Appendix I
LIST OF SYMBOLS

Table A-1. List of symbols.

Symbol	Unit	Meaning
A_{bulk}	-	BSIM parameter: bulk charge effect (see chapter 3.6)
af	-	SPICE noise parameter: current exponent (see chapter 3.6)
A_P	-	Available power gain of amplifier
C	F (F/cm^2)	Capacitance (per unit area)
C_{box}	F/cm^2	Capacitance per unit area of buried oxide (SOI)
C_d	F/cm^2	Depletion layer capacitance per unit area
C_{dm}	F/cm^2	Depletion layer capacitance per unit area when the depletion layer has its maximum width W_{dm}
C_{fb}	F/cm^2	Flat-band capacitance per unit area
C_{fox}	F/cm^2	Capacitance per unit area of front gate oxide (SOI)
C_G	F	Gate capacitance
C_{GS}	F	Gate-to-source capacitance
C_{it}	F/cm^2	Interface trap capacitance per unit area
C_{ox}	F/cm^2	Gate oxide capacitance per unit area
C_{Si}	F/cm^2	Capacitance per unit area of Si body (SOI)
d	m	Lattice constant
D_{it}	cm^{-2}eV^{-1}	Density of interface states (traps)
E	J, eV	Energy
E	V/cm	Electric field
$\mathrm{E_C}$	eV	Conduction band edge energy
E_{eff}	V/cm	Effective electric field (effective vertical field in inversion layer)
ef	-	BSIM and SPICE noise parameter: frequency exponent (see 3.6)
$\mathrm{E_F}$	eV	Fermi energy level

Symbol	Unit	Meaning
$E_{F,n}$	eV	Quasi-Fermi energy level for electrons
$E_{F,p}$	eV	Quasi-Fermi energy level for holes
E_g	eV	Band gap energy
E_i	eV	Intrinsic Fermi energy level
E_T	eV	Trap energy level
E_V	eV	Valence band edge energy
F	-	Noise factor
f	Hz	Frequency
$f(E)$	-	Fermi-Dirac distribution function, gives the probability that an electronic state at energy E is occupied
$f(X)$	-	Probability density function of random variable X
FF	-	Fano factor
f_T	Hz	Transition (cut-off) frequency
f_0	Hz	Oscillation frequency of VCO
g	-	Degeneracy factor
$g(\tau)$		Trap distribution function (chapter 1.3.5)
g_{ch}	S = A/V	Channel conductance
$g_{ch,0}$	S	Channel conductance at zero drain-source voltage
g_m	S	Transconductance
h	Js	Planck's constant (= 6.63×10^{-34} Js)
I	A	Current
i	A	Small-signal current
I_D	A	Drain current
$I_{D,sat}$	A	Drain current in saturation
I_G	A	Gate leakage current
i_n	A	Noise current
$\overline{i_n^2}$	A^2	Mean square of noise current
I_S	A	Source current
I_0	A	Diode saturation current
J	A/cm^2	Current density
k	J/K	Boltzmann's constant (= 1.38×10^{-23} J/K)
k	-	Dielectric constant (relative permittivity)
KF		SPICE noise parameter (see chapter 3.6)
L	cm	Gate length (length)
$L(\Delta f)$	dBc/Hz	Phase noise at offset Δf from carrier
L_{eff}	m	BSIM parameter: effective gate length (see chapter 3.6)
m	-	MOSFET body-effect coefficient
M	-	Avalanche multiplication factor
m^*	kg	Electron (hole) effective mass
N	-	Number of carriers

