

Index

Symbols

$\mathbf{A}(f : x, \delta)$, 70
 $\mathbf{A}(u : y, \rho)$, 16
 A^0 , 331
 A^f , 242
 $\mathcal{A}(x)$, 431
 $\mathcal{A}(\mathcal{F})$, 191
 A^\pm , 331
 $B_{x,\rho}$, 1
 \mathcal{B}_x , 127
 $\mathcal{B}(X)$, 5
 C , 312
 $C(n, a, b)$, 331
 $C(\Gamma)$, 180
 $C(\Gamma, E)$, 195
 $C_\infty^0(\mathbb{R}^n)$, 3
 $C_0^0(\Omega)$, 3
 $C^0(\Omega^-)$, 3
 $C^k(\Omega)$, 3
 $C^k(\Omega^-)$, 3
 $C_0^k(\Omega)$, 3
 $C^k(\Omega \cup \Sigma)$, 323
 $C_{z,\alpha}$, 102
 C_* , 181
 C^* , 181
 \mathcal{C} , 181
 C_a , 428
 $\text{cl} A$, 242
 D_i , 3
 D_{ij} , 3
 D_{x_i} , 3
 $D_{x_i x_j}$, 3
 $D_{(x)}^\beta$, 3
 D^β , 3

$D^j u$, 299
 $D_v^+ u$, 366
 $\mathcal{D}(A)$, 6
 $\mathcal{D}_s \mu$, 102
 $\mathcal{D} \mu$, 106
 d , 290
 $d(\Omega)$, 290
 $d(x, y)$, 289
 $d(F)$, 106
 $d(\mathcal{F}_0)$, 115
 d_x , 290
 $\tilde{d}(x)$, 303
 \tilde{d}_x , 303
 \tilde{d}_{xy} , 303
 $\tilde{d}(x, y)$, 303
 $\text{div } v$, 10
 \mathcal{E} , 268
 \mathcal{E}^+ , 260
 F^- , 1
 ∂F , 1
 $\text{cl } F$, 1
 $\text{int } F$, 1
 $\sim F$, 1
 $f \circ g$, 37
 $\mathcal{F}(\Gamma)$, 277
 f^* , 39
 $f\text{-lim}$, 242
 $f\text{-limsup}$, 242
 \mathbf{G}_Ω^Δ , 206
 $\mathbf{G}_\Omega f$, 146
 $\mathbf{G}_{1/2}$, 396
 $\mathbf{G}_\Omega \mu$, 138
 $G_\Omega^\Delta(x, \cdot)$, 206
 G , 125
 G_Ω , 125

