APPENDIX A: ELASTIC SYSTEM MODEL FOR THE IMMOBILE UNLOADED STATE

Here we give the entire procedure for deriving the mathematical model of dynamics of an elastic system as a part of a cooperative system composed of m six-DOF rigid manipulators, handling an object whose motion in three-dimensional space proceeds without any constraint (Figure 12).

It is assumed that the connections of the object and manipulators are elastic and the object is either rigid or elastic. For both cases, we assume that each connection or part of the manipulated object in the neighborhood of the contact point, can be represented by a rigid body at the MC where the forces of contact, gravitation, damping, and elasticity act. The object with the connections forms an elastic system of $m + 1$ elastically interconnected rigid bodies (Figure 12). It is assumed that elastic properties are such that a linear relationship can be established between each relative displacement of any part of the elastic system. Each body is allowed to have six DOFs of motion. Gravitational and contact forces are considered as a system of the external forces acting at the MC.

The coordinates of the MC displacements with respect to the unloaded state y_i defined by (69), are adopted as generalized coordinates.

Potential energy Π of the elastic system is equal to the deformation work

$$\begin{aligned} 2\Pi &= y^T K y = (\delta^T \mathcal{A}^T) K \begin{pmatrix} \delta \\ \mathcal{A} \end{pmatrix}, \\ K &= K^T \in R^{(6m+6)\times(6m+6)}, \quad \text{rank}\, K \le 6m, \end{aligned} \tag{364}$$

so that the derivative with respect to the coordinate is equal to elastic force and is given by

$$F_e = \frac{\partial \Pi}{\partial y} = K y \in R^{6m+6}, \tag{365}$$

$$\frac{\partial \Pi}{\partial y} = \begin{bmatrix} \frac{\partial \Pi}{\partial y_o} \\ \cdots \\ \frac{\partial \Pi}{\partial y_i} \\ \cdots \\ \frac{\partial \Pi}{\partial y_m} \end{bmatrix} = \begin{bmatrix} K_0 y \\ \cdots \\ K_i y \\ \cdots \\ K_m y \end{bmatrix} \Rightarrow \frac{\partial \Pi}{\partial y_i} = K_i y \in R^6, \quad K_i \in R^{6\times(6m+6)},$$

where K_i are the submatrices composed of $6i + 1$ to $6i + 6$ rows of the matrix K.

Relations between the angular velocity $\omega_i = \text{col}(p_i, q_i, r_i)$ measured along the body i main inertia axes and the derivatives of orientation given by $\dot{\mathcal{A}}_i = \text{col}(\dot{\psi}, \dot{\theta}, \dot{\varphi})$ are

$$\begin{bmatrix} p_i \\ q_i \\ r_i \end{bmatrix} = \begin{bmatrix} \dot{\psi} \sin\theta_i \sin\varphi_i + \dot{\theta}_i \cos\varphi_i \\ \dot{\psi} \sin\theta_i \cos\varphi_i - \dot{\theta}_i \sin\varphi_i \\ \dot{\psi} \cos\theta_i + \dot{\varphi}_i \end{bmatrix} = \begin{bmatrix} \sin\theta_i \sin\varphi_i & \cos\varphi_i & 0 \\ \sin\theta_i \cos\varphi_i & -\sin\varphi_i & 0 \\ \cos\theta_i & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{\psi} \\ \dot{\theta}_i \\ \dot{\varphi}_i \end{bmatrix},$$

$$\omega_i = \text{col}(p_i,\ q_i,\ r_i) = L_\omega(\mathcal{A}_i) \cdot \dot{\mathcal{A}}_i.$$

Motion velocity in expanded form is

$$v_{ia} = \begin{bmatrix} \dot{x}_i \\ \dot{y}_i \\ \dot{z}_i \\ \dot{p}_i \\ \dot{q}_i \\ \dot{r}_i \end{bmatrix} = \begin{bmatrix} \dot{r}_{ia} \\ \omega_{ia} \end{bmatrix} = \begin{bmatrix} I_{3\times3} & 0_{3\times3} \\ 0_{3\times3} & L_\omega(\mathcal{A}_i) \end{bmatrix} \cdot \begin{bmatrix} \dot{\delta}_i \\ \dot{\mathcal{A}}_i \end{bmatrix} = L_v(\mathcal{A}_i)\dot{y}_i = L_v(y_i)\dot{y}_i,$$

i.e.

$$v_i = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \sin\theta_i \sin\varphi_i & \cos\varphi_i & 0 \\ 0 & 0 & 0 & \sin\theta_i \cos\varphi_i & -\sin\varphi_i & 0 \\ 0 & 0 & 0 & \cos\theta_i & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \dot{x}_i \\ \dot{y}_i \\ \dot{z}_i \\ \dot{\psi} \\ \dot{\theta}_i \\ \dot{\varphi}_i \end{bmatrix} = L_v(y_i)\dot{y}_i.$$

The total kinetic energy is defined by

$$T = T_0 + T_1 + \cdots + T_i + \cdots + T_m,$$

while the kinetic energy of the ith part is

$$2T_i = m_i \dot{\delta}_i^T \dot{\delta}_i + \omega_i^T I_i \omega_i = m_i \dot{\delta}_i^{x\,2} + m_i \dot{\delta}_i^{y\,2} + m_i \dot{\delta}_i^{z\,2} + A\dot{p}_i^2 + B\dot{q}_i^2 + C\dot{r}_i^2,$$

where m_i is the mass and A_i, B_i, C_i are the ith object's main moments of inertia.

