, Sur les lignes singulières des fonctions analytiques, *Thèse*, Gauthier-Villars, Paris, 1887.
- [PEE] J. Peetre, The Berezin transform and Ha-Plitz operators, *J. Operator Theory* 24(1990), 165–186.
- [PET] P. Petersen, *Riemannian Geometry*, Springer, New York, 2009.
- [PHS] D. H. Phong and E. M. Stein, Hilbert integrals, singular integrals, and Radon transforms, *Acta Math.* 157(1986), 99–157.
- [PHS1] D. H. Phong and E. M. Stein, Hilbert integrals, singular integrals, and Radon transforms. I. *Acta Math.* 157(1986), 99–157.
- [PHS2] D. H. Phong and E. M. Stein, Hilbert integrals, singular integrals, and Radon transforms. II. *Invent. Math.* 86(1986), 75–113.
- [PIN] S. Pinchuk, The scaling method and holomorphic mappings, *Several Complex Variables and Complex Ggeometry*, Part 1 (Santa Cruz, CA, 1989), 151–161, Proc. Sympos. Pure Math., 52, Part 1, Amer. Math. Soc., Providence, RI, 1991.
- [PIH] S. Pinchuk and S. V. Hasanov, Asymptotically holomorphic functions (Russian), *Mat. Sb.* 134(176) (1987), 546–555.
- [PIT] S. Pinchuk and S. I. Tsyganov, Smoothness of CR-mappings between strictly pseudoconvex hypersurfaces. (Russian) *Izv. Akad. Nauk SSSR Ser. Mat.* 53(1989), 1120–1129, 1136; translation in *Math. USSR-Izv.* 35(1990), 457–467.
- [RAM1] I. Ramadanov, Sur une propriété de la fonction de Bergman. (French) *C. R. Acad. Bulgare Sci.* 20(1967), 759–762.