- G_B , 19
 $G_{1/2}$, 392
 $\mathcal{G}(A)$, 468
 $\text{ghm}_{\Omega}h$, 132
 \underline{H}_f , 88
 \overline{H}_f , 88
 H_f , 89
 $H_{f,g,h}$, 442
 $\overline{H}_{f,g,h}$, 442
 $H_{f,g,h}^+$, 442
 $H_{k+\alpha}(\Omega)$, 286
 $H_{k+\alpha}^{(b)}(\Omega)$, 303
 $H_{k+\alpha,\text{loc}}(\Omega)$, 286
 $\mathcal{H}_{f,g}$, 378
 $\mathcal{H}_{f,g}^+$, 378
 $\mathcal{H}_{f,g}^-$, 378
 $H_{2+\alpha}^{(b)}(\Omega; \phi)$, 290
 $H_{k+\alpha}^{*(b)}(\Omega)$, 449
 h , 392
 $\text{int}_f A$, 242
 $J(x^*, x)$, 39
 J_h , 82
 \mathbf{K} , 323
 K^+ , 152
 $K_B(x, z)$, 55
 $K_{z,\alpha}$, 102
 K_0^+ , 152
 $K^+(\Omega)$, 152
 $\mathcal{K}(\Omega)$, 180
 $k(x, y)$, 321
 $k(|x - y|)$, 331
 \mathbf{L} , 353
 \mathbf{L}_0 , 356
 $\mathbf{L}(f : x, \delta)$, 70
 $\mathbf{LS}(x, B, g)$, 375
 $\mathbf{LS}(x, U, \phi)$, 433
 $\mathbf{L}(u : y, \rho)$, 16
 L_x , 114
 $\mathcal{L}(\Omega)$, 71
 \mathfrak{L}_f , 87
 \mathfrak{L}_f^{Ω} , 88
 $\mathfrak{L}(f, g)$, 377, 434
 $\mathfrak{L}(f, g, h)$, 436
 \mathbf{M} , 353
 \mathbf{M}_0 , 392
 M , 358
 m , 358
 m, M , 356
 N^{∞} , 191
 $\mathcal{N}(x)$, 429
 $\mathcal{N}(F, \epsilon)$, 107
 \bar{n} , 191
 $\mathbf{n}(x)$, 10
 $\mathcal{O}(\Omega)$, 180
 $\mathbf{PI}(c, \mu, \Omega)$, 42
 $\mathbf{PI}(f : B)$, 30
 $\mathbf{PI}(c, f, \Omega)$, 43
 $\mathbf{PI}(f, \sim B_{y,\rho})$, 219
 $\mathbf{PI}(\mu : B)$, 30
 $\mathbf{PI}(\mu : y, \rho)$, 30
 $P_{m,n}$, 60
 $P(d(\Omega))$, 296
 \mathbf{R} , 180
 \mathbf{R}_{Γ} , 180
 $\hat{\mathbf{R}}_{\Gamma}$, 180
 $\hat{\mathbf{R}}$, 180
 $\hat{\mathbf{R}}_{\Delta}^u$, 175
 \mathbf{R}_{Δ}^u , 175
 $\mathcal{R}(A)$, 6
 R^n , 1
 R_{∞}^n , 215
 R_0^n , 331
 R_{\pm}^n , 331
 $\hat{R}_{\Delta}(u, \cdot)$, 206
 (r, θ) , 1
 $\mathcal{S}(\Omega)$, 71
 $\mathcal{S}^+(\Omega)$, 71
 $S_{x,\rho}$, 1
 $S_{z,\theta}$, 112
 $\|T(t)\|$, 468
 $\mathbf{U}f$, 336
 \mathbf{U}^{μ} , 27, 143
 \mathfrak{U}_f , 87
 \mathfrak{U}_f^{Ω} , 88
 $\mathfrak{U}(f, g)$, 377, 434
 $\mathfrak{U}(f, g, h)$, 436
 \hat{u} , 4
 u_{∞} , 132
 $u_{\infty,\Lambda}$, 132
 u_B, u^B , 80, 376
 u_U , 436
 $u \sim_B$, 224
 $u(|x - y|), u_y(x)$, 12
 $u_y(x)$, 12
 $\check{V}_{x,u}$, 434
 v^U , 436
 w_j^+ , 379
 w_j^- , 379
 w_j^{\pm} , 379
 x , 1
 $x \cdot y$, 1
 (x', x_n) , 222
 x' , 222
 x^* , 18, 38
 $Y_{n,m}$, 60

