Answers and Hints to Selected Exercises

EXERCISES 1.1

1 (ii). $\sigma(\mathbf{a}) = \sqrt{2}$, $\sigma(\mathbf{b}) = \sigma(\mathbf{c}) = 0$, $\sigma(\mathbf{d}) = 1$.

EXERCISES 1.3

2 (i). Hint: Note that $\tau_{ij} \equiv (1/2)(\tau_{ij} + \tau_{ji}) + (1/2)(\tau_{ij} - \tau_{ji})$.

EXERCISES 2.1

2. $\mathscr{X}^i(t) = \pm\sin(t - c^i)$.

EXERCISES 2.2

1. $S(\gamma) = 2$.

EXERCISES 2.4

1. Hint: $\tau^{i_1\cdots i_r}{}_{j_1\cdots j_s}(x) = a^{i_1}{}_{k_1}\cdots a^{i_r}{}_{k_r}\ell^{m_1}{}_{j_1}\cdots\ell^{m_s}{}_{j_s}\hat{\tau}^{k_1\cdots k_r}{}_{m_1\cdots m_s}(\hat{x})$.

EXERCISES 3.2

2. Hint: Note that the 4×4 identity matrix does not belong to this subset.

EXERCISES 3.4

3(i). Hint: $\dfrac{\partial}{\partial x} = \dfrac{\partial}{\partial v} + \dfrac{\partial}{\partial u}, \dfrac{\partial}{\partial t} = \dfrac{\partial}{\partial v} - \dfrac{\partial}{\partial u}$.

EXERCISES 4.1

1. Hint: Assuming $\hat{x}^\alpha = -x^\alpha$ prove that $UPU^\dagger + P = 0$, or $U\sigma_\alpha + \sigma_\alpha U = 0$. (The last equation leads to a contradiction.)

EXERCISES 4.2

1(i). Hint: Compare the problem 2(ii) of Exercises 1.3.

EXERCISES 4.3

2. $\mathscr{T} = \gamma^4\gamma^5$.

EXERCISES 5.1

1. $H(\theta, p_\theta) = (p_\theta^2/2m\ell^2) + mg\ell(1 - \cos\theta)$.

EXERCISES 5.3

1. $M^\alpha{}_\beta(v) = m(1 - \delta_{\mu\nu}v^\mu v^\nu)^{-1/2}[\delta^\alpha{}_\beta + (1 - \delta_{\rho\sigma}v^\rho v^\sigma)^{-1}v^\alpha v_\beta]$.

EXERCISES 5.4

2. Hint: $\dfrac{\partial\Lambda(x,u)}{\partial x^j} = \dfrac{\partial\hat{X}^k(x)}{\partial x^j}\dfrac{\partial\hat{\Lambda}(\hat{x},\hat{u})}{\partial\hat{x}^k} + u^b\dfrac{\partial^2\hat{X}^k(x)}{\partial x^b\partial x^j}\dfrac{\partial\hat{\Lambda}(\hat{x},\hat{u})}{\partial\hat{u}^k}$,

$$\frac{\partial\Lambda(x,u)}{\partial u^j} = \frac{\partial\hat{X}^k(x)}{\partial x^j}\frac{\partial\hat{\Lambda}(\hat{x},\hat{u})}{\partial\hat{u}^k}.$$

EXERCISES 6.1

1. Hint: $\mathscr{L}(x, \zeta, \bar{\zeta}, \zeta_k, \bar{\zeta}_k) = -d^{kj}\zeta_k\bar{\zeta}_j + k\cos\zeta$.

EXERCISES 6.2

4. Hint: $\dfrac{\partial\psi}{\partial x^1} = (\cos\alpha)\dfrac{\partial h}{\partial\omega}, \quad \dfrac{\partial\psi}{\partial x^2} = \left[(\sin\alpha)\dfrac{\partial h}{\partial\omega} + \dfrac{\partial h}{\partial\xi}\right]$,

$$\frac{\partial\psi}{\partial x^3} = i\left[\frac{\partial h}{\partial\omega} + (\sin\alpha)\frac{\partial h}{\partial\xi}\right], \quad \frac{\partial\psi}{\partial x^4} = (\cos\alpha)\frac{\partial h}{\partial\xi}.$$