- [RAM2] I. Ramadanov, A characterization of the balls in  $\mathbb{C}^n$  by means of the Bergman kernel, *C. R. Acad. Bulgare Sci.* 34(1981), 927–929.
- [RAMI] E. Ramirez, Divisions problem in der komplexen analysis mit einer Anwendung auf Rand integral darstellung, *Math. Ann.* 184(1970), 172–187.
- [RAN] R. M. Range, A remark on bounded strictly plurisubharmonic exhaustion functions, *Proc. A.M.S.* 81(1981), 220–222.
- [ROM] S. Roman, The formula of Faà di Bruno, *Am. Math. Monthly* 87(1980), 805–809.
- [ROW] B. Rodin and S. Warschawski, Estimates of the Riemann mapping function near a boundary point, in *Romanian-Finnish Seminar on Complex Analysis*, Springer Lecture Notes, vol. 743, 1979, 349–366.
- [ROS] J.-P. Rosay, Sur une characterization de la boule parmi les domaines de  $\mathbb{C}^n$  par son groupe d'automorphismes, *Ann. Inst. Four. Grenoble* XXIX(1979), 91–97.
- [ROSE] P. Rosenthal, On the zeroes of the Bergman function in doubly-connected domains, *Proc. Amer. Math. Soc.* 21(1969), 33–35.
- [RUD1] W. Rudin, *Principles of Mathematical Analysis*, 3rd ed., McGraw-Hill, New York, 1976.
- [RUD2] W. Rudin, *Function Theory in the Unit Ball of  $\mathbb{C}^n$* , Grundlehren der Mathematischen Wissenschaften in Einzeldarstellungen, Springer, Berlin, 1980.
- [SEM] S. Semmes, A generalization of Riemann mappings and geometric structures on a space of domains in  $\mathbb{C}^n$ , *Memoirs of the American Mathematical Society*, 1991.
- [SIB] N. Sibony, A class of hyperbolic manifolds, *Ann. of Math. Stud.* 100(1981), 357–372.
- [SIU] Y.-T. Siu, Non Hölder property of Bergman projection of smooth worm domain, *Aspects of Mathematics—Algebra, Geometry, and Several Complex Variables*, N. Mok (ed.), University of Hong Kong, 1996, 264–304.
- [SKW] M. Skwarczynski, The distance in the theory of pseudo-conformal transformations and the Lu Qi-King conjecture, *Proc. A.M.S.* 22(1969), 305–310.
- [STE1] E. M. Stein, *Singular Integrals and Differentiability Properties of Functions*, Princeton University Press, Princeton, NJ, 1970.
- [STE2] E. M. Stein, *Boundary Behavior of Holomorphic Functions of Several Complex Variables*, Princeton University Press, Princeton, 1972.
- [STW] E. M. Stein and G. Weiss, *Introduction to Fourier Analysis on Euclidean Space*, Princeton University Press, Princeton, NJ, 1971.
- [STR] K. Stromberg, *An Introduction to Classical Real Analysis*, Wadsworth, Belmont, 1981.
- [SUY] N. Suita and A. Yamada On the Lu Qi-Keng conjecture, *Proc. A.M.S.* 59(1976), 222–224.
- [SZE] G. Szegő, Über Orthogonalsysteme von Polynomen, *Math. Z.* 4(1919), 139–151.
- [TAN] N. Tanaka, On generalized graded Lie algebras and geometric structures, I, *J. Math. Soc. Japan* 19(1967), 215–254.
- [THO] G. B. Thomas, *Calculus*, 7<sup>th</sup> ed., Addison-Wesley, Reading, MA, 1999.
- [TRE] F. Trèves, *Introduction to Pseudodifferential and Fourier Integral Operators*, Vol. II, Plenum Press, New York 1982.
- [WAR1] S. Warschawski, On the boundary behavior of conformal maps, *Nagoya Math. J.* 30(1967), 83–101.
- [WAR2] S. Warschawski, On boundary derivatives in conformal mapping, *Ann. Acad. Sci. Fenn. Ser. A I* no. 420(1968), 22 pp.
- [WAR3] S. Warschawski, Hölder continuity at the boundary in conformal maps, *J. Math. Mech.* 18(1968/9), 423–7.
- [WEB1] S. Webster, Biholomorphic mappings and the Bergman kernel off the diagonal, *Invent. Math.* 51(1979), 155–169.
- [WEB2] S. Webster, On the reflection principle in several complex variables, *Proc. Amer. Math. Soc.* 71(1978), 26–28.
- [WHW] E. Whittaker and G. Watson, *A Course of Modern Analysis*, 4th ed., Cambridge Univ. Press, London, 1935.

- [WIE] J. Wiegerinck, Domains with finite dimensional Bergman space, *Math. Z.* 187(1984), 559–562.
- [WON] B. Wong, Characterization of the ball in  $\mathbb{C}^n$  by its automorphism group, *Invent. Math.* 41(1977), 253–257.
- [YAU] S.-T. Yau, Problem section, *Seminar on Differential Geometry*, S.-T. Yau ed., *Annals of Math. Studies*, vol. 102, Princeton University Press, 1982, 669–706.
- [ZHU] K. Zhu, *Spaces of Holomorphic Functions in the Unit Ball*, Springer, New York, 2005.

# Index

## A

- admissible
  - approach region, 98
  - limits, 98
- Ahlfors map, 83
- almost-the-shortest connector, 261
- analytic
  - polyhedron, 83
  - type, 158
- annulus, 43
- Arazy, J., 95
- Aronszajn, Nachman, 25, 35
- automorphism, 14, 54, 99
  - uniform bounds on derivatives, 153
- automorphism group, 14, 69, 147
  - compact, 151
  - is a real Lie group, 184
  - semicontinuity of, 151
  - topologies on, 163
  - transitive action, 100
- automorphism-invariant metric, 153

