Appendix A
Stochastic Processes

Probability Space The outcome of a phenomenon will be denoted by ξ. ξ belongs to a set Ω which contains all the possible outcomes of a phenomenon. Therefore, Ω is the *sure event*. One denotes the *impossible event* by ϕ. The set of events is denoted by $\mathcal{F}$. A probability measure P is associated to Ω, i.e., for subsets of Ω probability of occurrence is assigned. The triplet $(\Omega, \mathcal{F}, P)$ is called a *probability space*.

A σ-algebra $\mathcal{F}$ is a set of subsets of Ω which contains the empty set ϕ and is closed under complements and countable unions.

A random variable generates a σ-algebra $\mathcal{F}$ since all possible outcomes define subsets in the event space Ω.

Axioms of Probability Consider two events a and b belonging to Ω. To each event one assigns a probability $P(\cdot)$ satisfying the following axioms:

1. $P(a) \geq 0;\ \forall a \in \Omega$
2. $P(\Omega) = 1$
3. $P(a \cup b) = P(a) + P(b)$ if $a \cap b = \emptyset$

Random Variable A *random variable* X is a function from the *event space* Ω to R^1 and will be denoted by $X(\xi)$ with $\xi \in \Omega$. (The argument will be omitted in general.)

Induced Probability Since to each subset (event) of Ω, a probability has been assigned and since x is a function of ξ, there is a probability that X takes a value in a subset of R^1, i.e.:

$$P\{X(\xi) \leq x_1\} \quad \text{or} \quad P\{x_1 \leq X(\xi) \leq x_2\}$$

Distribution Function

$$F_X(x) = P\{X \leq x\}; \quad -\infty < x < \infty$$

is called the *distribution function*.

Probability Density Function

$$F_X(x) = P\{X \leq x\} = \int_{-\infty}^{x} f_X(x)dx$$

I.D. Landau et al., *Adaptive Control*, Communications and Control Engineering,
DOI 10.1007/978-0-85729-664-1,

where $f_X(x)$ is the *probability density function* (i.e., $F_X(x)$ is continuous and the derivatives with respect to x exist).

Gaussian (Normal) Distribution

$$f_X(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-x^2/2\sigma^2}$$

Expectation (Mean Value) of a Random Variable

$$m(X) = \mathbf{E}\{X(\xi)\} = \int_{-\infty}^{\infty} f_X(x)dx$$

Covariance of a Random Variable

$$\operatorname{cov} X = \mathbf{E}[X - m(x)]^2 = \mathbf{E}\{[X(\xi) - m(x)]^2\}$$

Independent Random Variables Two random variables X and Y are characterized by the *joint distribution function*

$$F_{XY}(xy) = P\{X \le x; Y \le y\}$$

The random variables X and Y are called *independent* if:

$$F_{XY}(x, y) = F_X(x)F_X(y) = P\{X \le x\}P\{Y \le y\}$$

Uncorrelated Random Variables Two random variables X and Y are uncorrelated if:

$$\mathbf{E}\{XY\} = \mathbf{E}\{X\}\mathbf{E}\{Y\}$$

Note: independence implies uncorrelation

Orthogonal Random Variables Two random variables X and Y are orthogonal if:

$$\mathbf{E}\{XY\} = 0$$

Sequence of Independent Random Variables Given n real random variables $X_1, X_2, \ldots, X_n$ they are independent if:

$$P\{X_1 \le x_1, \ldots, X_n \le x_n\} = P\{X_1 \le x_1\}, \ldots, P\{X_n \le x_n\}$$

Discrete-Time Stochastic Process A *discrete-time stochastic process* is defined as a sequence of random variables defined on a common probability space and indexed by the discrete time $t = 0, 1, 2 \ldots$. A stochastic process x is a function of t and ξ: $X(t, \xi)$. For a fixed t, one obtains a random variable. For each ξ, one obtains a *realization* of the stochastic process.

