

# Bibliography

- [1] Morton L. Curtis, *Matrix Groups*, second edition, Universitext Series, Springer-Verlag New York, Inc. (1984).
- [2] Paul R. Halmos, *Finite-Dimensional Vector Spaces*, second edition, Undergraduate Texts in Mathematics Series, Springer-Verlag New York, Inc. (1974).
- [3] F. Reese Harvey and H. Blaine Lawson, Jr., "Calibrated Geometries," *Acta. Math.* 148 (1982), 47-157.
- [4] I.N. Herstein, *Topics in Algebra*, second edition, John Wiley & Sons (1975).
- [5] Serge Lange, *Linear Algebra*, second edition, Addison -Wesley Publishing Co., Inc. (1971).
- [6] Seymour Lipschutz, *Linear Algebra*, Schaum's Outline Series, McGraw-Hill Book Co. (1968).

# Index

- abelian group, 2
- absolute value, 6
- adjoint, 129
- algebra, 18, 56
  - alternative, 154
  - associative, 18, 57
  - commutative, 57
  - division, 155
  - normed, 155
- algebraic multiplicity, 79, 105
- algebraically closed field, 4
- annihilator, 45
- associator, 60, 145
  
- basis, 22
- bilinear form, 113
- block diagonal matrix, 103
  
- Cayley numbers, 151
- Cayley-Dickson formula, 156
- Cayley-Dickson Process, 157
- Cayley-Hamilton Theorem, 84, 95
- change-of-basis matrix, 53, 116
- characteristic of field, 4
- characteristic polynomial, 83, 95
- classical adjoint, 71
- coefficients of a polynomial, 87
- coefficients of linear system, 72
- cofactor, 71
- column, 48
- column space, 55
  
- commutator, 60, 145
- comparable, 44
- complementary subspaces, 28
- complex numbers, 6
- conjugate, 53
  - in normed algebra, 145
  - of complex number, 6
  - of a matrix, 125
  - of quartenion, 144
- coordinate vector, 51
- coset, 32
- curve, 133
  - differentiable, 133
  - product, 133
  
- degree of polynomial, 87
- determinant, 63
- diagonal matrix, 14, 65
- diagonalizable, 104, 106
- dimension, 24, 36
- direct sum
  - external, 28
  - internal, 30
- distance, 111
- divide, 90
- Division Theorem, 88
- divisor of zero, 3
- dual basis, 42
- dual space, 41
  
- echelon form, 69

- eigenspace, 39
- eigenvalue, 39, 76, 84
- eigenvector, 39
- elementary column operations, 70
- elementary matrix, 69
- elementary row operations, 68
- elimination, 73
- endomorphism, 18
- $\text{End}(V)$ , 18
- epic, 16
- exact sequence, 31
- extension field, 4
- exterior algebra
  - on  $\mathbb{R}^3$ , 58
  - on  $\mathbb{R}^n$ , 61
  
- Factor Theorem**, 89, 90
- field, 2
- finite-dimensional, 22
- finite set, 22
- functional, 41, 120
- fundamental theorem
  - of algebra, 4
  
- general linear group**, 19, 65
- generates, 12
- geometric multiplicity, 39, 105
- Gram-Schmidt orthonormalization**
  - process, 118
- greatest common divisor, 90
- group, 1
  
- Hamel basis**, 24
- Hermitian form, 124
- Hermitian inner product, 123
- Hermitian linear, 124
- Hermitian matrix, 128
- Hermitian symmetric, 124
- Hilbert Space, 117
  
- homogeneous system
  - of equations, 76
- homomorphism, 82
- Hurwitz Theorem, 160
  
- $i, j$  entry, 48
- ideal, 82
- idempotent, 77, 97
- idempotent map, 20
- identity map, 15
- identity matrix, 52, 57
- image, 16
- imaginary part, 143, 145
- indeterminate, 87
- infinite-dimensional, 22
- infinite set, 22
- injective, 16
- inner product, 111
- inner product space
  - complex, 124
  - real, 116
- integral domain, 85
- inverse, 52
- irreducible, 91
- isomorphism of
  - vector spaces, 16
  
- Jacobi identity**, 60
- join, 36
- Jordan block, 103
- Jordan form, 104
  
- kernel, 16
  
- $\ell_2$ -space, 117
- Laplace expansion, 67
- left multiplication, 149
- left value, 89

- Lie Algebra, 60, 131
- Lie Group, 132
- line, 36
- linear combination, 11
- linear geometry of  $\mathbb{R}^3$ , 36
- linear map, 15
- linearly dependent, 21
- linearly independent, 21
  
- m**atrix, 14, 47
- matrix algebra, 58
- matrix multiplication, 50
- matrix representation
  - of a bilinear form, 115
  - of a linear map, 48
- maximal element, 44
- method of elimination, 13
- metric, 111
- minimal polynomial, 92
- monic map, 16
- monic polynomial, 7, 87
  
  
- n**atural map, 43
- nilpotent, 28, 77
- nondegenerate, 112
- nonsingular matrix, 15
- nonsingular, 52
- norm, 111, 125
- normal matrix, 138
- normed algebra, 141
- normed vector space, 120, 125
  
  
- o**ctonions, 151
- operator, 18
- orthogonal, 118, 125
- orthogonal complement, 122
- orthogonal group, 130
- orthogonal map, 118
- orthogonal matrix, 118, 130
  
  
- orthonormal, 118
- orthonormal basis, 118
  
  
- p**arallel, 36
- parallelogram law, 120
- partially ordered set, 43
- permutation, 6
- plane, 36
- point, 36
- polar form, 120
- polynomial function, 87
- positive definite bilinear form, 114
- positive definite matrix, 128
- preimage, 16
- projection, 20
  
  
- q**uadratic form, 116
- quaternions, 144, 151
- quotient space, 33
  
  
  
  
  
  
  
  
  
  
- r**ank, 55, 115
- Rank Theorem, 25
- real inner product space, 116
- real part, 143, 145
- relatively prime, 91
- Remainder Theorem, 90
- ring, 81
  - associative, 81
  - commutative, 81
  - homomorphism, 82
- root, 4, 87
- rotation group, 130
- row, 48
- row equivalent, 68
- row reduced, 69
- row space, 55
  
  
  
  
  
  
  
  
  
  
- scalar, 9

- Schwarz Inequality, 112, 115, 124
- similar matrices, 53
- simply ordered set, 44
- singular matrix, 15
- skew-Hermitian matrix, 128, 131
- skew-symmetric matrix, 130, 131
- span, 11, 12
- special linear group, 133
- special orthogonal group, 130
- special unitary group, 130
- Spectral Theorem, 98, 101
- split exact sequence, 31
- splitting, 31
- square matrix, 52, 57
- stable subspace, 37
- standard basis, 23
- subfield, 4
- subgroup, 5
- submatrix, 56
- subring, 82
- subspace, 11
- sum of subspaces, 14
- superdiagonal, 101
- surjective, 16
- symmetric bilinear form, 114
- symmetric matrix, 68
- symplectic group, 153
- symplectic matrix, 153
- system of equations, 72
- terms of a polynomial, 87
- torsion, 3
- trace, 54
- transpose, 65
- triangle property, 111
- triangular matrix, 65
- trivial solution, 72
  
- unit, 19, 81
- unit element, 18, 57, 81
- unit vector, 118
- unitary group, 125
- unitary map, 125
- unitary matrix, 125, 130, 134
- upper bound, 44
- upper triangular, 14, 65
  
- value, 89
- vector, 9
- vector space
  - over  $k$ , 10
  - real, 9
  
- Zorn's Lemma, 44