Symbol	Unit	Meaning
N^*	m^{-2}	BSIM noise parameter (see chapter 3.6)
n	cm^{-3}	Electron concentration (per unit volume)
N_a	cm^{-3}	Acceptor doping concentration
N_{body}	cm^{-3}	Doping concentration in Si body of SOI substrate
N_d	cm^{-3}	Donor doping concentration
NF	dB	Noise figure
n_i	cm^{-3}	Intrinsic carrier concentration (= 1.5×10^{10} cm^{-3} for Si at 300K)
N_l	m^{-2}	BSIM parameter: charge density at drain side (see chapter 3.6)
$NOIA$	$m^{-3}eV^{-1}$	BSIM noise parameter (see chapter 3.6)
$NOIB$	$m^{-1}eV^{-1}$	BSIM noise parameter (see chapter 3.6)
$NOIC$	$m\cdot eV^{-1}$	BSIM noise parameter (see chapter 3.6)
N_{ox}	cm^{-2}	Oxide charge density (= Q_{ox}/q)
N_{OX}	-	Number of oxide charges
n_s	cm^{-3}	Surface carrier concentration
N_s	cm^{-2}	Channel carrier density (= Q_i/q)
N_{sub}	cm^{-3}	Doping concentration in the substrate
N_t	$cm^{-3}eV^{-1}$	Oxide trap density (per unit volume)
N_T		Number of oxide traps
N_0	m^{-2}	BSIM parameter: charge density at source side (see chapter 3.6)
p	cm^{-3}	Hole concentration (per unit volume)
P	W	Power
P	-	Probability (chapter 1.2.2)
P_n	W	Available noise power
q	C	Electronic charge (= 1.602×10^{-19} C)
Q_d	C/cm^2	Depletion charge per unit area
Q_i	C/cm^2	Inversion charge per unit area
Q_m	C/cm^2	Charge on gate per unit area
$q_{\max}$	C	Maximum charge displacement of tank capacitor in VCO
Q_{ox}	C/cm^2	Oxide charge per unit area
R	Ω	Resistance
$R(s)$		Autocorrelation function (chapter 1.2.2)
r_{ch}	Ω	Channel resistance
R_D	Ω	Drain series resistance
R_{in}	Ω	Input resistance
R_L	Ω	Load resistance
R_n	Ω	Noise resistance (Eq. 1-12)
r_n	Ω	Equivalent noise resistance (Eq. 2-7)
R_S	Ω	Source series resistance
R_{SD}	Ω	Source-drain series resistance ($R_S + R_D$)
r_π	Ω	Dynamic resistance for pn-junction
S		Power spectral density

Symbol	Unit	Meaning
S_I	A^2/Hz	Power spectral density of current fluctuations
S_{I_D}	A^2/Hz	Power spectral density of drain current noise
$S_{I_{D,ch}}$	A^2/Hz	Power spectral density of drain current noise in the channel
$S_{I_{D,th}}$	A^2/Hz	Power spectral density of drain current thermal noise
$S_{I_{D,1/f}}$	A^2/Hz	Power spectral density of drain current $1/f$ noise
S_{I_G}	A^2/Hz	Power spectral density of gate current noise
$S_{I_{R_D}}$	A^2/Hz	Power spectral density of current noise generated in R_D
$S_{I_{R_S}}$	A^2/Hz	Power spectral density of current noise generated in R_S
$S_{I_{R_{SD}}}$	A^2/Hz	Power spectral density of current noise generated in R_{SD}
S_{lim}	A^2/Hz	BSIM noise parameter (see chapter 3.6)
S_N	1/Hz	Power spectral density of carrier number fluctuations
$S_{Q_{ox}}$	C^2/cm^4Hz	Power spectral density of oxide charge density fluctuations
S_R	Ω^2/Hz	Power spectral density of resistance fluctuations
S_V	V^2/Hz	Power spectral density of voltage fluctuations
$S_{V_{fb}}$	V^2/Hz	Power spectral density of flat-band voltage noise
S_{V_G}	V^2/Hz	Power spectral density of equivalent input gate voltage noise
S_{wi}	A^2/Hz	BSIM noise parameter (see chapter 3.6)
SS	V/decade	Subthreshold slope
T	K	Absolute temperature
T	s	Time (constant)
t	s	Time
t_{box}	cm	Thickness of buried oxide (SOI)
t_{EOT}	cm	Equivalent oxide thickness
T_n	K	Noise temperature (Eq. 1-11)
t_{ox}	cm	Gate oxide thickness
t_{Si}	cm	Thickness of Si body in SOI substrate
T_0	K	Standard noise temperature (= 290 K)
V	cm^3	Volume
V	V	Voltage
v	V	Small-signal voltage
V_B	V	Substrate voltage (Bulk terminal voltage)
V_{BS}	V	Substrate-to-source voltage
v_d	cm/s	Carrier drift velocity
V_d	V	Applied voltage across pn-junction
V_{DD}	V	Power supply voltage
V_{DS}	V	Drain-to-source voltage
$V_{DS,sat}$	V	MOSFET drain-to-source saturation voltage
V_{fb}	V	Flat-band voltage
V_G	V	Gate voltage