Mean Value of a Stochastic Process

$$m_X(t) = \mathbf{E}\{X(t, \xi)\}$$

Covariance Function

$$R_{XY}(t, l) = \mathbf{E}\{[X(t) - m_X(t)][Y(l) - m_Y(l)]\}$$

Discrete-Time (Gaussian) White Noise $\{e(t)\}$ It is a sequence of independent equally distributed (normal) random variables of zero mean ($m(x) = 0$) and variance σ^2. Often the sequence will be denoted $\{e(t)\}$ and will be characterized by the parameters $(0, \sigma)$ where 0 corresponds to zero mean and σ is the standard deviation (square root of the variance). This sequence has the following properties:

1. $\mathbf{E}\{e(t)\} = 0$
2. $R(t,l) = \mathbf{E}\{e(t)e(l)\} = \begin{cases} \sigma^2 & l = t \\ 0 & l \neq t \end{cases}$

Weakly Stationary Stochastic Processes They are characterized by:

1. $m_X(t) = m_X$
2. $R_{XX}(t,l) = R_{XX}(t-l)$

and the Fourrier transform of R_{XX} can be defined:

$$\phi_X(\omega) = \frac{1}{2\pi} \sum_{t=-\infty}^{t=\infty} R_{XX}(t)e^{-j\omega t}$$

which is called the *spectral density function*. For the discrete-time white noise:

$$\phi_e(\omega) = \frac{\sigma^2}{2\pi}$$

Conditional Probability Given an event b with non zero probability ($P(b) > 0$), the conditional probability of the event a assuming that b has occurred is defined by:

$$P(a|b) = \frac{P(a \cap b)}{P(b)}$$

This definition is extended to events $x_1, x_2, \ldots, x_n \in \mathcal{F}$ and more generally to $\mathcal{F}$.

Conditional Expectation

$$\mathbf{E}\{a|b\} = \frac{\mathbf{E}\{a \cap b\}}{\mathbf{E}\{b\}}$$

This can be extended to the case where b is the σ-algebra $\mathcal{F}_t$ generated by the sequence of random variables up to and including t. The conditional expectation of X with respect to $\mathcal{F}_t$ is written: $\mathbf{E}\{X|\mathcal{F}_t\}$. An increasing σ-algebra $\mathcal{F}_t$ has the property: $\mathcal{F}_{t-1} \subset \mathcal{F}_t$.

Martingale Sequence Let $(\Omega, \mathcal{F}, P)$ be a probability space and suppose that for each t there is a sub-σ-algebra $\mathcal{F}$ of $\mathcal{F}$ such that $\mathcal{F}_{t-i} \subset \mathcal{F}_t$ for $i \geq 0$. Then the sequence $X(t)$ is said to be adapted to the sequence of increasing σ-algebras $\mathcal{F}_t$ if $X(t)$ is $\mathcal{F}_t$ measurable for every t (every stochastic process is adapted to its own past).

Under the above conditions $X(t)$ is a *martingale* sequence provided that:

1. $\mathbf{E}\{|X(t)|\} < \infty$ almost sure
2. $\mathbf{E}\{X(t+1)|\mathcal{F}_t\} = 0$

Convergence w.p.1 (a.s.) If $X(t,\xi)$ is a sequence of random variables defined on a probability space $(\Omega, \mathcal{F}, P)$ such that:

$$\text{Prob}\left\{\lim_{t\to\infty} X(t) = x^*\right\} = 1$$

we say that: "$X(t)$ converges to x^* with probability 1 (w.p.1)" or "$X(t)$ converges to x^* almost sure (a.s.)".

Ergodic Stochastic Process (in the Mean) For any outcome ξ (with the exception of a subset with zero probability) one has:

$$P\left\{\lim_{N\to\infty}\frac{1}{N}\sum_{t=1}^{N}X(t,\xi)=\mathbf{E}\{X(t)\}\right\}=1$$

For ergodic stochastic process time average replaces set average.