By substituting the angular velocity, we obtain

$$2T_i = m_i \dot{\delta}_i^T \dot{\delta}_i + \dot{\mathcal{A}}_i^T L_\omega^T(\mathcal{A}_i) I_i L_\omega(\mathcal{A}_i) \dot{\mathcal{A}}_i$$

or together with

$$2T_i = \dot{y}_i^T L_v^T(y_i) M_i L_v(y_i) \dot{y}_i = \dot{y}_i W_i(y_i) \dot{y}_i,$$

$$W_i(y_i) = L_v^T(y_i) M_i L_v(y_i) \in R^{6\times 6}, \quad W_i(y_i) = W_i^T(y_i), \quad \det W_i(y_i) \neq 0,$$

where $I_i = \mathrm{diag}(A_i, B_i, C_i)$ and $M_i = \mathrm{diag}(m_i, m_i, m_i, A_i, B_i, C_i)$. In an expanded form $W_i(y_i)$ is given by

$$W_i(y_i) =$$

$$\begin{bmatrix} m_i & 0 & 0 & 0 & 0 & 0 \\ 0 & m_i & 0 & 0 & 0 & 0 \\ 0 & 0 & m_i & 0 & 0 & 0 \\ 0 & 0 & 0 & \sin^2\theta_i(A_i\sin^2\varphi_i + B_i\cos^2\varphi_i) + C_i\cos^2\theta_i & \frac{1}{2}(A_i - B_i)\sin\theta_i\sin 2\varphi_i & C_i\cos\theta_i \\ 0 & 0 & 0 & \frac{1}{2}(A_i - B_i)\sin\theta_i\sin 2\varphi_i & A_i\cos^2\varphi_i + B_i\sin^2\varphi_i & 0 \\ 0 & 0 & 0 & C_i\cos\theta_i & 0 & C_i \end{bmatrix}$$

or, in scalar form,

$$\begin{aligned} 2T_i = \;& m_i \dot{\delta}_i^{x\,2} + m_i \dot{\delta}_i^{y\,2} + m_i \dot{\delta}_i^{z\,2} \\ & + \dot{\psi}_i^2(\sin^2\theta_i(A_i\sin^2\varphi_i + B_i\cos^2\varphi_i) + C_i\cos^2\theta_i) \\ & + \dot{\psi}_i\dot{\theta}_i(A_i - B_i)\sin\theta_i\sin 2\varphi_i + +2\dot{\psi}_i\dot{\varphi}_i C_i\cos\theta_i \\ & + \dot{\theta}_i^2(A_i\cos^2\varphi_i + B_i\sin^2\varphi_i) + C_i\dot{\varphi}_i^2. \end{aligned}$$

Derivatives of kinetic energy with respect to velocity are defined by

$$\frac{\partial T_i}{\partial \dot{y}_i} = L_v^T(y_i) M_i L_v(y_i) \dot{y}_i = W_i(y_i)\dot{y}_i,$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial T_i}{\partial \dot{y}_i} = \dot{W}_i(y_i)\dot{y}_i + W_i(y_i)\ddot{y}_i.$$

The member $\dot{W}_i(y_i)\dot{y}_i$ is defined in the following way:

$$\dot{W}_i(y_i)\dot{y}_i =$$

$$\begin{bmatrix} 0 \\ 0 \\ 0 \\ \{\dot{\psi}_i\dot{\theta}_i \sin 2\theta_i (A_i \sin^2\varphi_i + B_i \cos^2\varphi_i - C_i) + \dot{\psi}_i\dot{\varphi}_i(A_i - B_i)\sin^2\theta_i \sin 2\varphi_i \\ +\dot{\theta}_i\dot{\varphi}_i \sin\theta_i((A_i - B_i)\cos 2\varphi_i - C_i) + \frac{1}{2}\dot{\theta}_i^2(A_i - B_i)\cos\theta_i \sin 2\varphi_i\} \\ \frac{1}{2}\dot{\psi}_i\dot{\theta}_i(A_i - B_i)\cos\theta_i \sin 2\varphi_i + \dot{\psi}_i\dot{\varphi}_i(A_i - B_i)\sin\theta_i \cos 2\varphi_i \\ -\dot{\theta}_i\dot{\varphi}_i(A_i - B_i)\sin 2\varphi_i - \dot{\psi}_i\dot{\theta}_i C_i \sin\theta_i \end{bmatrix}.$$

The partial derivative of kinetic energy with respect to the coordinate is the function of that coordinate and its derivative, and is given by

$$\frac{\partial T_i}{\partial y_i}(y_i, \dot{y}_i) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \frac{1}{2}\dot{\psi}_i^2 \sin 2\theta_i(A_i \sin^2\varphi_i + B_i\cos^2\varphi_i - C_i) \\ +\frac{1}{2}\dot{\psi}_i\dot{\theta}_i(A_i - B_i)\cos\theta_i \sin 2\varphi_i - \dot{\psi}_i\dot{\varphi}_i C_i \sin\theta_i \\ \frac{1}{2}\dot{\psi}_i^2(A_i - B_i)\sin^2\theta_i \sin 2\varphi_i + \dot{\psi}_i\dot{\theta}_i(A_i - B_i)\sin\theta_i\cos 2\varphi_i \\ -\frac{1}{2}\dot{\theta}_i^2(A_i - B_i)\sin 2\varphi_i \end{bmatrix}.$$