Symbol	Unit	Meaning
V_{GS}	V	Gate-to-source voltage
V_{GT}	V	Gate voltage overdrive (= $\lvert V_{GS} - V_T \rvert$)
v_i	cm/s	Individual carrier drift velocity
V_{max}	V	Maximum voltage swing over tank capacitor in VCO
v_n	V	Noise voltage
$\overline{v_n^2}$	V^2	Mean square of noise voltage
$v_{n,rms}$	V	RMS noise voltage
V_T	V	Threshold voltage
v_{th}	cm/s	Thermal velocity of electrons
W	cm	Gate width
W_d	cm	Depletion layer width
W_{dm}	cm	Maximum depletion layer width
W_{eff}	m	BSIM parameter: effective gate width (see chapter 3.6)
Y_S	S	Source admittance
z	cm	Distance in a direction vertical to the channel
Z_S	Ω	Source impedance
z_t	cm	Trap distance from gate oxide/channel interface
α	Vs/C	Scattering parameter of the correlated mobility fluctuations
α_C	Vs/C	Coulomb scattering parameter
α_H	-	Hooge parameter
$\alpha_{H,a}$	-	Hooge parameter of 1/*f* noise generated in scattering processes other than surface roughness scattering.
$\alpha_{H,ph}$	-	Hooge parameter of 1/*f* noise generated in the phonon scattering
$\alpha_{H,sr}$	-	Hooge parameter of 1/*f* noise generated in the surface roughness scattering
δ	m	Skin depth (Eq. 2-1)
Δf	Hz	Frequency separation from the oscillating frequency of a VCO
ΔL_{clm}	m	BSIM parameter: channel length reduction (see chapter 3.6)
Δx or δx		Fluctuation in x
ε_{ox}	F/cm	Permittivity of SiO_2 (= 3.45×10^{-13} F/cm)
ε_{Si}	F/cm	Silicon Permittivity (= 1.04×10^{-12} F/cm)
Φ_B	J, eV	Energy barrier height
ϕ_{ms}	V	Work-function difference between the gate material and the substrate material
γ	-	Frequency exponent
γ	-	MOSFET thermal noise coefficient
η	-	Electric field parameter (in Eq. 3-66)
η_v	-	MOSFET parameter describing the relative degree of drain saturation (in Eqs. 3-21 and 3-22)
κ_D	-	Gate leakage current partitioning coefficient at drain side

Symbol	Unit	Meaning
κ_S	-	Gate leakage current partitioning coefficient at source side
λ	cm	Tunneling attenuation length
λ_e	m	Phonon mean free path
μ	cm^2/Vs	Carrier mobility
μ_a	cm^2/Vs	Carrier mobility limited by other mechanisms than surface roughness scattering
μ_{ac}	cm^2/Vs	Mobility limited by scattering with surface acoustic phonons
μ_b	cm^2/Vs	Mobility limited by scattering with bulk phonons
μ_C	cm^2/Vs	Mobility limited by Coulomb scattering
$\mu_{C,imp}$	cm^2/Vs	Mobility limited by Coulomb scattering from impurities
$\mu_{C,ox}$	cm^2/Vs	Mobility limited by Coulomb scattering from oxide charges
μ_{C0}	cm/Vs	Screened Coulomb scattering parameter
μ_{eff}	cm^2/Vs	Effective mobility in MOSFET inversion layer
μ_i	cm^2/Vs	Individual carrier mobility
μ_{ph}	cm^2/Vs	Mobility limited by scattering with phonons, both bulk phonons and surface acoustic phonons.
μ_r		Relative permeability (in Eq. 2-1)
μ_{sr}	cm^2/Vs	Mobility limited by surface roughness scattering
μ_0	cm^2/Vs	Low-field mobility (Eq. 3-78)
μ_0	H/m	Permeability of free space ($= 4\pi\times10^{-7}$ H/m), used in Eq. (2-1)
θ	rad	Phase
θ	V^{-1}	Mobility attenuation coefficient (in Eq. 3-78)
$\rho_{1,2}$	-	Correlation coefficient
σ	$\Omega^{-1}cm^{-1}$	Conductivity
σ		Standard deviation (in Eq. 1-5)
σ_e	cm^2	Capture cross section for electrons
σ_h	cm^2	Capture cross section for holes
σ_{N_t}	$cm^{0.5}eV^{0.5}$	Relative standard deviation of the trap density
τ	s	CMOS inverter delay
τ	s	Time constant of g-r noise
τ_c	s	Capture time for electrons (holes)
τ_e	s	Emission time for electrons (holes)
τ_h	s	Time in high level of two-state RTS
τ_l	s	Time in low level of two-state RTS
$\tau_{ph\text{-}ph}$	s	Relaxation time for phonon-phonon scattering
τ_{th}	s	Time constant of thermally activated traps
τ_0	s	Tunneling time constant (usually taken as 10^{-10} s)
ω_0	rad/s	Angular frequency of oscillation
ψ_B	V	Difference between Fermi level and intrinsic level potentials
ψ_s	V	Surface potential

1 eV = 1.602×10^{-19} J
1 Å = 10^{-10} m
log means logarithm function in base 10
ln means natural logarithm (base *e*)

Appendix II
LIST OF ACRONYMS

Table A-2. List of Acronyms.