Spectral Factorization Theorem (Åström 1970) Consider a stationary stochastic process with a spectral density $\phi(e^{j\omega})$ which is rational in $\cos\omega$.

1. There exists a linear system with the pulse transfer function

$$H(z^{-1})=\frac{C(z^{-1})}{D(z^{-1})}$$

with poles in $|z|<1$ and zeros in $|z|\le 1$ such that:

$$\phi(e^{j\omega})=H(e^{-j\omega})H(e^{j\omega})$$

2. The spectral density of the output of $H(z^{-1})$ when the input is a discrete-time Gaussian white noise is a stationary stochastic process with spectral density $\phi(e^{j\omega})$.

Remarks

1. Without the restriction on poles and zeros the factorization of $\phi(e^{j\omega})$ is not unique.
2. Spectral factorization theorem allows to consider all the stationary stochastic processes with a rational spectral density as being generated by an asymptotically stable linear system driven by a white noise.
3. Positive real lemma (see Appendix D) plays an important role in solving the factorization problem (see Faurre et al. 1979).

Innovation Process A consequence of the factorization theorem is that a stochastic process can be expressed as:

$$y(t+1)=f[y(t),y(t-1),\dots]+h(0)e(t+1)$$

where $e(t+1)$ is a white noise and $h(0)$ is the coefficient of the impulse response for $l=t+1-i=0$ ($y(t+1)=\sum_{i=-\infty}^{t+1}h(t+1-i)e(i)$). Therefore $y(t+1)$ can be expressed as a sum of two terms, a term which depends only on the past (measurable) quantities and a pure stochastic term which is the "true" new information unpredictable at t. This formulation can be further extended for $y(t+d)$, i.e.

$$y(t+d)=f_y[y(t),y(t-1),\dots]+f_e[e(t+d),e(t+d-1),\dots,e(t+1)]$$

The term $f_e[e(t+d),e(t+d-1)\dots]$ is called the *innovation*. As a consequence:

$$\mathbf{E}\{y(t+d)|y(t),y(t-1),\dots\}=f_y[y(t),y(t-1),\dots]$$
$$\mathbf{E}\{f_y\cdot f_e\}=0$$

The innovation representation plays an important role in prediction problems.

Appendix B
Stability

We will consider free (unforced) discrete-time systems of the form:

$$x(t+1) = f[x(t), t] \tag{B.1}$$

where $x(t)$ denotes the n-dimensional state vector of the system. In the linear time invariant case (B.1) becomes:

$$x(t+1) = Ax(t) \tag{B.2}$$

The solutions of the system (B.1) will be denoted by $x(t, x_0, t_0)$ where x_0 is the initial condition at time t_0.

Definition B.1 A state x_e of the system (B.1) is an *equilibrium state* if:

$$f(x_e, t) = 0; \quad \forall t \geq t_0$$

Definition B.2 (Stability) An equilibrium state x_e of the system (B.1) is *stable* if for arbitrary t_0 and $\varepsilon > 0$, there exists a real number $\delta(\varepsilon, t_0)$ such that:

$$\|x_0 - x_e\| \leq \delta(\varepsilon, t_0) \quad \Longrightarrow \quad \|x(t, x_0, t_0) - x_e\| \leq \varepsilon; \quad \forall t \geq t_0$$

Definition B.3 (Uniform Stability) An equilibrium state x_e of the system (B.1) is *uniformly stable* if it is stable and if $\delta(\varepsilon, t_0)$ does not depend on t_0.