The preceding equations for kinetic energy can be given concisely as

$$\begin{aligned} 2T &= \sum_{i=0}^{m}\left(\sum_{j=1}^{\infty} dm_j v_j\right) = \sum_{i=0}^{m}\left(\sum_{j=1}^{\infty} dm_j \frac{\mathrm{d}}{\mathrm{d}t}(y_i + b_j)\right), \\ 2T &= \sum_{i=0}^{m} m_i\delta_i^2 + \sum_{i=0}^{m} I_i\omega_i^2 \\ &= \sum_{i=0}^{m} v_i^T M_i v_i = v^T M v = \dot{y}^T L_v^T(y) M L_v(y)\dot{y}, \end{aligned} \tag{366}$$

where

$$\begin{aligned} M &= \mathrm{diag}(M_0, M_1, \ldots, M_n) \in R^{(6m+6)\times(6m+6)}, \\ M_i &= \mathrm{diag}(m_i, m_i, m_i, A_i, B_i, C_i) \in R^{6\times 6}, \\ v &= \mathrm{col}(v_0, v_1, \ldots, v_m) \in R^{(6m+6)\times 1}, \\ v_i &= \mathrm{col}(\dot{\delta}_i, \omega_i(\mathcal{A}_i)) = L_{vi}(y_i)\dot{y}_i \in R^{6\times 1}, \\ \omega_i &= L_{\omega i}(\mathcal{A}_i)\dot{\mathcal{A}}_i \in R^{3\times 1}, \\ L_{vi}(y_i) &= \mathrm{diag}(I_{3\times 3}, L_{\omega_i}(\mathcal{A}_i)) \in R^{6\times 6}, \\ L_v(y) &= \mathrm{diag}(L_{v0}, L_{v1} \ldots . L_{vm}) \in R^{(6m+6)\times(6m+6)}. \end{aligned} \tag{367}$$

If the connections have dissipative properties, the dissipation energy can be expressed as

$$2\mathcal{D} = -\dot{y}^T D\dot{y}, \quad D = D^T \geq 0, \quad D \in R^{(6m+6)\times(6m+6)}, \tag{368}$$

where $D \in R^{6(m+1)\times 6(m+1)}$ is the matrix with damping coefficients corresponding to velocities. The derivative of the dissipation energy with respect to velocity is given by

$$\frac{\partial \mathcal{D}}{\partial \dot{y}} = -D\dot{y}, \quad \frac{\partial \mathcal{D}}{\partial \dot{y}} = \begin{bmatrix} \dfrac{\partial \mathcal{D}}{\partial \dot{y}_o} \\ \cdots \\ \dfrac{\partial \mathcal{D}}{\partial \dot{y}_i} \\ \cdots \\ \dfrac{\partial \mathcal{D}}{\partial \dot{y}_m} \end{bmatrix} = \begin{bmatrix} D_0\dot{y} \\ \cdots \\ D_i\dot{y} \\ \cdots \\ D_m\dot{y} \end{bmatrix} \Rightarrow \frac{\partial \mathcal{D}}{\partial \dot{y}_i} = -D_i\dot{y}, \quad D_i \in R^{6\times(6m+6)},$$

where D_i are the submatrices composed of the rows atarting from $6i+1$ to $6i+6$ inclusive of the matrix D.

By substituting these expressions into the Langrange equations we obtain

$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial T}{\partial \dot{y}_i} - \frac{\partial T}{\partial y_i} - \frac{\partial \mathcal{D}}{\partial \dot{y}_i} + \frac{\partial \Pi}{\partial y_i} = Q_i, \quad i = 0, 1, \ldots, m, \tag{369}$$

where

$$\begin{aligned}
Q_i &= \mathrm{col}(Q_i^1, \ldots, Q_i^6) = G_i(m_i g) + F_i, \\
Q_i^j &= \sum_{k=0}^{m} \frac{\partial y_k}{\partial y_i^j} Q_k = \sum_{k=0}^{m} \frac{\partial y_k}{\partial y_i^j}(G_k(m_k g) + F_k), \quad i = 0, 1, \ldots, m, \quad j = 1, \ldots, 6, \\
Q_i^j &= \sum_{k=0}^{m} \left(F_{ku}^x \frac{\partial \delta_k^x}{\partial y_i^j} + F_{ku}^y \frac{\partial \delta_k^y}{\partial y_i^j} + F_{ku}^z \frac{\partial \delta_k^z}{\partial y_i^j} + F_{ku}^\psi \frac{\partial \psi_k}{\partial y_i^j} + F_{ku}^\theta \frac{\partial \theta_k}{\partial y_i^j} + F_{ku}^\varphi \frac{\partial \varphi_k}{\partial y_i^j} \right) \\
&= F_{iu}^j \\
&\Rightarrow F_{iu} = G_i(m_i g) + F_i = \mathrm{col}(F_{iu}^1, \ldots, F_{iu}^6) \in R^{6\times 1}, \quad j = x, y, z, \psi_i, \theta_i, \varphi_i,
\end{aligned}$$

and $y_i^j, \quad j = 1, \ldots, 6$ are the individual components of the vector $y_i = \mathrm{col}(y_i^1, y_i^2, y_i^3, y_i^4, y_i^5, y_i^6) = \mathrm{col}(\delta_i^x, \delta_i^y, \delta_i^z, \psi_i, \theta_i, \varphi_i)$.

