Appendix A
Sample Quizzes

A.1 Number Theory

Short Answer Section

The following questions are worth 1 point each. Answer the questions in your *Maple* file.

1. What is e^{13} evaluated to 23 significant figures?
2. Factorize $x^{12} + 3\,x^{10} - 23\,x^8 - 51\,x^6 + 94\,x^4 + 120\,x^2$.
3. Convert $\dfrac{e^{2x} - 1}{xe^{2x} + x}$ into an expression involving trig functions.
4. What is the partial fraction decomposition of the rational polynomial.

$$\frac{x^7 + 4\,x^6 + 25\,x^4 - 20\,x^3 + 53\,x^2 - 42\,x + 33}{x^8 - 2\,x^7 + 7\,x^6 - 12\,x^5 + 18\,x^4 - 24\,x^3 + 20\,x^2 - 16\,x + 8}$$

5. What are the first 20 terms of the sequence $\left\{\dfrac{1}{k(k+1)}\right\}_{k=1}^{\infty}$?
6. Evaluate $\displaystyle\sum_{k=1}^{\infty} \frac{1}{k(k+1)}$.
7. Evaluate $\displaystyle\prod_{k=1}^{\infty} \left(1 - \frac{1}{2x^2}\right)$.
8. The following variable names are all names that *Maple* treats as Greek letters. Which of them are protected?

delta, Delta, gamma, Gamma, GAMMA, pi, Pi, PI, zeta, Zeta, ZETA

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Long Answer Section

The following questions are worth 3 points each. Points are given for working. Answer the questions in your *Maple* file.

9. Let $a_n = 2n - 1$ and $s_n = n^2$ and define the sequences

$$A := \{a_n\}_{n=1}^{\infty} \quad S := \{s_n\}_{n=1}^{\infty}$$

It should be clear then that $A = \{1, 3, 5, \dots\}$ and $S = \{1, 4, 9, 16, \dots\}$.

a. Calculate the first 20 terms of the sequence

$$\{s_{n+1} - s_n\}_{n=1}^{\infty}$$

What is $s_{n+1} - s_n$?

b. Calculate the first 20 terms of the sequence

$$\left\{\sum_{k=1}^{n} a_k\right\}_{n=1}^{\infty}$$

What is $\sum_{k=1}^{n} a_k$?

10. Recall that $\sum k^{-1}$ diverges. It may be shown that $\sum k^{-(1+\epsilon)}$ converges for any $\epsilon > 0$. For this question we let $\epsilon = \frac{1}{100}$

a. Evaluate the series $\sum_{k=1}^{\infty} 1/k^{\frac{101}{100}}$, and obtain a decimal approximation.

b. Calculate decimal approximations of the partial sums

$$\sum_{k=1}^{N} \frac{1}{k^{101/100}} \text{ for } N = 10, 100, 1000, 10000, 100000$$

and measure how much time each takes to calculate.

Notice that this series converges very slowly.

11. Let $\{f_n\}$ be the Fibonacci-like sequence defined by

$$f_n := f_{n-1} + f_{n-2} \quad f_1 = -2, f_2 = 3$$

a. Write a *Maple* function (using arrow notation or a procedure as you choose) to calculate the terms of this sequence.

b. What are the first 10 terms of this sequence?

c. What is the largest number in this sequence less than 1,000,000, and what is its index?

12. Let s be the first-order nonlinear recurrence relation defined by

$$s_n = n\, s_{n-1}^2$$

a. Let $s_0 = C$ and calculate the first 5 or so terms of the recurrence.

b. Solve the recurrence.

c. Verify the solution for at least 20 terms of the sequence, and in general if you can.

A.2 Calculus

Short Answer Section

The following questions are worth 1 point each. Answer the questions in your *Maple* file.

1. What is the limit of $\dfrac{x+\sin x}{\pi\, x}$ at $x \to \infty$?
2. Find $\lim\limits_{x\to 0^+} \dfrac{-\cosh x}{x}$.
3. What is the derivative of $\dfrac{\cos x}{x}$?
4. What is the slope of the tangent to the curve $y = \dfrac{\cos x}{x}$ at $x = \frac{1}{3}\pi$?
5. Evaluate $\displaystyle\int_0^1 \log x\, dx$.
6. Find a function whose derivative is $\tanh x$.
7. Find the first partial derivatives of $z = x^2 - y^2$.
8. How many critical points does $z = x^2 - y^2$ have, and what kind of critical points are they?

Long Answer Section

The following questions are worth 3 points each. Points are given for working. Answer the questions in your *Maple* file.

9. A length of wire 10 meters long is cut in two. One of the pieces is bent into a square, the other into an equilateral triangle. Let x be the length of wire that is bent into the square (meaning that $10 - x$ is the length of wire bent into the triangle). Let A_s be the area of the square and let A_t be the area of the triangle.
   a. Define A to be the formula for the total area of the two shapes (i.e., $A := A_s + A_t$). Plot A.
   b. How much wire should be used for the square to maximize the total area?
   c. How much wire should be used for the square to minimize the total area?
10. The Airy functions $\mathrm{Ai}(z), \mathrm{Bi}(z)$ are the two independent solutions to the differential equation
$$y'' - z\,y = 0 \tag{A.1}$$
   a. Solve the differential equation (A.1), and verify the solution.
   b. Plot the Airy functions together on the same axes. Make sure to show good detail of what the functions are doing.
   c. Find and plot a third solution, other than $y = \mathrm{Ai}(z)$ and $y = \mathrm{Bi}(z)$, to equation (A.1).
11. Use solids of revolution to verify the following volumes.
   a. The volume of a sphere with radius r ($4/3\pi r^3$)
   b. The volume of a cone with height h and radius r ($1/3\pi r^2 h$)
12. Consider the surface $z = \sin(x)\cos(y)$.
   a. Plot the surface z.
   b. Find a general formula, or formulae, for the critical points.
   c. Which critical points are maxima, which are minima, and which are saddle points?