Acronym	Meaning
ALD	Atomic layer deposition
B	Bulk
BSIM	Berkeley short channel IGFET model
CMOS	Complementary metal-oxide-semiconductor
CVD	Chemical vapour deposition
D	Drain
DC	Direct current
DCR	Direct conversion receiver
DIBL	Drain-induced barrier lowering
DT	Dynamic threshold
DUT	Device-under-test
EOT	Equivalent oxide thickness
FD	Fully depleted
FFT	Fast Fourier Transform
FOM	Figure-of-merit
FUSI	Fully silicided
G	Gate
g-r	Generation-recombination
HF-clean	Clean in Hydrofluoric acid
ISF	Impulse sensitivity function
ITRS	International technology roadmap of semiconductors
I-V	Current-voltage
JFET	Junction field-effect transistor

Acronym	Meaning
LC	Inductance-capacitance
LNA	Low-noise amplifier
MBE	Molecular beam epitaxy
MOCVD	Metal-organic chemical vapour deposition
MOSFET	Metal-oxide-semiconductor field-effect transistor
nMOSFET	n-channel MOS transistor
PD	Partially depleted
pMOSFET	p-channel MOS transistor
PSD	Power spectral density
PVD	Physical vapour deposition
RF	Radio frequency
RTS	Random-telegraph-signal
S	Source
SCE	Short channel effect
SNR	Signal-to-noise ratio
SOI	Silicon-on-insulator
TCAD	Technology computer aided design
TEM	Transmission electron microscopy
VCO	Voltage controlled oscillator
WKB	Wentzel-Kramers-Brillouin

Appendix III
SOLUTIONS TO PROBLEMS

Solution problem 1-1.

We calculate the mean square noise voltage for the different alternatives.

A: Only thermal noise

$$\overline{v_n^2} = 4kTR\Delta f = 4\times1.38\times10^{-23}\times300\times5000\times10^{6}\ \text{V}^2 = 8.28\times10^{-11}\ \text{V}^2.$$

B: $T_n = 500$ K

$$\overline{v_n^2} = 4kT_nR\Delta f = 1.38\times10^{-10}\ \text{V}^2.$$

C: $v_{n,rms} = 15\ \mu\text{V}$

$$\overline{v_n^2} = v_{n,rms}^2 = 2.25\times10^{-10}\ \text{V}^2.$$

D: Power spectral density $S_V = 2\times10^{-16}\ \text{V}^2/\text{Hz}$

$$\overline{v_n^2} = S_V\times\Delta f = 2\times10^{-16}\times10^{6}\ \text{V}^2 = 2\times10^{-10}\ \text{V}^2.$$

E: $R_n = 10\ \text{k}\Omega$

$$\overline{v_n^2} = 4kTR_n\Delta f = 1.656\times10^{-10}\ \text{V}^2.$$

Answer: A, B, E, D, C.

Solution problem 1-2

The thermal noise from the resistor is

$$S_{I,th} = 4kT/R = 4\times1.38\times10^{-23}\times300/200\ \mathrm{A^2/Hz} = 8.28\times10^{-23}\ \mathrm{A^2/Hz}.$$

The thermal noise and the $1/f$ noise are uncorrelated. Thus

$$S_{I,tot}(f) = 8.28\times10^{-23} + 2.5\times10^{-19}/f\ \ \mathrm{A^2/Hz}.$$

The noise power is given by integrating the PSD over the bandwidth

$$\overline{i_n^2} = \int_1^{10^4} S_{I,tot}df = 8.28\times10^{-23}\times(10^4-1) + 2.5\times10^{-19}\times\ln\left(\frac{10^4}{1}\right)\ \mathrm{A}^2 =$$

$$8.3\times10^{-19} + 2.3\times10^{-18}\ \mathrm{A}^2 = 3.1\times10^{-18}\ \mathrm{A}^2 \Rightarrow i_{n,rms} = \sqrt{\overline{i_n^2}} = 1.8\ \mathrm{nA}.$$

Solution problem 1-3

The noise equivalent circuit is shown in Fig. A-1.