For the elastic system performing general motion about the immobile unloaded state 0 under the action of the external forces F, a general form of the model is obtained as

$$W_i(y_i)\ddot{y}_i + \dot{W}_i(y_i)\dot{y}_i - \frac{\partial T_i}{\partial y_i}(y_i, \dot{y}_i) + D_i\dot{y} + K_i y = G_i(m_i g) + F_i, \quad i = 0, 1, \ldots, m$$

or, in short form,

$$W_i(y_i)\ddot{y}_i + w_i(y,\dot{y}) = F_i, \quad i = 0, 1, \ldots, m,$$

where

$$\begin{aligned} w_i(y,\dot{y}) &= \dot{W}_i(y_i)\dot{y}_i - \frac{\partial T_i}{\partial y_i}(y_i,\dot{y}_i) \\ &\quad + D_i\dot{y} + K_i y - G_i(m_i g) \in R^{6\times 1}, \quad i = 0, 1, \ldots, m. \end{aligned}$$

Taking into account that $G_i(m_i g) = (0, 0, -m_i g, 0, 0, 0)^T$, the member $w_i(y,\dot{y})$ has the expanded form

$$w_i(y,\dot{y}) = \begin{bmatrix} \sum_{j=0}^m D_{(6i+1)j}\dot{y} + \sum_{j=0}^m K_{(6i+1)j}y \\ \cdots\cdots\cdots\cdots\cdots \\ \sum_{j=0}^m D_{(6i+2)j}\dot{y} + \sum_{j=0}^m K_{(6i+2)j}y \\ \cdots\cdots\cdots\cdots\cdots \\ \sum_{j=0}^m D_{(6i+3)j}\dot{y} + \sum_{j=0}^m K_{(6i+3)j}y + m_i g \\ \cdots\cdots\cdots\cdots\cdots \\ \dot{\psi}_i\dot{\theta}_i \sin 2\theta_i (A_i \sin^2\varphi_i + B_i\cos^2\varphi_i - C_i) + \dot{\psi}_i\dot{\varphi}_i(A_i - B_i)\sin^2\theta_i \sin 2\varphi_i + \\ \dot{\theta}_i\dot{\varphi}_i \sin\theta_i((A_i - B_i)\cos 2\varphi_i - C_i) + \frac{1}{2}\dot{\theta}_i^2(A_i - B_i)\cos\theta_i \sin 2\varphi_i \\ + \sum_{j=0}^m D_{(6i+4)j}\dot{y} + \sum_{j=0}^m K_{(6i+4)j}y \\ \cdots\cdots\cdots\cdots\cdots \\ -\frac{1}{2}\dot{\psi}_i^2 \sin 2\theta_i (A_i\sin^2\varphi_i + B_i\cos^2\varphi_i C_i) + \dot{\psi}_i\dot{\varphi}_i\sin\theta_i((A_i - B_i)\cos 2\varphi_i + C_i) \\ -\dot{\theta}_i\dot{\varphi}_i(A_i - B_i)\sin 2\varphi_i + \sum_{j=0}^m D_{(6i+5)j}\dot{y} + \sum_{j=0}^m K_{(6i+5)j}y \\ \cdots\cdots\cdots\cdots\cdots \\ -\frac{1}{2}\dot{\psi}_i^2(A_i - B_i)\sin^2\theta_i \sin 2\varphi_i - \dot{\psi}_i\dot{\theta}_i \sin\theta_i((A_i - B_i)\cos 2\varphi_i + C_i) \\ +\frac{1}{2}\dot{\theta}_i^2(A_i - B_i)\sin 2\varphi_i + \sum_{j=0}^m D_{6(i+1)j}\dot{y} + \sum_{j=0}^m K_{6(i+1)j}y \end{bmatrix}.$$

By uniting all $6m + 6$ equations, we obtain

$$W(y)\ddot{y} + w(y,\dot{y}) = F, \tag{370}$$

where

$$W(y) = \mathrm{diag}(W_0(y_0)W_1(y_1)\ldots W_m(y_m)) \in R^{(6m+6)\times(6m+6)}, \quad W(y) = W^T(y),$$

$$\det W(y) \neq 0, \quad w(y,\dot{y}) = \mathrm{col}(w_o(y,\dot{y}), \ldots, w_m(y,\dot{y})) \in R^{(6m+6)\times 1}.$$

From $6m+6$ equations (370), the number of independent equations is exactly equal to the stiffness matrix rank (rank K).

Equation (370) can be presented in such a way that the descriptions of connections motion and manipulation object are separated

$$W_c(y_c)\ddot{y}_c + w_c(y,\dot{y}) = F_c,$$

$$W_0(y_0)\ddot{y}_0 + w_0(y,\dot{y}) = 0, \tag{371}$$

where the subscript c designates the quantities related to the contact points, and the subscript 0 designates the quantities related to the manipulated object. Hereby

$$y_c = \mathrm{col}(y_1, y_2, \ldots, y_m) \in R^{6m\times 1},\ y_o \in R^{6\times 1},$$

$$F_c = \mathrm{col}(F_1, F_2, \ldots., F_m) \in R^{6m\times 1},\ F_0 = 0 \in R^{6\times 1},$$

$$W_c(y_c) = \mathrm{diag}(W_1(y_1) \ldots W_m(y_m)) \in R^{6m\times 6m},$$

$$W_c(y_c) = W_c^T(y_c),\ \det W_c(y_c) \neq 0,$$

$$w_c(y, \dot{y}) = \mathrm{col}(w_1(y, \dot{y}), \ldots, w_m(y, \dot{y})) \in R^{6m\times 1},$$

where y_c denotes the expanded contact position vector in the $6m$-dimensional space and F_c is the expanded vector of the contact forces, adjoint to that vector. Let us mention that at the manipulated object MC, no contact force acts directly, hence $F_0 = 0$. Equations (370) and (371) represent the final form of equations of the elastic system behavior which, under the action of external contact forces F_c, performs the general motion about the immobile unloaded state 0.