y^r , 45
 β_t , 393
 $\beta!$, 3
 β_n , 393
 γ_j , 327
 χ_C , 115
 $\delta_\Omega(x, \cdot)$, 206
 $\delta_\Omega^\Delta(\cdot, u)$, 211
 $\delta_\Omega^\Delta(\mu, \cdot)$, 211
 $\delta_\Omega^\Delta(x, \cdot)$, 206
 ε , 1
 ϵ_j , 291
 θ_r , 462
 Λ^r , 45
 λ_0 , 414
 $\mu, \mu^+, \mu^-, |\mu|$, 5
 μ_n , 5
 $\mu_{\mathcal{F}}$, 275
 μ_Γ , 180
 $\mu_{\mathcal{F}(\Gamma)}$, 277
 μ_x^Ω , 235
 μ_x , 114
 ν_n , 2
 $\nu_n(\rho)$, 2
 ν , 414
 Ω , 3
 Ω_δ , 290, 327
 $\widetilde{\Omega}_\delta$, 303
 Ω_0 , 345
 Ω_\pm , 345
 Ω , 462
 $\partial_\infty \Omega$, 215, 216
 Φ_Λ^u , 174
 ψ , 349
 σ_n , 2
 $\sigma_n(\rho)$, 2
 $\sigma(\mathfrak{Q})$, 474
 $|f|_{k+\alpha, \Omega}^{*(b)}$, 449
 $|g|_{k+\alpha, \Sigma}^{*(b)}$, 449
 $\|f\|_{0, \Omega}$, 3
 $\|f\|_k$, 286
 $\|f\|_\infty$, 3
 $|g|_{1+\alpha, \Sigma}^{(1+b)}$, 305
 $|g; d|_{1+\alpha, \Sigma}$, 400
 $|g|_{1+\alpha, \Sigma}$, 288
 $|u|_{k+0, \Omega}$, 286
 $|u|_{k+\alpha, \Omega}$, 286
 $|u|_{k, \Omega}$, 286
 $[u]_{k+\alpha, \Omega}$, 286
 $[u; \phi]_{k+0, \Omega}^{(b)}$, 289
 $[u; \phi]_{k+\alpha, \Omega}^{(b)}$, 289
 $[u; \phi]_{k, \Omega}^{(b)}$, 289

$|u; \phi|_{k+\alpha, \Omega}$, 290
 $[u]_{k+\alpha, \Omega}^{(b)}$, 303
 $|x|$, 1
 $|\beta|$, 3
 $|\lambda|_e^2$, 268
 $|\mu|_e$, 268
 $|\mu|_e^2$, 259
 $\|\cdot\|_{\mathfrak{X}}$, 6
 Δu , 11
 $\Delta(x)$, 11
 \wedge , 303
 ∞ , 215
 $[\lambda_1, \lambda_2]_e$, 268
 $[\mu, \nu]_e$, 259
 ∇ , 3
 $\theta - \lim_{x \rightarrow z} u(x)$, 112

A

a.e., 76
 Adapted, 465
 Admissible, 419
 Admissible neighborhood, 431
 Alternating method, 231
 Analytic, 191
 Approximate barrier, 379
 Approximate lower barrier, 379
 Approximate upper barrier, 379
 Apriori inequality, 391
 Arzelá-Ascoli theorem, 5
 Averaging principle, 16

B

Bôcher's theorem, 123
 Balayage, 131
 Banach space, 6
 Barrier, 93, 229
 Barrier, approximate, 379
 Barrier, approximate lower, 379
 Barrier, approximate upper, 379
 Borel measure, 3, 5
 Borel sets, 5
 Bouligand, 93, 170, 233
 Boundary point lemma, 366, 414, 431
 Bounded linear operator, 354
 Bounded map, 354
 Bounded variation, 5
 Brelot, 118, 171, 219, 250, 252, 257

C

Capacitable, 181
 Capacity distribution, 180

Capacitary potential, 180
 Capacity, 180, 181
 Capacity, inner, 181
 Capacity, outer, 181
 Cartan, 152, 155, 183, 256, 274
 Cauchy initial value problem, 454
 Choquet, 194
 Choquet's lemma, 67
 Closable linear operator, 468
 Closed linear operator, 468
 Concave, 84
 Conservative, 7
 Continuity method, 354
 Continuum, 97
 Contraction map, 245, 354
 Contraction semigroup, 468
 Convex, 74, 275
 Countably strongly subadditive, 186

D

d-system, 115
 Daniell-Stone theorem, 474
 Derivate, 106
 Diffeomorphism, 428
 Differentiable of class C_a , 428
 Dini's formula, 49
 Directed set, 5
 Dirichlet problem, 29, 86
 Dirichlet region, 92
 Dirichlet solution, 89
 Divergence, 10
 Divergence theorem, 10
 Domination principle, 133, 182
 Dynkin system, 115