## B

- ball and polydisc are biholomorphically inequivalent, 83
- balls
  - nonisotropic, 111
- Banach-Alaoglu theorem, 57
- Barrett's counterexample, 236
- Barrett's theorem, 191, 202, 204, 236
- Barrett, David, 74, 191
- Bell's theorem
  - localization of, 191
- Bell, Steven R., 73, 141
- Bell–Boas condition for mappings, 14

- Bell–Krantz proof of the Fefferman's theorem, 73
- Berezin
  - kernel, 91
  - transform, 90, 91
- Bergman
  - transformation law, 89
- Bergman basis
  - new, 36
- Bergman distance, 11
- Bergman kernel, 4
  - as a Hilbert integral, 246
  - asymptotic expansion for, 205
  - boundary asymptotics, 205
  - boundary behavior, 208
  - boundary localization of, 59
  - boundary singularity, 41
  - calculation of, 14
  - constructed with partial differential equations, 20
  - for a Sobolev space, 54, 221
  - for the annulus, 12
  - for the annulus, approximate formula, 44
  - for the annulus, special basis, 44
  - for the disc, 18
  - for the disc by conformal invariance, 23
  - for the polydisc, 19
  - for the unit disc, 12
  - in an increasing sequence of domains, 57
  - invariance of, 2, 92
  - is conjugate symmetric, 4
  - on a domain in several complex variables, 64
  - on a smooth, finitely connected, planar domain, 62
  - on multiply connected domains, 54

- Bergman kernel (*cont.*)  
 on the annulus, 43  
 on the ball, 15  
 positivity of on the diagonal, 10  
 singularities of, 35  
 singularity of, 41  
 smoothness of, 34  
 smoothness to the boundary of, 13  
 sum formula for, 5  
 uniqueness of, 5  
 with a logarithmic term, 219
- Bergman metric, 2, 10, 71, 183  
 boundary behavior, 184  
 boundary behavior of, 81  
 by conformal invariance, 23  
 completeness of, 263  
 for the ball, 18  
 for the disc, 18  
 isometry of, 11, 100  
 length of a curve in, 82  
 localization of holomorphic sectional curvature, 261  
 near a strictly pseudoconvex point, 262
- Bergman metric holomorphic sectional curvature  
 localization of, 261
- Bergman projection, 6, 30, 141  
 and the Neumann operator, 141  
 on the worm does not satisfy Condition  $R$ , 206
- Bergman projection and the Neumann operator, 208
- Bergman representative coordinates, 87, 164  
 biholomorphic maps are linear in, 87  
 definition of, 87
- Bergman space, 1, 3  
 completeness of, 1  
 dimension of, 174  
 is a Hilbert space, 4  
 of harmonic functions, 41  
 on a manifold, 181  
 real, 40
- Bergman theory  
 on a manifold, 178
- Bergman, Stefan, 1  
 basis, 263  
 kernel, 263  
 metric, 261, 263  
 metric is positive definite, 88  
 representative coordinates, 87
- Berndtsson, B., 196
- Bessel potential  
 tangential, 143
- bigraded spherical harmonics, 130
- biholomorphic  
 to ball, sufficient condition for, 171
- biholomorphic mapping  
 is linear in Bergman representative coordinates, 87, 89  
 of pseudoconvex domains, 189  
 of the worm, 192
- biholomorphic mappings, 2, 9, 11
- biholomorphic self-mapping, 99
- Boas, Harold, 13, 91
- Boas–Straube theorem, 194
- Bochner, Salomon, 1
- Bochner–Martinelli formula, 2, 25, 27
- boundary orbit accumulation points are pseudoconvex, 251
- boundary regularity for the Dirichlet problem  
 for the invariant Laplacian, 139
- boundary smoothness and Dirichlet’s problem, 226
- bounded plurisubharmonic exhaustion function, 195
- Boutet de Monvel, Louis, 32
- Bremermann, M., 83
- Bungart, Lutz, 1
- Bun Wong’s theorem, 152
- Burns, D., 71
- Burns–Shnider–Wells theorem, 71
- C**
- Caffarelli, L., 229
- Caffarelli–Kohn–Nirenberg–Spruck theorem, 229
- Carathéodory  
 metric, 83
- Cartan’s theorem, 148
- Cartan, Henri, 147
- Catlin, David, 91
- Cauchy  
 formula, 29  
 integral formula, 115  
 kernel, 2, 115  
 transform, 2, 177
- Charpentier, P., 196
- Chern, S. S., 14, 72
- Chern–Moser–Tanaka invariants, 72
- Christ’s theorem, 241
- Christ, Michael, 81  
 Condition  $R$  fails on the worm, 191
- Coifman, R. R., 96
- commutator, 156, 158
- complex  
 manifold, 181