The result would be also obtained by using the d'Alembert principle by replacing the components of inertial, damping and gravitational forces on the left-hand side of Equation (365).

APPENDIX B: ELASTIC SYSTEM MODEL FOR THE MOBILE UNLOADED STATE

Let the geometrical figure move from the state 0. Stress of the elastic system takes place in the same way as when the state 0 is at rest. In other words, the elastic system stress is still regarded only with respect to the mobile state 0, while this motion of state 0 influences only the members which do not reflect the elastic properties of the elastic system. Simply, the potential energy and dissipation energy of the elastic system connections are determined by displacement of the system relative to the mobile unloaded state, while the other quantities are defined for the elastic system absolute coordinates.

Kinetic energy is defined by the absolute velocities as

$$
\begin{aligned}
2T_a &= \sum_{i=0}^{m}\left(\sum_{j=1}^{\infty}\mathrm{d}m_j v_j\right) = \sum_{i=0}^{m}\left(\sum_{j=1}^{\infty}\mathrm{d}m_j \frac{\mathrm{d}}{\mathrm{d}t}(r_{ia}+b_j)\right),\\
2T_a &= \sum_{i=0}^{m} m_i \dot{r}_{ia}^2 + \sum_{i=0}^{m} I_i \omega_{ia}^2\\
&= \sum_{i=0}^{m} v_{ia}^T M_i v_{ia} = v_a^T M v_a = \dot{Y}^T L_{va}^T(Y) M L_{va}(Y)\dot{Y},\\
2T_a &= \dot{Y}^T W_a(Y)\dot{Y}, \qquad (372)
\end{aligned}
$$

where

$$
\begin{aligned}
Y_i &= \mathrm{col}(r_{ia}, \mathcal{A}_{ia}) \in R^{6\times 1}, \quad Y = \mathrm{col}(Y_0, Y_1, \ldots, Y_m) \in R^{(6m+6)\times 1},\\
v_{ia} &= \mathrm{col}(\dot{r}_{ia}, \omega_{ia}(\mathcal{A}_{ia})) = L_{via}(Y_i)\dot{Y}_i \in R^{6\times 1},\\
\omega_{ia} &= L_{\omega ia}(\mathcal{A}_{ia})\dot{\mathcal{A}}_{ia} \in R^{3\times 1},
\end{aligned}
$$

$$\begin{aligned}
v_a &= \mathrm{col}(v_{0a}, v_{1a}, \ldots, v_{ma}) \in R^{(6m+6)\times 1},\\
L_{via}(Y_i) &= \mathrm{diag}(I_{3\times 3}, L_{\omega ia}(\mathcal{A}_{ia})) \in R^{6\times 6},\\
L_{va}(Y) &= \mathrm{diag}(L_{v0a}, L_{v1a} \ldots, L_{vma}) \in R^{(6m+6)\times(6m+6)},\\
W_a(Y) &= \mathrm{diag}(W_{0a}, W_{1a}, \ldots, W_{ma})\\
&= L_{va}^T(Y) M L_{va}(Y) \in R^{(6m+6)\times(6m+6)}. \qquad (373)
\end{aligned}$$

The mathematical form of W_{ia} is identical to the mathematical form of W_i, whereby instead of the subscript i, the subscript ia is used as the designation of absolute coordinates. Hence, the derivatives of the kinetic energy

$$\frac{\partial T_{ia}}{\partial \dot{Y}_i} = W_{ia}(Y_i)\dot{Y}_i, \quad \frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial T_{ia}}{\partial \dot{Y}_i} = \dot{W}_{ia}(Y_i)\dot{Y}_i + W_{ia}(Y_i)\ddot{Y}_i, \quad \frac{\partial T_{ia}}{\partial Y_i}(Y_i, \dot{Y}_i),$$

are also identical to the expressions obtained in Appendix A, in which the subscripts should be changed, i.e. instead of the displacement coordinates marked by i, the absolute coordinates with the subscript ia should be used.

It has already been mentioned that the elastic system potential energy is equal to the elastic system deformation work.

Total potential energy due to the linear and rotational displacement of the body i relative to the body j, $i, j = 0, 1, \ldots, m$ is defined by

$$\begin{aligned}
\Pi_a = \Pi_{01a} + \Pi_{02a} + \Pi_{03a} + \cdots + \Pi_{0ma} +\\
+\Pi_{12a} + \Pi_{13a} + \cdots + \Pi_{1ma} +\\
+\Pi_{23a} + \cdots + \Pi_{2ma} +\\
\cdots\cdots\\
+\Pi_{(m-1)m}.
\end{aligned}$$

The arbitrary member Π_{ija} of this sum for $y_{ij} = y_{ij}^D$ is defined by

$$2\Pi_{ija} = (y_{ij}^D)^T K_{ija} y_{ij}^D = (Y_i - Y_j)^T \Lambda_{ij}(Y_i, Y_j) K_{ija} \Lambda_{ij}(Y_i, Y_j)(Y_i - Y_j),$$

$$2\Pi_{ija} = (Y_i - Y_j)^T \pi_{ij}(Y_i - Y_j) = Y_i^T \pi_{ij} Y_i - 2Y_i^T \pi_{ij} Y_j + Y_j^T \pi_{ij} Y_j,$$

where $\pi_{ij} = \Lambda_{ij}(Y_i, Y_j) K_{ija} \Lambda_{ij}$,

$$\pi_{ij} = \pi_{ji}$$

$$= \begin{bmatrix} c_{ij}^x\left(1-\frac{\|\rho_{ij0}\|}{\|r_{ia}-r_{ja}\|}\right)^2 & 0 & 0 & 0 & 0 & 0 \\ 0 & c_{ij}^y\left(1-\frac{\|\rho_{ij0}\|}{\|r_{ia}-r_{ja}\|}\right)^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & c_{ij}^z\left(1-\frac{\|\rho_{ij0}\|}{\|r_{ia}-r_{ja}\|}\right)^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & c_{ij}^\psi & 0 & 0 \\ 0 & 0 & 0 & 0 & c_{ij}^\theta & 0 \\ 0 & 0 & 0 & 0 & 0 & c_{ij}^\varphi \end{bmatrix}.$$

