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Historical and Bibliographical Remarks

Chapter 1

The basic ideas of optimal control theory are linked to sequential analysis and the theory of statistical decision functions which originated and were developed by A. Wald (see, e.g. [43]). R. Bellman developed this theory for more specific cases under the name of dynamic programming (see [3], [4]). Further development of multi-staged Markov processes was stimulated by Howard [26], [27] who also studied the problem of the controlled Markov chain. The general results on optimal controlled Markov chains with arbitrary sets of states were obtained by D. Blackwell [7], [8], [9], R. E. Strauch [53], and N. W. Krylov [28], [29]. A detailed exposition of the theory of controlled Markov chains is contained in Kushner's monograph [34] and a more general theory may be found in E. B. Dynkin and A. A. Yushkevich's memoir [14].

The general description of controlled stochastic objects presented in the first chapter of the present book and the constructions of optimal and ε-optimal controls are based on A. V. Skorohod's paper [50].

The bibliography of works devoted to the problem of optimal stopping is quite substantial. A general solution of this problem is due to Snell [51]. A more detailed exposition of optimal stopping theory is presented in A. N. Shiryaev's [40] and in Y. S. Chow *et al.*'s monographs [46].

Chapter 2

The definition and construction of arbitrary controlled (in general non-Markov) objects with continuous time given in the second chapter is along the lines of A. V. Skorohod's paper [50] cited above as well as N. I. Portenko and A. V. Skorohod's work [37]; however, our definition is wider in scope and more far reaching. This construction of the representation of a controlled object appears in the literature presumably for the first time.

Optimal controlled semi-Markov processes with continuous time are discussed by L. D. Stone [52].

Chapter 3

The theory of stochastic differential equations is studied in more detail in the following books: [19] and [20] vol 3, for example.

Initially the development of the theory of optimal control for solutions of stochastic differential equations was concerned solely with linear systems with a square cost of control and with problems related to the theory of "Bellman's equation".

Section 5 deals with the problem of control for a solution of a linear system of stochastic differential equations with complete observations of a somewhat more general type than usually considered in the literature, namely the random perturbations affecting the system may be general continuous processes with independent increments and finite moments of the second order.

A review of the results dealing with the theory of linear systems is presented for instance in A. W. Wonham's paper [45].

In connection with Bellman's equation and its justification for stochastic processes with continuous time, the reader is referred to the works of H. J. Kushner [33], W. Fleming [16], [17], and N. W. Krylov [30–32], where the results of the theory of parabolic equations with partial derivatives are substantially utilized. The most far-reaching study of controlled diffusion processes based on a direct probabilistic approach to Bellman's equation is presented in N. W. Krylov's monograph [49]; see also W. H. Fleming and R. W. Rishel's memoir [47].

The convergence of finite-difference approximations in the problem of optimal control for solutions of stochastic differential equations was considered in the papers of A. V. Skorohod [50], A. P. Gatun and I. I. Gihman [48].

The material in Sections 6 and 7 is based on M. H. A. Davis and P. P. Varaiya's [11] and M. H. A. Davis' contributions [10] (see also V. E. Benes [5–6].)

In this monograph only the case of complete observations is studied. A more general case of controlling incomplete observations is discussed in [11]. The construction of ε-optimal Markov controls is based on N. I. Portenko and A. V. Skorohod's exposition [37].

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