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Notational Conventions

Algebra As usual, $\mathbf{R}$ and $\mathbf{C}$ denote the fields of real and complex numbers, respectively, and $\mathbf{Z}$ the ring of integers. Let

$$\mathbf{R}^+ = \{t \in \mathbf{R} : t \geq 0\}, \quad \mathbf{Z}^+ = \mathbf{Z} \cap \mathbf{R}^+ .$$

If $\alpha \in \mathbf{C}$, $\operatorname{Re}\alpha$ denotes the real part of α, $\operatorname{Im}\alpha$ its imaginary part, $|\alpha|$ its modulus.

If G is a group, $A \subset G$ a subset and $g \in G$ an element, we put

$$A^g = \{gag^{-1} : a \in A\}\,, g^A = \{aga^{-1} : a \in A\}\,.$$

The group of real matrices leaving invariant the quadratic form

$$x_1^2 + \cdots + x_p^2 - x_{p+1}^2 - \cdots - x_{p+q}^2$$

is denoted by $\mathbf{O}(p,q)$. We put $\mathbf{O}(n) = \mathbf{O}(o,n) = \mathbf{O}(n,o)$, and write $\mathbf{U}(n)$ for the group of $n \times n$ unitary matrices. The group of isometries of Euclidean n-space $\mathbf{R}^n$ is denoted by $\mathbf{M}(n)$.

Geometry The $(n-1)$-dimensional unit sphere in $\mathbf{R}^n$ is denoted by $\mathbf{S}^{n-1}$, $\Omega_n = 2\pi^{n/2}/\Gamma(n/2)$ denotes its area. The n-dimensional manifold of hyperplanes in $\mathbf{R}^n$ is denoted by $\mathbf{P}^n$. If $0 < d < n$ the manifold of d-dimensional planes in $\mathbf{R}^n$ is denoted by $\mathbf{G}(d,n)$; we put $\mathbf{G}_{d,n} = \{\sigma \in \mathbf{G}(d,n) : o \in \sigma\}$. In a metric space, $B_r(x)$ denotes the open ball with center x and radius r; $S_r(x)$ denotes the corresponding sphere. For $\mathbf{P}^n$ we use the notation $\beta_A(0)$ for the set of hyperplanes $\xi \subset \mathbf{R}^n$ of distance $< A$ from 0, σ_A for the set of hyperplanes of distance $= A$. The hyperbolic n-space is denoted by $\mathbf{H}^n$ and the n-sphere by $\mathbf{S}^n$.

Analysis If X is a topological space, $C(X)$ (resp. $C_c(X)$) denotes the sphere of complex-valued continuous functions (resp. of compact support). If X is a manifold, we denote:

$$\begin{aligned}
C^m(X) &= \left\{ \begin{array}{l} \text{complex-valued } m\text{-times continuously} \\ \text{differentiable functions on } X \end{array} \right\} \\
\mathcal{C}^\infty(X) &= \mathcal{E}(X) = \cap_{m>0} C^m(X)\,. \\
\mathcal{C}_c^\infty(X) &= \mathcal{D}(X) = C_c(X) \cap \mathcal{C}^\infty(X)\,. \\
\mathcal{D}'(X) &= \{\text{distributions on } X\}\,. \\
\mathcal{E}'(X) &= \{\text{distributions on } X \text{ of compact support}\}\,. \\
\mathcal{D}_A(X) &= \{f \in \mathcal{D}(X) : \text{ support } f \subset A\}\,. \\
\mathcal{S}(\mathbf{R}^n) &= \{\text{rapidly decreasing functions on } \mathbf{R}^n\}\,. \\
\mathcal{S}'(\mathbf{R}^n) &= \{\text{tempered distributions on } \mathbf{R}^n\}\,.
\end{aligned}$$

The subspaces $\mathcal{D}_H$, $\mathcal{S}_H$, $\mathcal{S}^*$, $\mathcal{S}_o$ of $\mathcal{S}$ are defined in Ch. I, §§1–2.

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While the functions considered are usually assumed to be complex-valued, we occasionally use the notation above for spaces of real-valued functions.

The Radon transform and its dual are denoted by $f \to \widehat{f}$, $\varphi \to \check{\varphi}$, the Fourier transform by $f \to \widetilde{f}$ and the Hilbert transform by $\mathcal{H}$.

I^α, I_-^λ, I_o^λ and I_+^λ denote Riesz potentials and their generalizations. M^r the mean value operator and orbital integral, L the Laplacian on $\mathbf{R}^n$ and the Laplace-Beltrami operator on a pseudo-Riemannian manifold. The operators $\square$ and Λ operate on certain function spaces on $\mathbf{P}^n$; $\square$ is also used for the Laplace-Beltrami operator on a Lorentzian manifold, and Λ is also used for other differential operators.