Therefore, after substitution, the total potential energy is

$$\begin{aligned} 2\Pi_a = {} & Y_0^T(\pi_{01}+\pi_{02}+\cdots+\pi_{0m})Y_0 - Y_0^T\pi_{01}Y_1 - Y_0^T\pi_{02}Y_2 - \cdots \\ & - Y_0^T\pi_{0m}Y_m - Y_1^T\pi_{10}Y_0 + Y_1^T(\pi_{10}+\pi_{12}+\cdots+\pi_{1m})Y_1 \\ & - Y_1^T\pi_{12}Y_2 - \cdots - Y_1^T\pi_{1m}Y_m \\ & \cdots\cdots\cdots\cdots\cdots \\ & - Y_m^T\pi_{m0}Y_0 - Y_m^T\pi_{m1}Y_1 - \cdots + Y_m^T(\pi_{m0}+\pi_{m1}+\cdots+\pi_{m(m-1)})Y_m \end{aligned}$$

or, in comprised form,

$$2\Pi_a = Y^T\pi_a(Y)Y, \tag{374}$$

where, due to $\pi_{ij} = \pi_{ji}$,

$$\pi_a(Y) = \pi_a^T(Y)$$

$$= \begin{bmatrix} \sum_{k=0,k\neq 0}^n \pi_{0k} & -\pi_{01} & -\pi_{02} & \cdots & -\pi_{0m} \\ -\pi_{01} & \sum_{k=0,k\neq 1}^n \pi_{1k} & -\pi_{12} & \cdots & -\pi_{1m} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ -\pi_{0m} & -\pi_{1m} & -\pi_{2m} & \cdots & \sum_{k=0,k\neq m}^n \pi_{km} \end{bmatrix}.$$

From this, the derivative of potential energy with respect to the coordinate is

$$\frac{\partial \Pi_a}{\partial Y} = \frac{1}{2}\frac{\partial Y^T\bar{\pi}_a Y}{\partial Y} + \pi_a(Y)Y,$$

$$\frac{\partial \Pi_a}{\partial Y} = \begin{bmatrix} \frac{\partial \Pi_a}{\partial Y_0} \\ \cdots \\ \frac{\partial \Pi_a}{\partial Y_i} \\ \cdots \\ \frac{\partial \Pi_a}{\partial Y_m} \end{bmatrix} = \begin{bmatrix} \frac{1}{2}\frac{\partial Y^T\bar{\pi}_a Y}{\partial Y_0} + \pi_{0a}(Y)Y \\ \cdots \\ \frac{1}{2}\frac{\partial Y^T\bar{\pi}_a Y}{\partial Y_i} + \pi_{ia}(Y)Y \\ \cdots \\ \frac{1}{2}\frac{\partial Y^T\bar{\pi}_a Y}{\partial Y_m} + \pi_{ma}(Y)Y \end{bmatrix},$$

$$\Rightarrow \frac{\partial \Pi_a}{\partial Y_i} = \frac{1}{2}\frac{\partial Y^T\bar{\pi}_a Y}{\partial Y_i} + \pi_{ia}(Y)Y \in R^6,$$

where $\pi_{ia}(Y) \in R^{6\times(6m+6)}$ are the submatrices composed of the rows starting from $6i+1$ to $6i+6$ inclusive of the matrix $\pi_a(Y)$, and $\partial(Y^T\bar{\pi}_a Y)/\partial Y_i$ is the vector of the quadratic form (scalar) derivative $Y^T\pi_a Y$ with respect to the vector Y_i, whereby the macron designates that partial derivation is carried out over the matrix π_a. In expanded form, this vector is

$$\frac{\partial Y^T\bar{\pi}_a Y}{\partial Y_i} = \begin{bmatrix} \dfrac{\partial Y^T\bar{\pi}_a Y}{\partial Y_i^1} \\ \dfrac{\partial Y^T\bar{\pi}_a Y}{\partial Y_i^2} \\ \cdots \\ \dfrac{\partial Y^T\bar{\pi}_a Y}{\partial Y_i^6} \end{bmatrix} = \begin{bmatrix} Y^T \dfrac{\partial \pi_a}{\partial Y_i^1} Y \\ Y^T \dfrac{\partial \pi_a}{\partial Y_i^2} Y \\ \cdots \\ Y^T \dfrac{\partial \pi_a}{\partial Y_i^6} Y \end{bmatrix}.$$

Total dissipation energy consumed in the course of linear and rotational displacement of the body i relative to the body j, $i, j = 0, 1, \ldots, m$ is defined by

$$\begin{aligned} \mathcal{D}_a = \mathcal{D}_{01a} + \mathcal{D}_{02a} + \mathcal{D}_{03a} + \cdots + \mathcal{D}_{0ma} + \\ + \mathcal{D}_{12a} + \mathcal{D}_{13a} + \cdots + \mathcal{D}_{1ma} + \\ + \mathcal{D}_{23a} + \cdots + \mathcal{D}_{2ma} + \\ \cdots\cdots \\ + \mathcal{D}_{(m-1)m}. \end{aligned}$$