E

Elliptic, 358
 Elliptic operator, 353
 Elliptic, strictly, 358
 Energizable, 263
 Energy, 259, 268
 Energy principle, 266
 Equicontinuity, 4
 Evans and Vasilescu, 161
 Exterior Dirichlet problem, 215
 Exterior sphere condition, 96

F

Fatou, 113
 Fine closure, 242
 Fine interior, 242

Fine limit, 242
 Fine limsup, 242
 Fine neighborhood, 242
 Fine topology, 241
 Fine topology on R_∞^n , 255
 Force field, 6
 Fourier series, 47
 Frostman, 264
 Fundamental harmonic function, 12

G

Gauss, 264
 Gauss' averaging principle, 27
 Gauss' integral theorem, 13
 Gauss's integral, 266
 Gauss-Frostman, 264
 Gradient, 3
 Greatest harmonic minorant, 132
 Green function, 19, 121, 125
 Green function, half space, 221
 Green function, Neumann problem, 55
 Green function, second kind, 55
 Green potential, 138
 Green's identity, 11
 Green's representation theorem, 13
 Greenian set, 121, 125

H

Hölder continuous, 327
 Hölder spaces, 286
 Hörmander, 294
 Hörmander's convexity theorem, 296
 Harmonic, 12, 216
 Harmonic function, 12
 Harmonic measure, 235
 Harmonic minorant, 131
 Harnack's inequality, 64
 Herglotz, 29
 Herglotz theorem, 36
 Hervé, 380
 Hille-Yosida theorem, 468
 Homogeneity, 449
 Hopf, 366–368
 Hyperharmonic, hypoharmonic, 87, 224

I

Image, 130
 Indicator function, 115
 Infinitesimal generator, 468
 Inner capacity, 181
 Inner polar set, 166

Inner q.e., 166
 Inner quasi everywhere, 166
 Interior estimates, 358
 Interior sphere condition, 366
 Interpolate, 294
 Interpolation inequalities, 299
 Interpolation, global, 298
 Interpolation, local, 306
 Inverse, 18
 Inversion, 18, 38
 Irregular boundary point, 92, 144

J

Jacobian, 39
 Jensen's inequality, 74

K

Kelvin transform, 39
 Kelvin transformation, 38, 225
 Kernel, 321

L

L-regular boundary point, 380
 l-resolutive, 378
 l.s.c., 4
 Laplacian, 11
 Lebesgue, 97
 Lebesgue measure, 5
 Lebesgue points, 111
 Lebesgue spine, 144
 Left-directed, 66
 Lift, 436
 Lifting, 80, 376, 436
 Linear map, 354
 Linear operator, 6, 468
 Lipschitz continuous, 327
 Local barrier, 93, 229
 Local interpolation, 306
 Local property, 169
 Local solution, 433
 Locally Hölder continuous, 327
 Locally integrable, 5
 Locally super-mean-valued, 70
 Logarithmic potential, 143
 Lord Kelvin, 18
 Lower, 436
 Lower class, 87
 Lower regularization, 4
 Lower semicontinuous, 4
 Lowered, 223
 Lowering, 80, 376, 436

M

Maria-Frostman, 182
 Markov process, 463
 Markov semigroup, 473
 Martingale, 465
 Maximum principle, 23
 Maximum principle, strong, 367, 432
 Maximum principle, weak, 365
 Mean value property, 15
 Mean value theorem, 4
 Mean valued, 16, 70
 Measurable map, 245
 Method of continuity, 354, 373, 425
 Method of images, 130
 Method of separation of variables, 99
 Miller, 386
 Minimum principle, 23
 Mokobodzki, 202
 Mollifier, 81
 Multi-index, 2
 Mutual energy, 259, 268
 Myrberg's theorem, 199

N

Net, 5
 Neumann problem, 47
 Neumann problem for a ball, 51
 Newton's Law, 121
 Newtonian potential, 143, 335
 Nonnegative definite, 364
 Nontangential limit, 112
 Norm, 5, 6
 Normed linear space, 6
 Nucleus, 191