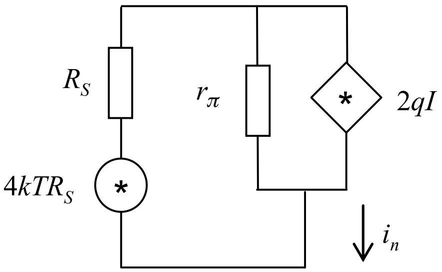

Figure A-1. Noise equivalent circuit in example 1-3. $r_\pi = kT/qI$.

The two noise sources are uncorrelated. We use the superposition principle to calculate the noise current from each source.

From the thermal noise source using Ohm's law:

$$S_{I_n,1} = \frac{4kTR_S}{\left(R_S + r_\pi\right)^2}.$$

From the shot noise source using the current-division principle

$$S_{I_n,2} = 2qI\frac{r_\pi^2}{\left(R_S + r_\pi\right)^2}.$$

<u>Answer:</u> The total PSD is according to the superposition principle

$$S_{I_n} = S_{I_n,1} + S_{I_n,2}.$$

Solution problem 1-4

The measured noise consists of superimposed $1/f$ noise and white noise (thermal noise). From the graph:

$$S_V = 2\times10^{-13}/f + 5\times10^{-16}\ \text{V}^2/\text{Hz}.$$

The resistance can be calculated from the thermal noise level

$$R = \frac{5\times10^{-16}}{4kT}\ \Omega = 30\ \text{k}\Omega.$$

The Hooge noise model (see Eq. 1-37)

$$\frac{S_R}{R^2} = \frac{S_V}{V^2} = \frac{\alpha_H}{fN} \quad (V = RI \Rightarrow S_V = S_R I^2 \Rightarrow S_V/V^2 = S_R/R^2).$$

We need to determine the number of (free) electrons N
N = electron concentration × sample volume ⇒

$$N = 10^{17}\ \text{cm}^{-3}\times10^{-3}\ \text{cm}\times10^{-2}\ \text{cm}\times10^{-4}\ \text{cm} = 10^8.$$

Answer: $\alpha_H = \frac{fNS_V}{V^2} = \frac{10^8 \times 2 \times 10^{-13}}{\left(30 \times 10^3 \times 16.6 \times 10^{-6}\right)^2} = 8\times10^{-5}.$

Solution problem 2-1

The noise equivalent circuit is shown in Fig. A-2.

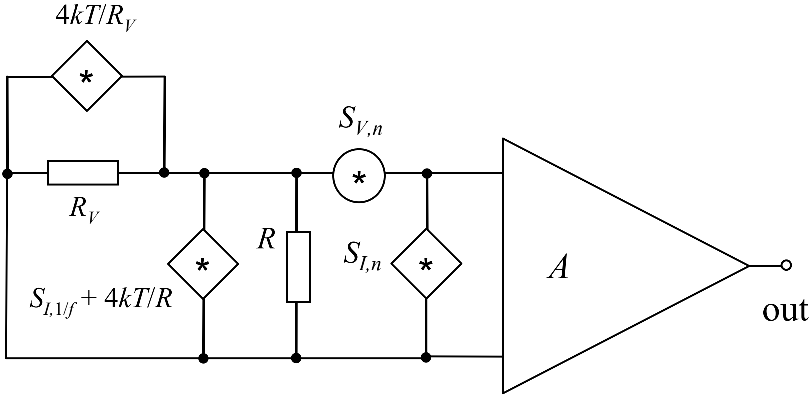

Figure A-2. Noise equivalent circuit in example 2-1.

Answer:
The voltage noise at the output can be written according to the superposition principle:

$$S_{V,out} = \left(\frac{4kT/R_V \cdot R_V^2 R^2}{(R+R_V)^2} + \frac{(S_{I,1/f} + 4kT/R)\cdot R_V^2 R^2}{(R+R_V)^2} \right.$$

$$\left. + S_{V,n} + \frac{S_{I,n} \cdot R_V^2 R^2}{(R+R_V)^2} \right) \cdot A^2$$

$$= \left(\frac{4kTR_V R}{(R+R_V)} + \frac{S_{I,1/f} R_V^2 R^2}{(R+R_V)^2} + S_{V,n} + \frac{S_{I,n} \cdot R_V^2 R^2}{(R+R_V)^2} \right) \cdot A^2.$$