An arbitrary member $\mathcal{D}_{ija}$ of that sum is given by

$$\begin{aligned} -2\mathcal{D}_{ija} &= (\dot{\delta}_{ij}^D)^T D_{ij}^{\delta} \dot{\delta}_{ij}^D + (\dot{\mathcal{A}}_{ia} - \dot{\mathcal{A}}_{ja})^T D_{ij}^{\mathcal{A}} (\dot{\mathcal{A}}_{ia} - \dot{\mathcal{A}}_{ja}) \\ &= (\dot{r}_{ia} - \dot{r}_{ja})^T \mathcal{G}_{ija}(r_{ia}, r_{ja}) D_{ij}^{\delta} \mathcal{G}_{ija}(r_{ia}, r_{ja})(\dot{r}_{ia} - \dot{r}_{ja}) \\ &\quad + (\dot{\mathcal{A}}_{ia} - \dot{\mathcal{A}}_{ja})^T D_{ij}^{\mathcal{A}} (\dot{\mathcal{A}}_{ia} - \dot{\mathcal{A}}_{ja}) \\ &= ((\dot{r}_{ia} - \dot{r}_{ja})^T \mid (\dot{\mathcal{A}}_{ia} - \dot{\mathcal{A}}_{ja})^T) \\ &\times \begin{bmatrix} \mathcal{G}_{ija}(r_{ia}, r_{ja}) D_{ij}^{\delta} \mathcal{G}_{ija}(r_{ia}, r_{ja}) & 0_{3\times3} \\ 0_{3\times3} & D_{ij}^{\mathcal{A}} \end{bmatrix} \begin{bmatrix} \dot{r}_{ia} - \dot{r}_{ja} \\ \dot{\mathcal{A}}_{ia} - \dot{\mathcal{A}}_{ja} \end{bmatrix} \\ &= (\dot{Y}_i - \dot{Y}_j)^T D_{ij} (\dot{Y}_i - \dot{Y}_j) \\ &= \dot{Y}_i^T D_{ij} \dot{Y}_i - 2\dot{Y}_i^T D_{ij} \dot{Y}_j + \dot{Y}_j^T D_{ij} \dot{Y}_j, \end{aligned}$$

where

$$D_{ij} = D_{ij}^T = D_{ji} = \operatorname{diag}(\mathcal{G}_{ija}(r_{ia}, r_{ja}) D_{ij}^{\delta} \mathcal{G}_{ija}(r_{ia}, r_{ja}), D_{ij}^{\mathcal{A}}) \in R^{6\times6},$$

wherefrom, after substitution, the total dissipation energy is determined by

$$\begin{aligned}
-2\mathcal{D}_a = {} & \dot{Y}_0^T(D_{01} + D_{02} + \cdots + D_{0m})\dot{Y}_0 - \dot{Y}_0^T D_{01}\dot{Y}_1 - \cdots - \dot{Y}_0^T D_{0m}\dot{Y}_m \\
& -\dot{Y}_1^T D_{10}\dot{Y}_0 + \dot{Y}_1^T(D_{10} + D_{12} + \cdots + D_{1m})\dot{Y}_1 - \cdots - \dot{Y}_1^T D_{1m}\dot{Y}_m \\
& \cdots\cdots\cdots\cdots\cdots \\
& -\dot{Y}_m^T D_{m0}\dot{Y}_0 - \dot{Y}_m^T D_{m1}\dot{Y}_1 - \cdots + \dot{Y}_m^T(D_{m0} + D_{m1} + \cdots + D_{m(m-1)})\dot{Y}_m.
\end{aligned}$$

In united quadratic form with respect to the absolute coordinates derivatives, the dissipation energy is given by

$$2\mathcal{D}_a = -\dot{Y}^T D_a(Y)\dot{Y}, \tag{375}$$

where, because of $D_{ij} = D_{ji}$

$$\begin{aligned}
D_a(Y) &= D_a^T(Y) \\
&= \begin{bmatrix} \sum_{k=0,k\neq 0}^{n} D_{0k} & -D_{01} & -D_{02} & \cdots & -D_{0m} \\ -D_{01} & \sum_{k=0,k\neq 1}^{n} D_{1k} & -D_{12} & \cdots & -D_{1m} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ -D_{0m} & -D_{1m} & -D_{2m} & \cdots & \sum_{k=0,k\neq m}^{n} D_{km} \end{bmatrix}.
\end{aligned}$$

From this, the derivative with respect to velocity is defined by

$$\begin{aligned}
\frac{\partial \mathcal{D}_a}{\partial \dot{Y}} &= -D_a(Y)\dot{Y}, \\
\frac{\partial \mathcal{D}_a}{\partial \dot{Y}} &= \begin{bmatrix} \frac{\partial \mathcal{D}_a}{\partial \dot{Y}_o} \\ \cdots \\ \frac{\partial \mathcal{D}_a}{\partial \dot{Y}_i} \\ \cdots \\ \frac{\partial \mathcal{D}_a}{\partial \dot{Y}_m} \end{bmatrix} = \begin{bmatrix} D_{0a}(Y)\dot{Y} \\ \cdots \\ D_{ia}(Y)\dot{Y} \\ \cdots \\ D_{ma}(Y)\dot{Y} \end{bmatrix} \Rightarrow \frac{\partial \mathcal{D}_a}{\partial \dot{Y}_i} = -D_{ia}(Y)\dot{Y} \in R^{6\times 1},
\end{aligned}$$

where $D_{ia}(Y) \in R^{6\times 6(m+1)}$ are the submatrices composed of the rows starting from $6i + 1$ to $6i + 6$ inclusive of the matrix $D_a(Y)$.