EXERCISES 6.3

2. Hint: Use special Minkowski coordinates such that

$$F_{34}(x) = F_{12}(x) \equiv 0.$$

Index of Symbols

$\mathscr{A}(F)$ action functional of a relativistic field
$A_k(x)$ electromagnetic field potential
$\mathbf{a}\cdot\mathbf{b}$ inner product between two vectors
$A \times B$ Cartesian product of two sets
$M \times N$ Kronecker product of two matrices
$B^A{}_i(x)$ gauge potential components
$\mathscr{C}^2(D;\mathbb{R})$; $\mathscr{C}^2(D;S)$ the set of twice differentiable functions from D into $\mathbb{R}$, S respectively
$C^A{}_{BD}$ structure constants of a Lie algebra
$\chi(p) = x = (x^1, x^2, x^3, x^4)$ local coordinates of a point p in the manifold
(χ, U) chart in a differentiable manifold
$D = [d_{ij}] \equiv \text{diag}[1, 1, 1, -1]$ Lorentz metric; a domain $D \subset \mathbb{R}^n$
∂D boundary of a domain D
$\square$ d'Alembertian or wave operator, also denotes the completion of an *example*
$\blacksquare$ Q.E.D.
$D_a\psi^K(x)$ gauge covariant derivative
$\delta(k - k')$ Dirac delta function
$\delta^a{}_b$ Kronecker delta
$\Delta\tilde{\omega}$ perihelion shift per revolution
$\partial(\hat{x}^1, \hat{x}^2, \hat{x}^3, \hat{x}^4)/\partial(x^1, x^2, x^3, x^4)$ Jacobian of a coordinate transformation
$\mathbb{E}_3$ three-dimensional Euclidean space
$\{\mathbf{e}_i\}_1^4 \equiv \{\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3, \mathbf{e}_4\}$ a basis set or tetrad
$\mathbf{e}_i \cdot \mathbf{e}_j = d_{ij}$ an M-orthonormal tetrad
$E(\mathbf{p})$ frequency associated with Dirac plane wave
$\in$ belongs to
ε_{ijkl} totally antisymmetric permutation symbol (Levi–Civita)
η_{ijkl} totally antisymmetric pseudotensor (Levi–Civita)
$F^i(x, u)$ four-force components
F_{ij} electromagnetic tensor components (field)
F^*_{ij} Hodge dual of electromagnetic tensor components
$F^A{}_{ij}(x)$ gauge field components

ϕ^A spinor (field) components
G a group
g_{ij} metric tensor components
$g_{ij}(x, u)$ Finsler metric components
$\gamma\colon x^i = \mathscr{X}^i(t)$ a parameterized curve on the manifold
γ^k Dirac matrices
$\gamma^{ku}{}_v$ entries of Dirac matrices, also numerical bispinor-tensor components
$\gamma_{ab}(x)$ linearized gravitational ten-potential components
$H(t, \mathbf{x}, \mathbf{p})$ (prerelativistic) Hamiltonian function
$I = [\delta^i{}_j]$ the identity matrix
$J[\gamma]$ prerelativistic action functional
$J_h[\tilde{\gamma}]$ prerelativistic canonical action functional
$J_L[\gamma]$ relativistic action functional
$J_H[\tilde{\gamma}]$ relativistic canonical action functional
$\mathscr{J}^i{}_{ab}(x)$ relativistic angular momentum tensor density
$j^k(x)$ charge-current four-vector density
J_{ab} the total relativistic angular momentum tensor
$\mathbf{L}$, $L \equiv [l^i{}_j]$ a linear (or Lorentz) mapping and the corresponding matrix
L^T the transposed matrix
$L(t, \mathbf{x}, \mathbf{v})$ prerelativistic Lagrangian
$L'(t, \mathbf{x}, t', \mathbf{x}')$ prerelativisitc Lagrangian in space *and* time
$\mathscr{L}(x, y^A, y^A{}_i)$ the Lagrangian of a field
λ a scalar (a real or complex number)
$\Lambda(x, u)$ relativistic Lagrangian of a particle
$\mathscr{L}_4$ general Lorentz group
$\mathscr{L}_{4+}^{+}$ proper, orthochronous subgroup of $\mathscr{L}_4$
$\mathbf{M}_4$ (flat) Minkowski space–time manifold
$\mathscr{N}_{x_0}$ Null cone with vertex at x_0
$\nu(\mathbf{k})$ frequency associated with electromagnetic plane wave
$\omega(\mathbf{k})$ frequency associated with Klein–Gordon plane wave
$\Omega(x, p) = 0$ constraining mass hypersurface, Super-Hamiltonian
$\mathbf{P}$ space reflection mapping
P_a the total four-momentum vector
p_k four-momentum components
$\mathscr{P}_4$ Poincaré group
$\mathscr{Q}$ the total charge
$\mathbb{R}$ the set of real numbers
$\mathbb{R}^n \equiv \mathbb{R} \times \cdots \times \mathbb{R}$ (n factors) Cartesian product of the set $\mathbb{R}$
$S(\gamma)$ separation functional on the curve γ
$\mathrm{SU}(N)$ the set of unitary unimodular matrices
$\sigma(\mathbf{v})$ separation number of the vector $\mathbf{v}$
σ_α Pauli matrices
$\sigma^{k\bar{A}B}$ numerical spinor-tensor
$\psi(x)$ Klein–Gordon field
ψ^u, $\psi^u(x)$ bispinor, Dirac field components

$\Sigma_1, \Sigma_2, \Sigma_3$ 1-flat, 2-flat, 3-flat submanifolds
T time reversal mapping
$T^i{}_j(x)$ canonical energy-momentum-stress tensor density
$\tau^{i_1 \cdots i_r}_{j_1 \cdots j_s}$ Minkowski tensor components
$\tau^{i_1 \cdots i_r}_{j_1 \cdots j_s}(x)$ Minkowski tensor field components
$\theta_{ij}(x)$ symmetrized energy-momentum-stress tensor density
$\tilde{u}, u_i$ covariant vector (or covector) and components
$u^i(s)$ four-velocity components
$V(\mathbf{x})$ the potential energy
$\mathbf{V}_4$ Minkowski vector space
$\mathbf{v}, v^i$ Minkowski vector, components
$z \equiv x^1 + ix^2$ a complex variable
$\bar{z} \equiv x^1 - ix^2$ the complex conjugate variable
$\mathbb{Z}$ the set of integers
$\mathbb{Z}^+$ the set of positive integers

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