Monge–Ampère equation, 233  
   tangential vector fields, 158  
 Condition  $R$ , 81, 141, 190  
 conformal mappings  
   boundary smoothness, 224  
 conformality  
   in higher dimensions, 189  
 conjugating diffeomorphism, 155  
 connector, 261  
   almost-the-shortest, 261  
 constructible integral formula, 29  
 constructing holomorphic functions, 29  
 conversion by scaling, 262

**D**

D’Angelo, John, 91  
 $\bar{\partial}$ -Neumann conditions, 241  
 $\bar{\partial}$ -Neumann operator, 34, 191  
 $\bar{\partial}$ -Neumann problem, 73  
 defining function  
   plurisubharmonic, 228  
 density of  $A(\Omega)$  in  $A^2(\Omega)$ , 92  
 derivatives  
   uniform bounds on, 162  
 Diederich, Klas, 74  
 Diederich–Fornæss worm domain, 13, 35, 187, 188  
 differences between  $n = 1$  and  $n > 1$ , 72  
 differential forms, 7, 179  
 dilations, 265  
 Dini–Kummer test, 133  
 Dirichlet problem, 61, 138, 226  
   and boundary smoothness, 226  
   for the invariant Laplacian, 109  
   for the Laplace–Beltrami operator, 109, 111  
   regularity for, 224  
 disc  
   characterization of by scaling argument, 267  
 domain, 1, 147  
   with smooth boundary, 224

**E**

Ebin, David, 151  
   theorem, 151, 152  
   theorem for manifolds with boundary, 155  
 elliptic operator, 226  
 ellipticity, 228  
 Engliš, M., 95  
 Euclidean volume, 100

**F**

failure of Condition  $R$ , 241  
 Fefferman asymptotic expansion, 98, 99  
 Fefferman’s approximation argument, 215  
 Fefferman’s theorem, 72, 190  
   new proof of, 229  
 Fefferman, Charles, 14  
 finite analytic type, 158  
 finite geometric type, 158  
 finite type, 81, 156, 161  
   in dimension two, 156  
 first-order commutator, 156  
 Folland, G. B., 114, 141  
 formula of Kohn, 190  
 Fornæss, John Erik, 74  
 functional analysis, 1  
 functions agreeing to order  $k$ , 75

**G**

Gamelin, T., 229  
 Garabedian, Paul, 59  
 Garnett, J. B., 109  
 Gauss, Carl F., 135  
 Gegenbauer polynomial, 126  
 geodesic  
   normal coordinates, 90  
 geometric type, 158  
 Gleason, Andrew, 1  
 Godement, R., 103  
 Graham, C. R., 114  
 Grauert, H., 2  
 Green’s function, 21, 60  
   of the unit disc, 23  
 Greene, R. E., 13, 14, 71, 99  
 Greene–Krantz  
   conjecture, 251  
   invariant metric, 154  
   semincontinuity theorem, 152  
 Greene–Krantz theorem, 72, 73  
 group action, 152

**H**

Haar measure, 212  
 Hardy space, 25  
 Hartogs, F., 187  
   triangle, 187  
 Hausdorff measure, 25  
 Henkin, G. M., 2  
 Hilbert integral, 246  
   higher-dimensional version, 247