Substituting the obtained expressions into Langrange's equations

$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial T_a}{\partial \dot{Y}_i^j} - \frac{\partial T_a}{\partial Y_i^j} - \frac{\partial \mathcal{D}_a}{\partial \dot{Y}_i^j} + \frac{\partial \Pi_a}{\partial Y_i^j} = Q_{ia}^j, \quad i = 0, 1, \ldots, m, \quad j = 1, \ldots, 6, \tag{376}$$

where the generalized forces for the individual components Y_i^j of the vector Y_i are given by

$$\begin{aligned} Q_{ia}^j &= \sum_{k=0}^{m} \frac{\partial Y_k}{\partial Y_i^j}(G_k(m_k g) + F_k) \\ &= G_i^j(m_i g) + F_i^j, \quad i = 0, 1, \dots, m, \quad j = 1, \dots, 6, \end{aligned}$$

for the elastic system performing general motion under the action of a system of external forces about the mobile unloaded state 0, which is also performing general motion, the general form is obtained as

$$\begin{aligned} W_{ia}(Y_i)\ddot{Y}_i \;+\; \dot{W}_{ia}(Y_i)\dot{Y}_i - \frac{\partial T_{ia}}{\partial Y_i}(Y_i, \dot{Y}_i) + D_{ia}\dot{Y} \\ +\; \frac{1}{2}\frac{\partial Y^T \bar{\pi}_a Y}{\partial Y_i} + \pi_{ia}(Y)Y = G_i(m_i g) + F_i, \end{aligned}$$

for $i = 0, 1, \dots, m$ or, in short form,

$$W_{ia}(Y_i)\ddot{Y}_i + w_{ia}(Y, \dot{Y}) = F_i, \quad i = 0, 1, \dots, m,$$

where

$$\begin{aligned} w_{ia}(Y, \dot{Y}) \;=\; \dot{W}_{ia}(Y_i)\dot{Y}_i - \frac{\partial T_{ia}}{\partial Y_i}(Y_i, \dot{Y}_i) + D_{ia}\dot{Y} \\ + \frac{1}{2}\frac{\partial Y^T \bar{\pi}_a Y}{\partial Y_i} + \pi_{ia}(Y)Y - G_i(m_i g) \in R^{6\times 1}, \end{aligned}$$

for $i = 0, 1, \dots, m$. By putting together all $6m + 6$ equations, we obtain

$$W_a(Y)\ddot{Y} + w_a(Y, \dot{Y}) = F, \tag{377}$$

where

$$\begin{aligned} W_a(Y) &= \mathrm{diag}(W_{0a}(Y_0), W_{1a}(Y_1), \dots, W_{ma}(Y_m)) \in R^{(6m+6)\times(6m+6)}, \\ W_a(Y) &= W_a^T(Y),\ \det W_a(Y) \neq 0, \\ w_a(Y, \dot{Y}) &= \mathrm{col}(w_{0a}(Y, \dot{Y}), \dots, w_{ma}(Y, \dot{Y})) \in R^{(6m+6)\times 1}. \end{aligned}$$

From $6m + 6$ equations (377), only rank K equations are independent.

Equation (377) can be presented in such way that the descriptions of connections and manipulated object motion are divided:

$$W_{ca}(Y_c)\ddot{Y}_c + w_{ca}(Y, \dot{Y}) = F_{ca},$$

$$W_{0a}(Y_0)\ddot{Y}_0 + w_{0a}(Y, \dot{Y}) = 0, \tag{378}$$

where the subscript c designates the quantities related to the contact points, and the subscript 0 quantities related to the manipulated object. Here

$$\begin{aligned}
Y_{ca} &= \operatorname{col}(Y_{1a}, Y_{2a}, \ldots., Y_{ma}) \in R^{6m\times 1},\ Y_{0a} \in R^{6\times 1},\\
F_{ca} &= \operatorname{col}(F_{1a}, F_{2a}, \ldots., F_{ma}) \in R^{6m\times 1},\ F_{0a} = 0 \in R^{6\times 1},\\
W_{ca}(Y_{ca}) &= \operatorname{diag}(W_{1a}(Y_1) \ldots W_{ma}(Y_m)) \in R^{6m\times 6m},\\
W_{ca}(Y_c) &= W_{ca}^T(Y_c),\ \det W_{ca}(Y_c) \neq 0,\\
w_{ca}(Y, \dot{Y}) &= \operatorname{col}(w_{1a}(Y, \dot{Y}), \ldots, w_{ma}(Y, \dot{Y})) \in R^{6m\times 1},
\end{aligned}$$

where the expanded position vector of the contact position in the $6m$-dimensional space is denoted by Y_{ca} and the expanded vector of the contact forces acting at the contact point in that space is denoted by F_{ca}. It should be noted that no contact force acts directly at the manipulated object MC, hence $F_{0a} = 0$. Equations (377) and (378) represent the final form of the equations of elastic system behavior under the action of external forces F_{ca}, while the system performs general motion about the unloaded state 0, which also performs general motion.

The general motion of elastic system motion is described by (377), i.e. $6m + 6$ relations are defined, of which rank K are independent. This means that for the unique definition of elastic system position during the motion, it is necessary to prescribe $6m + 6 - \operatorname{rank} K$ absolute generalized coordinates and their derivatives.

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