Definition B.4 (Asymptotic Stability) An equilibrium state x_e of the system (B.1) is *asymptotically stable* if:

1. x_e is stable
2. there exists a real number $\delta(t_0) > 0$ such that:

$$\|x_0 - x_e\| \leq \delta(t_0) \quad \Longrightarrow \quad \lim_{t \to \infty} \|x(t, x_0, t_0) - x_e\| = 0$$

Definition B.5 (Uniformly Asymptotically Stable) An equilibrium state x_e of the system (B.1) is *uniformly asymptotically stable* if:

I.D. Landau et al., *Adaptive Control*, Communications and Control Engineering,
DOI 10.1007/978-0-85729-664-1, © Springer-Verlag London Limited 2011

1. x_e is uniformly stable
2. there exists a real number $\delta > 0$ independent of t_o such that:

$$\|x_0 - x_e\| \leq \delta \quad \Longrightarrow \quad \lim_{t\to\infty} \|x(t, x_0, t_0) - x_e\| = 0$$

Definition B.6 (Global Asymptotic Stability) An equilibrium state x_e of the system (B.1) is *globally asymptotically stable* if for all $x_0 \in R_n$:

1. x_e is stable
2. $\lim_{t\to\infty} \|x(t, x_0, t_0) - x_e\| = 0$

If, in addition, x_e is uniformly stable then x_e is *uniformly globally asymptotically stable*.

Theorem B.1 (Lyapunov) *Consider the free dynamic system* (B.1) *with*:

$$f(0,t) = 0; \quad -\infty < t < +\infty \ (x_e = 0 \textit{ is an equilibrium state})$$

If there exists a real scalar function $V(x,t)$ *such that*:

1. $V(0,t) = 0 \ -\infty < t < \infty$.
2. $V(x,t) \geq \alpha(\|x\|) > 0, \ \forall x \neq 0, x \in R_n$ *and* $\forall t$ *where* $\alpha(\cdot)$ *is a continuous non-decreasing scalar function such that* $\alpha(0) = 0$ ($V(x,t)$ *is positive definite*).
3. $V(x,t) \to \infty$ *with* $\|x\| \to \infty$ ($V(x,t)$ *is radially unbounded*).
4. $\Delta V(x,t) = V(x,t+1) - V(x,t) \leq -\mu(\|x\|) < 0, \ \forall x \neq 0, \ x \in R_n, \ \forall t$ *along the trajectories of the system* (B.1) *where* $\mu(\cdot)$ *is a continuous non-decreasing scalar function such that* $\mu(0) = 0$. ($\Delta V(x,t)$ *is negative definite*.) *Then the equilibrium state* $x_e = 0$ *is* globally asymptotically stable *and* $V(x,t)$ *is a* Lyapunov function *for the system* (B.1).

    If, in addition:
5. $V(x,t) \leq \beta(\|x\|)$ *where* $\beta(\cdot)$ *is a continuous, non-decreasing scalar function such that* $\beta(0) = 0$, *then* $x_e = 0$ *is* uniformly globally asymptotically stable.

Corollary B.1 *If in Theorem* B.1, $V(x,t)$ *is replaced by* $V(x)$, *the equilibrium state* $x_e = 0$ *of the autonomous system*:

$$x(t+1) = f[x(t)] \tag{B.3}$$

is globally uniformly asymptotically stable *if*:

1. $V(0) = 0$;
2. $V(x) > 0, \ \forall x \neq 0, \ x \in R_n$;
3. $V(x) \to \infty$ *with* $\|x\| \to \infty$;
4. $\Delta V[x(t)] = V[x(t+1)] - V[x(t)] < 0$ *along the trajectories of* (B.3) *for all* $x \neq 0, \ x \in R_n$.

Corollary B.2 *In Corollary* B.1, *condition* 4 *can be replaced by*:

1. $\Delta V(x) \leq 0, \ \forall x \neq 0, \ x \in R_n$;
2. $\Delta V(x)$ *is not identically zero along any trajectory of* (B.3).