- Hilbert space with reproducing kernel, 25, 35, 90
- Hodge theory, 179
- holomorphic
- chain, 260
  - curve, 161
  - implicit function theorem, 8
  - Jacobian matrix, 7
  - local coordinates, 88
  - peak function, 210, 255
  - sectional curvature, 261
  - vector field, 157
- holomorphic mappings
- convergence of, 156
- Hopf's lemma, 82, 185, 225
- Hörmander's theorem about solvability of  $\bar{\partial}$ , 242
- Hörmander, Lars
- theorem, 209
- Hua, L., 90
- Hurwitz, A., 148
- theorem, 148
- hypergeometric
- equation, 131
  - functions, 132
- I**
- identity principle for power series, 129
- infinite analytic type, 158
- interior stability, 262
- invariant
- metrics, 182
- isometry, 100
- isometry group
- semincontinuity of, 151
- K**
- Kähler
- manifold, 172
- Kähler
- metric, 87, 100, 183
- Kellogg, Oliver, 73
- kernel forms, 179
- Kerzman, N., 2
- Kiselman, Christer, 189
- idea, 202
- Klembeck's theorem, 251, 261
- with stability, 261
- Klembeck, Paul, 14, 261
- Kobayashi metric, 83
- lower bound estimate, 257
- Kobayashi–Royden metric
- lower bound for, 257
- Kohn, Joseph J., 91, 142, 229
- projection formula, 34, 207
- Kontinuitätssatz, 83
- Koranyi, Adam, 105
- Krantz, S. G., 13, 14, 71, 99, 109
- L**
- Laplace–Beltrami operator, 100, 139
- Laplacian
- fundamental solution for, 20
  - invariance of, 226
- Lebesgue measure, 177
- lemma
- key, 3
  - normal-families type, 153
- Lempert, Laszlo, 81
- Leray form, 26
- Lie group
- real, 162
- Lieb, I., 2
- Ligocka, Ewa, 73, 141
- Liouville, J., 189
- local holomorphic peak functions, 255
- logarithmic capacity, 178
- logarithmically convex domain, 170
- Lopatinski, Y., 109
- Lu Qi-Keng
- conjecture, 12, 169
- Lu Qi-Keng
- theorem, 87, 152, 171
- M**
- Möbius transformation, 24
- mapping
- problem, 14
- mean-value property, 24
- metric
- double, 167
  - nonisotropic, 110
  - product, 154
- Möbius transformation, 177
- Monge–Ampère equation, 228, 233
- monomials
- with even index, 37
  - with index in an arithmetic sequence, 39
  - with odd index, 39
- Montel, Paul, 256
- theorem, 256
- Moser, Jürgen, 14, 72
- multiplicity, 161

**N**

- Nebenhülle, 187
- neighborhood basis in the  $C^k$  topology, 71
- Nirenberg, L., 229
- non-smooth worms, 200
- noncompact automorphism group, 251
- nonsingular
  - complex variety, 159
  - disc, 159
- nonzero component in the complex normal direction, 158
- normal
  - convergence of domains, 255
  - families, 153
  - set-convergence, 254
- normal convergence of domains, 254

**O**

- open mapping theorem, 269
- orthonormal basis
  - special for Bergman space, 10
- other bases for the Bergman space, 39

**P**

- parallels orthogonal to a vector, 123
- peak function, 13, 255
  - holomorphic, 255
  - local holomorphic, 255
  - plurisubharmonic, 257
- peak point, 255
  - local plurisubharmonic, 257
- Pinchuk, S., 81
- pluriharmonic function, 103, 139
- plurisubharmonic defining function, 228
  - that vanishes to high order at the boundary, 229
- plurisubharmonic peak functions, 256
- Poincaré metric on the disc, 18
- Poincaré-Bergman distance, 19
- Poincaré's theorem, 83
- point of finite type, 158
- Poisson
  - extension of a function, 31
  - integral formula, 259
  - kernel, 105, 115
- Poisson-Bergman
  - kernel, 91
- Poisson-Bergman kernel
  - asymptotic expansion for, 96
  - invariance of, 93
- Poisson-Bergman potentials
  - boundary limits of, 94