A.3 Linear Algebra

The following questions are worth 1 point each. Answer the questions in your *Maple* file.

1. Calculate the vector $\pi \cdot (8, 1, 5, 1, 9) + e \cdot (1, 2, 5, 6, 6)$.
2. Calculate the matrix product $\begin{bmatrix} 3 & 1 & 6 \\ 0 & 0 & 7 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 0 & 0 \\ 5 & 4 \end{bmatrix}$.
3. Calculate the dot product of the vectors $(3, 3, 2, 3, 3, 3)$ and $(3, 2, 3, 3, 2, 1)$.
4. Find the angle between the vectors $(3, 3, 2, 3, 3, 3)$ and $(3, 2, 3, 3, 2, 1)$.
5. Find a vector perpendicular to the vectors $(5, 5, 3)$ and $(5, 5, 5)$.
6. Create the 9×10 matrix M whose entries $m_{i,j} = 17\, i\, j$.
7. Find the elementary matrix that will add k multiplied by row 7 to row 9 of a 10×10 matrix.
8. How many solutions are there to the vector equation $M \cdot x = 0$ where

$$M := \begin{bmatrix} 9 & 4 & 7 & 8 & 5 \\ 4 & 7 & 3 & 4 & 8 \\ 7 & 7 & 0 & 5 & 7 \\ 7 & 4 & 6 & 4 & 4 \\ 7 & 6 & 2 & 2 & 6 \end{bmatrix}$$

Long Answer Section

The following questions are worth 3 points each. Points are given for working. Answer the questions in your *Maple* file.

9. This question refers to the following simultaneous equations.

$$\begin{aligned} y + 3z &= 2 \\ 8x + 2y + 2z &= 3 \\ 3x + 3y + 5z &= 2 \end{aligned}$$

   a. Solve the simultaneous equations. How many solutions are there?
   b. Plot the three surfaces in such a way that clearly shows the solution.

10. This question refers to the following three matrices

$$\begin{bmatrix} 1 & 1 & 7 & 2 \\ 2 & 2 & 0 & 9 \\ 7 & 7 & 2 & 5 \\ 0 & 0 & 1 & 5 \end{bmatrix}, \begin{bmatrix} 0 & 4 & 1 & 4 \\ 0 & 2 & 1 & 1 \\ 3 & 3 & 4 & 4 \\ 6 & 4 & 9 & 5 \end{bmatrix}, \begin{bmatrix} 0 & 1 & 1 & 1 \\ 6 & 1 & 0 & 0 \\ 2 & 2 & 6 & 6 \\ 9 & 4 & 4 & 4 \end{bmatrix}$$

   a. Which of the matrices may be expressed as a product of elementary matrices?
   b. For the matrices that may be expressed as a product of elementary matrices, find the sequence of elementary matrices whose product is that matrix. (Equivalently, you may find the sequence of row operations performed on the identity matrix.)

11. Let A be the matrix below, and let $p, q \in \mathbb{R}$.

$$A := \begin{bmatrix} p & q & 1-p-q \\ 1-p-q & p & q \\ q & 1-p-q & p \end{bmatrix}$$

   a. Create A in *Maple* as a function of p and q.
   b. By examining various numerical cases where $p > 0$, $q > 0$ and $1 - p - q > 0$, conjecture the behavior of the matrix A^n as $n \to \infty$.

12. This question refers to the following set of matrices

$$\begin{bmatrix} 7 & 2 & 9 \\ 1 & 2 & 6 \\ 2 & 4 & 8 \end{bmatrix}, \begin{bmatrix} 8 & 4 & 7 \\ 6 & 7 & 4 \\ 9 & 5 & 7 \end{bmatrix}, \begin{bmatrix} 0 & 6 & 0 \\ 2 & 4 & 0 \\ 5 & 2 & 5 \end{bmatrix}, \begin{bmatrix} 0 & 7 & 4 \\ 2 & 7 & 9 \\ 9 & 3 & 7 \end{bmatrix}$$
$$\begin{bmatrix} 0 & 0 & 9 \\ 3 & 7 & 1 \\ 0 & 0 & 3 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 1 \\ 7 & 3 & 9 \\ 0 & 0 & 4 \end{bmatrix}, \begin{bmatrix} 7 & 9 & 1 \\ 1 & 4 & 5 \\ 1 & 6 & 3 \end{bmatrix}, \begin{bmatrix} 9 & 7 & 4 \\ 2 & 1 & 4 \\ 5 & 9 & 0 \end{bmatrix}, \begin{bmatrix} 5 & 0 & 8 \\ 7 & 5 & 4 \\ 9 & 4 & 8 \end{bmatrix}$$

   a. Do the matrices form a basis for $M_3(\mathbb{R})$? Justify your answer.
   b. Find the coefficients of a linear combination of these matrices for an arbitrary 3×3 matrix.

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