Theorem B.2 (Lyapunov) *The equilibrium state* $x_e = 0$ *of a linear time invariant free system*:

$$x(t+1) = Ax(t) \tag{B.4}$$

is globally uniformly asymptotically stable if and only if, given a positive definite matrix Q, *there exists a positive definite matrix* P *which is the unique solution of the matrix equation*:

$$A^T PA - P = -Q \tag{B.5}$$

Consider the system:

$$x(t+1) = f[x(t), u(t)] \tag{B.6}$$

$$y(t) = h[x(t), t] \tag{B.7}$$

Definition B.7 The system is said to be weakly finite gain stable if:

$$\|y(t)\|_{2T} \le \kappa \|u(t)\|_{2T} + \beta[x(t_0)] \tag{B.8}$$

where $x(t_0) \in \Omega$, $0 < \kappa < \infty$, $|\beta[x(t_0)]| < \infty$ and $\|y(t)\|_{2T} = \sum_{t=t_0}^{T} y^2(t)$.

Consider the system:

$$x(t+1) = Ax(t) + Bu(t) \tag{B.9}$$

$$y(t) = Cx(t) + Du(t) \tag{B.10}$$

Lemma B.1 (Goodwin and Sin 1984) *Provided that the following conditions are satisfied*:

(i) $|\lambda_i(A)| \le 1;\ i = 1, \dots, n.$
(ii) *All controllable modes of* (A, B) *are inside the unit circle.*
(iii) *Any eigenvalues of* A *on the unit circle have a Jordan block of size* 1.

Then:

(a) *There exist constants* K_1 *and* K_2 *with* $0 \le K_1 < \infty$ *and* $0 \le K_2 < \infty$, *which are independent of* N *such that*:

$$\sum_{t=1}^{N} \|y(t)\|^2 \le K_1 \sum_{t=1}^{N} \|u(t)\|^2 + K_2; \quad \forall N \ge 1 \tag{B.11}$$

(*the system is finite gain stable*).

(b) *There exist constants* $0 \le m_3 < \infty$ *and* $0 \le m_4 < \infty$ *which are independent of* t *such that*:

$$|y(t)| \le m_3 + m_4 \max_{1 \le \tau \le N} \|u(\tau)\|; \quad \forall 1 \le t \le N \tag{B.12}$$

Lemma B.2 *If the system* (B.9), (B.10) *is a minimal state space realization and* $|\lambda_i(A)| < 1;\ i = 1, \ldots, n$, *then*:

$$\lim_{N\to\infty} \frac{1}{N} \sum_{t=1}^{N} \|y(t)\|^2 \leq K_1 \lim_{N\to\infty} \frac{1}{N} \sum_{t=1}^{N} \|u(t)\|^2 \tag{B.13}$$

For definitions and proofs concerning the stability of discrete time systems see Kalman and Bertram (1960), Mendel (1973).

Appendix C
Passive (Hyperstable) Systems

C.1 Passive (Hyperstable) Systems

The concept of passive (hyperstable) systems emerged in control through the seminal work of Popov, Kalman and Yakubovitch (the Popov-Kalman-Yakubovitch lemma) in connection with the problem of stability of nonlinear feedback systems (Popov 1963, 1964). The concept of *passive* (hyperstable) *system* can be viewed also as a particularization of the concept of *positive dynamic systems*. The term *positive dynamic system* is an extension of the mathematical concept of *positivity*. This is a necessary and sufficient condition for a mathematical object to be factorizable as a product (for example a *positive definite matrix* Q can be factored as $Q = LL^T$).