O

Oblique derivative problem, 354
 Outer capacity, 181
 Outer unit normal, 10

P

Partition of unity, 148
 Path, 6
 Path integral, 7
 Paved space, 191
 Paving, 191
 Peaks, 247
 Perron lower solution, 87
 Perron subsolution, 378, 442
 Perron supersolution, 378, 442

Perron upper solution, 87
 Perron-Wiener method, 86
 Picard, 23
 Poincaré, 96, 131, 381
 Point at infinity, 215
 Poisson integral, 30, 41, 219
 Poisson integral for half-space, 41
 Poisson integral formula, 20
 Poisson's equation, 335
 Polar cap, 102
 Polar representation, 385
 Polar set, 99, 166, 224
 Pole, 12
 Positive definite, 364, 430
 Positive semigroup, 468
 Potential, 138
 Potential function, 7, 121
 Pre-integral, 474
 Projection, 276
 Proof of Theorem 2.6.28, 250
 Property B, 182
 PWB method, unbounded regions, 222

Q

q.e., 166
 Quasi everywhere, 166

R

Random variable, 463
 Reciprocity law, 151
 Reduction, 132, 175
 Reflection, 45
 Reflection principle, 345
 Regular, 442
 Regular Borel measure, 5
 Regular boundary point, 92, 229
 Regular region, 92
 Regular, (\mathbf{L}, \mathbf{M}) -, 442
 Regularized reduction, 175
 Representation, 473
 Resolutive, 225
 Resolutive boundary function, 89
 Resolutive, (\mathbf{L}, \mathbf{M}) -, 442
 Riesz decomposition theorem, 159
 Riesz measure, 159
 Riesz potential, 322
 Riesz representation theorem, 159
 Right-directed, 66

S

Saturated, 86, 224
 Schauder interior estimates, 358

Schwarz, 31, 170, 231
 Schwarz's Reflection Principle, 345
 Semigroup, 467
 Seminorm, 6
 Separation of variables, 99
 Sequences of harmonic functions, 64
 Shift operator, 462
 Sibony, 202
 Sierpinski, 193
 Signed measure, 5
 Spherical cap, 306
 Spherical cap, admissible, 419
 Spherical coordinates, 1
 Spherical harmonic, 60
 Spine, Lebesgue, 144
 Stolz domain, 112
 Stone vector lattice, 473
 Stopping time, 463
 Straighten boundary, 427
 Stratifiable, 314
 Strictly elliptic, 358
 Strong barrier, 93
 Strong convergence, 271
 Strong Markov property, 464
 Strong maximum principle, 367, 432
 Strongly continuous semigroup, 468
 Strongly subadditive, 186
 Strongly subadditive, countably, 186
 Subfunction, 375, 434
 Subharmonic, 71, 216
 Submartingale, 465
 Subnewtonian kernel, 322
 Subsequence selection principle, 287
 Subsolution, 377
 Super-mean-valued, 70
 Superfunction, 375, 433
 Superharmonic, 70, 72, 73, 216
 Superharmonic, alternative definition, 72
 Superharmonic, yet another definition, 73
 Supermartingale, 465
 Supersolution, 377, 436
 Suslin scheme, 191
 Sweeping, 206
 Sweeping out, 131
 Swept measure, 206
 Symmetric deriviate, 102

T

Thin, 243, 255
 Tonelli, 27
 Total, 152
 Total variation, 5
 Transition function, 461

U

U.s.c., 4
Uniform equicontinuity, 4
Uniformly Hölder continuous, 327
Upper class, 87
Upper inner normal derivate, 366
Upper semicontinuous, 4

V

Vague convergence, 271
Vasilesco, 171
Vector field, 6
Vector lattice, 473
Vertex, 277
Vitali covering, 107

W

W^* -topology, 5
Weak convergence, 271
Weak maximum principle, 365
Wedge conditions, 446
Wiener, 92, 179
Wiener's test, 279

Z

Zaremba, 97
Zaremba's cone condition, 97, 283, 388
Zero-one law, 211