Solution problem 2-2

Using Eqs. (2-8) to (2-10), $p = 2$ for number fluctuation noise and

$g_{ch}(R_{SD} + R_L) << 1$ gives

$$S_{I_{D,tot}} = \frac{A \cdot I_D^2}{fWL(V_{GS} - V_T)^2} + g_{ch}^2 R_{SD}^2 \frac{B \cdot I_D^2}{f}$$

where A and B are constants. At a constant V_{DS} in the linear regime

$$g_{ch} = \left(\frac{dI_D}{dV_{DS}}\right) = \left\{ I_D = \frac{W}{L}\mu C_{ox}(V_{GS} - V_T)V_{DS} \right\} = I_D / V_{DS}.$$

Thus,

$$S_{I_{D,tot}} = \frac{A \cdot I_D^2}{fWL(V_{GS} - V_T)^2} + \frac{R_{SD}^2}{V_{DS}^2}\frac{B \cdot I_D^4}{f}.$$

It was given that the noise contributions from the S/D resistance and the channel are equally strong at $I_D = I_{D,1}$. Thus,

$$\frac{A \cdot I_{D,1}^2}{fWL(V_{GS,1} - V_T)^2} = \frac{R_{SD}^2}{V_{DS}^2}\frac{B \cdot I_{D,1}^4}{f} \Rightarrow$$

$$\frac{R_{SD}^2}{V_{DS}^2}\frac{B}{f} = \frac{A \cdot I_{D,1}^{-2}}{fWL(V_{GS,1} - V_T)^2}.$$

At $I_D = I_{D,1}/3$:

$$I_D \propto (V_{GS} - V_T) \Rightarrow (V_{GS} - V_T) = (V_{GS,1} - V_T)/3.$$

The relative contribution from the $1/f$ noise in the S/D resistance to the output drain current noise equals

$$\frac{\frac{R_{SD}^2}{V_{DS}^2}\frac{B\cdot I_D^4}{f}}{\frac{R_{SD}^2}{V_{DS}^2}\frac{B\cdot I_D^4}{f}+\frac{A\cdot I_D^2}{fWL(V_{GS}-V_T)^2}}=\frac{1}{1+\frac{\frac{A\cdot I_{D,1}^2/3^2}{fWL(V_{GS,1}-V_T)^2/3^2}}{\frac{A\cdot I_{D,1}^{-2}\cdot I_{D,1}^4/3^4}{fWL(V_{GS,1}-V_T)^2}}}$$

$$=\frac{1}{1+3^4}\approx 1.2\%.$$

Answer: The noise from the S/D resistance is only 1.2% of the total output drain current noise at $I_D = I_{D,1}/3$.

Solution problem 2-3

RTS noise can be observed on top of the $1/f$ noise when the RTS noise PSD is larger than the $1/f$ noise PSD.

Using Eq. (1-37) for the $1/f$ noise

$$S_{I,1/f}=\frac{\alpha_H\cdot I_D^2}{fN}$$

and Eq. (1-31) for the RTS noise with $N_T = 1$ (one trap)

$$S_{I,RTS}=\frac{I_D^2}{N^2}\frac{\tau}{1+(2\pi f)^2\tau^2}.$$

The RTS noise PSD is maximized relative to the $1/f$ noise when $\tau = 1/2\pi f$.

Answer: the condition to observe RTS noise

$$S_{I,RTS}>S_{I,1/f}\Rightarrow$$

$$\frac{I_D^2}{N^2}\frac{\tau}{2}>\frac{\alpha_H I_D^2 2\pi\tau}{N}$$

$$N<\frac{1}{4\pi\alpha_H}.$$

Solution problem 2-4

RTS noise can be observed if
(i) the number of traps is small *and*
(ii) the RTS noise is higher than the $1/f$ noise.

We make the assumption that RTS can be observed if there are less than 5 active traps.
Thus, using Eq. (2-12)

$$4kTWLN_t z < 5 \Rightarrow WL < \frac{5}{4kTN_t z}.$$

It was given that $z = 2$ nm and $N_t = 1\times10^{17}$ cm^{-3}eV^{-1}. Hence,

$$WL < 0.24\ \mu\text{m}^2.$$

The number of carriers N must also satisfy Eq. (2-11). For a MOSFET in inversion

$$N = WLC_{ox}(V_{GS} - V_T)/q$$

Thus,

$$N < \frac{1}{4\pi\alpha_H} \Rightarrow \alpha_H < \frac{q}{4\pi WLC_{ox}(V_{GS} - V_T)}.$$

This relation is valid in strong inversion. The RTS noise is easiest to discover when $V_{GS} - V_T$ is small. Therefore, we assume $V_{GS} - V_T = 0.1$ V. Furthermore, $C_{ox} = \varepsilon_{ox}/t_{ox} = 1.73\cdot10^{-6}$ F/cm^2. Thus, the requirement on α_{H} is found to be

$$\alpha_H < 3\cdot10^{-5}.$$

Answer: RTS noise is estimated to be observed in a device with a gate area $W{\cdot}L$ smaller than 0.24 μm^2 and with a Hooge parameter for background $1/f$ noise lower than $3\cdot10^{-5}$.