- Poisson-Szegő kernel, 31, 108, 138
  - for the ball, 33
  - for the polydisc, 33
  - solves the Dirichlet problem, 103
- positive kernels, 31
- product metric, 154
- proof of Fefferman's theorem, 81
- pseudodifferential operator, 226
- pseudohyperbolic metric, 20
- pseudolocality, 35
- pseudometric, 96
- pseudotransversal geodesic, 98
- pullback, 161

**Q**

- quantization of Kähler manifolds, 90

**R**

- Raabe's test, 133
- radial boundary limits, 98
- Ramadanov's theorem
  - for the Szegő kernel, 59
- Ramadanov, Ivan
  - theorem, 56, 262
- Ramirez, E., 2
- real
  - analytic boundary, 164
  - Jacobian matrix, 7
- Reinhardt domain
  - complete, 170
- reproducing kernel
  - explicit, 1
  - with holomorphic free variable, 2
- Riemann mapping theorem, 72, 189, 224
- Riemannian manifold
  - compact, 155
- Riemannian metric, 167
- Riesz representation theorem, 36
- Riesz-Fischer
  - theorem, 12
  - theory, 31
- Rochberg, Richard, 59
- Rosay, Jean-Pierre, 73

**S**

- scaling
  - argument, 165
  - conversion by, 262
  - map, 268
  - method, 251, 265, 268
  - nonisotropic, 252



one-dimensional method, 265  
 sequence, 270  
 theorem, 262  
 scaling method  
   an example, 265  
 Schmidt, Erhard, 1  
 semicontinuity  
   examples of, 168  
   of automorphism groups, 151  
   of type, 160  
   theorem, 166  
 Semmes, S., 72  
 Shnider, S., 71  
 Siegel half space, 253  
 singular integrals, 246  
 Sjöstrand, Johannes, 32  
 Skwarczyński, M., 169  
 smooth boundary continuation of conformal  
   mappings, 224  
 Sobolev  
   embedding theorem, 75, 142  
   space, 75  
 source domains, 255  
 space of homogeneous type, 96  
 spherical harmonics  
   in the complex domain, 130  
 spherical harmonics, 114, 117  
 Spruck, J., 229  
 square-integrable holomorphic form, 182  
 Stein neighborhood basis, 187  
 Stein, E. M., 103  
 Stokes's theorem, 22  
   in complex form, 21  
 strictly pseudoconvex domain, 2, 95,  
   98  
 strong type operator, 97  
 symmetry, 151  
   creating, 151  
   destroying, 151  
 synthesis, 270  
 Szegő kernel, 25  
   for the ball, 33

  for the polydisc, 33  
   on the disc, 30  
 Szegő projection, 30

## T

Tanaka, N., 72  
 target domains, 255  
 Taylor expansion in powers of  $\rho$ , 77  
 Tsyganov, S. I., 81  
 tubular neighborhood, 165

## V

vector fields  
   gradation of, 157

## W

Warschawski, S., 73  
 weak type operator, 97  
 Weiss, Guido, 96  
 well-posed boundary value problem, 109  
 Wells, R. O., 71  
 Wiegerinck, J., 174  
 Wong, Bun, 73  
 Wong–Rosay theorem, 266  
   by way of scaling, 254  
 worm  
   has a locally defined plurisubharmonic  
     defining function, 193  
   has no globally defined plurisubharmonic  
     defining function, 193  
   is pseudoconvex, 189  
   is smoothly bounded, 189  
   is strictly pseudoconvex except on a curve,  
     189  
   non-smooth versions of, 199

## Z

zonal harmonics, 118, 119, 127