Frequently Used Symbols

Ad:	adjoint representation of a Lie group, 259
ad:	adjoint representation of a Lie algebra, 259
$\mathfrak{a}$, $\mathfrak{a}_*$:	abelian subspaces, 272
$\mathfrak{a}'$:	regular set, 272
$\mathfrak{a}^+$:	Weyl chamber, 272
A_x:	antipodal set to x , 147
$A(r)$:	spherical area, 148
$B_r(p)$:	open ball with radius r, center p, 8
β_R:	ball in Ξ, 28
B:	Killing form, 260
$\mathbf{C}_n$:	special set, 241
$C(X)$:	space of continuous functions on X, 1
$\mathcal{D}(X)$:	$\mathcal{C}_c^\infty(X)$, 1
$\mathcal{D}'(X)$:	space of distributions on X, 70, 223
dg_k:	invariant measure on G/K, 69
$\mathcal{D}_K(X)$:	set of $f \in \mathcal{D}(X)$ with support in K, 221
$\mathbf{D}(G)$:	algebra of left-invariant differential operators on G, 256
$\mathbf{E}(X)$:	set of all differential operators on X, 253
$\mathcal{E}'(X)$:	space of compactly supported distributions on X, 70
$F(X)$:	space of rapidly decreasing functions, 179
$\mathcal{F}(X)$:	space of exponentially decreasing functions, 179
$f \to \dot{f}$:	mapping from G to G/K, 73
$f \to \widehat{f}$, $f \to \widehat{f}_p$:	Radon transforms, 77, 114
$\varphi \to \check{\varphi}$, $\varphi \to \check{\varphi}_p$:	dual transforms, 77, 114
φ_λ:	spherical function, 130
$\mathbf{G}(d,n)$:	manifold of d-planes in $\mathbf{R}^n$, 34
$\mathbf{G}_{d,n}$:	manifold of d-dimensional subspaces of $\mathbf{R}^n$, 34
$\mathbf{H}^n$:	hyperbolic space, 78, 118
$\mathcal{H}$:	Hilbert transform, 22, 57
I^α:	Riesz potential, 199, 236
Im :	imaginary part
$L^1(X)$:	space of integrable functions, 18
$L = L_X$:	Laplace-Beltrami operator, 3, 185
$L(g) = L_g$:	left translation by g, 254
Λ:	operator on $\mathbf{P}^n$, 22, 43
M_p:	tangent space to a manifold M at p, 253
M^r:	mean value operators, 8, 209
$\mathbf{M}(n)$:	group of isometries of $\mathbf{R}^n$, 3
$\mathbf{O}(n)$, $\mathbf{O}(p,q)$:	orthogonal groups, 9, 187
Ω_n:	area of $\mathbf{S}^{n-1}$, 9

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$\mathbf{P}^n$:	set of hyperplanes in $\mathbf{R}^n$, 2
$\mathfrak{p}$:	part of Cartan decomposition, 268
r^α:	special distribution, 237
Re:	real part
Res:	residue, 237
$\mathbf{S}^n$:	n-sphere, 3
$S_r(p)$:	sphere of radius r, center p, 8
$\mathcal{S}(\mathbf{R}^n)$:	space of rapidly decreasing functions, 5
$\mathcal{S}^*(\mathbf{R}^n)$, $\mathcal{S}_0(\mathbf{R}^n)$:	subspaces of $\mathcal{S}(\mathbf{R}^n)$, 10
$\mathcal{S}'(\mathbf{R}^n)$:	space of tempered distributions, 223
$\mathrm{sgn}(s)$:	signum function, 28
$\mathcal{S}_H(\mathbf{P}^n)$:	subspace of $\mathcal{S}(\mathbf{P}^n)$, 5
$\tau(x)$:	translation $gH \to xgH$ on G/H, 64
θ:	Cartan involution, 268
$U(\mathfrak{g})$:	universal enveloping algebra, 257
Ξ:	dual space, 63
x_+^α:	special distribution, 236
$\mathbf{Z}(G)$:	center of $\mathbf{D}(G)$, 259
$\sim$:	Fourier transform, 4, 226
$\widehat{\ }$:	Radon transform, incidence, 2, 68
$\vee$:	Dual transform, incidence, 2, 68
$\langle\ ,\ \rangle$:	inner product, 106
$\natural$:	K-invariance, 175
$\Box$:	operator, wave operator, 3, 196
$\lvert\ \rvert$:	norm, 4

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