Solution problem 3-1

Insert $V_{DS,sat} = (V_{GS} - V_T)/m$ in Eq. (3-50):

$$\frac{S_{I_D}}{I_D^2} = \frac{q\alpha_H \mu_{eff} V_{DS}}{fL^2 I_D} = \frac{q\alpha_H \mu_{eff}(V_{GS} - V_T)}{mfL^2 I_D}. \tag{A3-1}$$

Then insert Eq. (3-8) for I_D in the equation above

$$\begin{aligned} I_D &= \frac{W}{L}\mu_{eff} C_{ox} \frac{(V_{GS} - V_T)^2}{2m} \Rightarrow \\ S_{I_D} &= \frac{2q\alpha_H I_D^2}{fWLC_{ox}(V_{GS} - V_T)}. \end{aligned} \tag{A3-2}$$

Comparing with Eq. (3-80) for mobility fluctuation noise in the linear region, we see that the drain current noise is a factor of two higher in the saturation region. Why? The number of carriers in the channel is reduced to 2/3 of its value in the linear region at a certain gate voltage overdrive since the channel at the drain end is pinched off. By using

$$Q_i = C_{ox}(V_{GS} - V_T)\sqrt{1 - x/L}. \tag{A3-3}$$

in Eq. (3-50), the same expression is as in Eq. (A3-2) is found.

Solution problem 3-2

First, calculate the expression for g_m in the saturation region from Eq. (3-8)

$$g_{m,sat} = \left(\frac{dI_{D,sat}}{dV_{GS}}\right) = \frac{2I_D}{(V_{GS} - V_T)} = \sqrt{2I_D \frac{W}{L}\frac{\mu_{eff} C_{ox}}{m}}. \tag{A3-4}$$

Inserted in Eq. (3-79), this yields

$$S_{I_D} = \frac{q^2 kT\lambda N_t}{f^\gamma WLC_{ox}^2}\left(1+\frac{\alpha\mu_{eff}C_{ox}I_D}{g_m}\right)^2 g_m^2 =$$

$$\frac{q^2 kT\lambda N_t}{f^\gamma WLC_{ox}^2}\left(1+\frac{\alpha\mu_{eff}C_{ox}(V_{GS}-V_T)}{2}\right)^2 \frac{2I_D W}{L}\frac{\mu_{eff}C_{ox}}{m} = \tag{A3-5}$$

$$\frac{2q^2 kT\mu_{eff}I_D\lambda N_t}{f^\gamma L^2 C_{ox} m}\left(1+\frac{\alpha\mu_{eff}C_{ox}(V_{GS}-V_T)}{2}\right)^2.$$

This expression has the same functional dependence as Eqs. (3-68). The BSIM3 noise model is based on the number fluctuation noise theory. The SPICE2 model in Eq. (3-67) resembles the pure number fluctuation noise model (without correlated mobility fluctuations) in the saturation region.

Solution problem 3-3

(a) Mobility fluctuations in the linear or subthreshold region (see Eq. 3-80), or number fluctuations in the saturation region (see Eq. A3-5).
(b) Mobility fluctuations in the saturation region. See Eq. (3-51).
(c) Number fluctuations in the linear region. See Eq. (3-39) and note that $V_{GS} - V_T \propto I_D$.
(d) The output noise stems from noise in the drain series resistance. See Eq. (3-12) with

$$S_{I_{R_D}} >> S_{I_{D,ch}} \text{ and } r_{ch} > R_D.$$

Fig. 2-10 shows a simulation of the situation described above.
(e) Two alternatives: the noise stems (i) from number fluctuation noise in the channel when biased in the subthreshold region (see Eq. 3-40) or (ii) from noise in the drain series resistance when

$$S_{I_{R_D}} > S_{I_{D,ch}} \text{ and } R_D > r_{ch}.$$

Solution problem 4-1

First calculate the oxide capacitance C_{ox} and the transconductance g_m.

$$C_{ox} = 3.9\cdot 8.854\cdot 10^{-14}/(2.2\cdot 10^{-7})\,\text{F/cm}^2 = 1.57\cdot 10^{-6}\,\text{F/cm}^2.$$

Use Eq. (3-8) to find g_m

$$g_m = \frac{I_D}{\left[(V_{GS} - V_T) - mV_{DS}/2\right]} = 1.30\,\text{mS if we assume}\; m = 1\,.$$

(a) Answer: Eq. (3-80) $\Rightarrow \alpha_H = 7.9\times10^{-5}$.

The trap density is found from Eq. (3-79). The correlated mobility fluctuations can be neglected since the gate voltage overdrive is small (0.1 V). Thus,

$$N_t = \frac{10\cdot 10\cdot 10^{-4}\cdot 0.075\cdot 10^{-4}\cdot (1.57\cdot 10^{-6})^2}{1.602\cdot 10^{-19}\cdot 1.38\cdot 10^{-23}\cdot 300\cdot 10^{-8}\cdot (0.0013)^2}\cdot 1.4\cdot 10^{-17}$$

cm^{-3}eV^{-1}.

Here, $\lambda = 10^{-8}$ cm was used (see Table 4-4).

Answer: $N_t = 2.3\times10^{17}$ cm^{-3}eV^{-1}.
(note that in order to get the unit eV, multiplication with 1.602×10^{-19} is performed).

(b) Calculate the input gate voltage noise and normalize with gate area

$$WLS_{V_G} = WLS_{I_D} / g_m^2 = 6.2\ \mu\text{V}^2\mu\text{m}^2\text{/Hz (at } f = 10\text{ Hz).}$$

According to Table 4-1, the ITRS requires a noise level below 19 μV$^2\mu$m^2/Hz at f = 10 Hz for a device with L = 75 nm and t_{EOT} = 2.2 nm (note that the value is given at 1 Hz in the table).

Answer: The ITRS requirements are fulfilled.

Solution to problem 4-2

The input gate voltage noise is calculated in comparison with the reference device for all four technologies.

Answer:

i) $S_{V_G} = S_{V_{G,ref}} \dfrac{N_t / N_{t,ref} \cdot (t_{EOT} / t_{EOT,ref})^2}{L / L_{ref}} = S_{V_{G,ref}} \cdot \dfrac{6 \cdot (1/2)^2}{(1/2)} = 3S_{V_{G,ref}}$.

ii) Mobility fluctuation noise: $S_{V_G} = S_{V_{G,ref}} \cdot \dfrac{2 \cdot (3/4)}{(3/4)} = 2S_{V_{G,ref}}$.

iii) A front-back gate coupling factor $(1+N_{t,b}/N_{t,f})$ must be considered in the FD SOI device (see Eq. 4-16).

$$S_{V_G} = S_{V_{G,ref}} \cdot \frac{1}{(2/3)}\left(1+3\right) = 6S_{V_{G,ref}} .$$

iv) $S_{V_G} = S_{V_{G,ref}} \cdot 2^2 = 4S_{V_{G,ref}}$.

Solution to problem 5-1

Identifying Fourier coefficients: (see Eq. 5-1)
$c_0 = 0.2$, $c_1 = 1$.

The $1/f^3$-corner frequency is the frequency Δf where the first term and the second term in Eq. (5-2) are equal. Thus

$$0.2^2 \cdot 1 \cdot 10^{-18} / \Delta f = 1 \cdot 10^{-23} (0.2^2 + 1^2) \Rightarrow \Delta f = 3.8 \text{ kHz}.$$

Answer: The $1/f^3$-corner frequency is equal to 3.8 kHz.

Solution to problem 5-2

Denote the noise power (in dBm) due to phase noise with P_N, the power of the interfering signal P_I and the power of the signal in the desired channel P_C. Thus, in order to achieve a SNR of 15 dB

$$\text{SNR} = P_C - P_N = P_I - 40 \text{ dB} - P_N = 15 \text{ dB}. \quad \text{(A5-1)}$$

The phase noise with respect to the power of the interfering signal is denoted by L (dBc/Hz). The noise power is integrated over the channel bandwidth of 200 kHz

$$P_N = L + 10\log(200\cdot 10^3) + P_I\,. \qquad \text{(A5-2)}$$

Inserted in Eq. (A5-1), this yields

$$15 = -40 - L - 10\log(200\cdot 10^3) \Rightarrow L = -40 - 53 - 15 = -108\ \text{dBc/Hz}.$$

<u>Answer:</u> The phase noise at 2 MHz offset must be lower than –108 dBc/Hz.

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