The study of passive (hyperstable) systems and their relations on the one hand with positive dynamic systems and on the other hand with the properties of passive network have been initiated in Popov (1964). The terms "hyperstable" has been coined to describe some input-output properties of a system which appears to be a generalization of the property of passive systems, which requires that at every moment the stored energy be equal or less than the initial energy stored in the system, plus the energy supplied to the system's terminal over the considered time horizon. (This generalization has been done more or less in the same way as for Lyapunov functions which are positive definite functions, and for a particular state representation they can be related with the energy of the system.) The main use of this property was to treat stability problems related to the interconnection of hyperstable systems. A deep treatment of the subject can be found in Popov (1966, 1973) as well as in Anderson and Vongpanitlerd (1972). Through the work of Willems (1971), Desoer and Vidyasagar (1975), the term *passive* systems (instead of hyperstability) became widely accepted, even if in this context, *passive* is not directly related to the energy supplied to a system. References for this subject include Zames (1966), Anderson (1967), Hitz and Anderson (1969), Hill and Moylan (1980), Faurre et al. (1979).

I.D. Landau et al., *Adaptive Control*, Communications and Control Engineering,
DOI 10.1007/978-0-85729-664-1,

C.2 Passivity—Some Definitions

The norm of a vector is defined as:

$$\|x(t)\| = (x^T x)^{1/2}; \quad x \in R_n$$

The norm L_2 is defined as:

$$\|x(t)\|_2 = \left(\sum_0^{\infty} x^T(t)x(t)\right)^{1/2}$$

where $x(t) \in \mathbb{R}_n$ and t is an integer (it is assumed that all signals are 0 for $t < 0$). To avoid the assumption that all signals go to zero as $t \to \infty$, one uses the extended L_2 space denoted L_{2e} which contains the *truncated* sequences:

$$x_T(t) = \begin{cases} x(t) & 0 \le t \le T \\ 0 & t > T \end{cases}$$

Consider a system S with input u and output y (of same dimension). Let us define the input-output product:

$$\eta(0, t_1) = \sum_{t=0}^{t_1} y^T(t)u(t)$$

Definition C.1 A system S is termed *passive* if:

$$\eta(0, t_1) \ge -\gamma^2; \quad \gamma^2 < \infty; \ \forall t_1 \ge 0$$

Definition C.2 A system S is termed *input strictly passive* if:

$$\eta(0, t_1) \ge -\gamma^2 + \kappa\|u\|_{2T}^2; \quad \gamma^2 < \infty; \ \kappa > 0; \ \forall t_1 \ge 0$$

Definition C.3 A system S is termed *output strictly passive* if:

$$\eta(0, t_1) \ge -\gamma^2 + \delta\|y\|_{2T}^2; \quad \gamma^2 < \infty; \ \delta > 0; \ \forall t_1 \ge 0$$

Definition C.4 A system S is termed *very strictly passive* if:

$$\eta(0, t_1) \ge -\gamma^2 + \kappa\|u\|_{2T}^2 + \delta\|y\|_{2T}^2; \quad \gamma^2 < \infty; \ \delta > 0; \ \kappa > 0; \ \forall t_1 \ge 0$$

In all the above definitions, the term γ^2 will depend upon the initial conditions. If a state space representation (state vector: x) can be associated to the system S, one can introduce also the following lemmas:

Lemma C.1 *The system resulting from the parallel connection of two passive blocks is passive.*

Lemma C.2 *The system resulting from the negative feedback connection of two passive blocks is passive.*

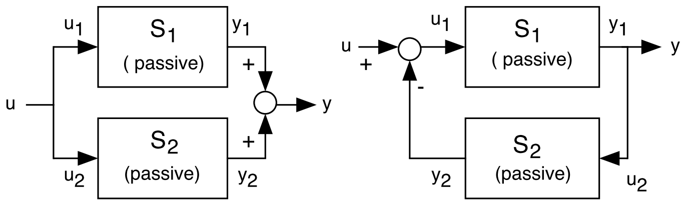


Fig. C.1 Parallel and feedback connections of two passive blocks

Proof The resulting systems are illustrated in Fig. C.1. The proof is based on the observation that in the *parallel* connection, one has for the resulting system (input u, output y):

$$u = u_1 = u_2; \qquad y = y_1 + y_2$$

and in the case of *feedback* connection, one has:

$$u = u_1 + y_2; \qquad y = y_1 = u_2$$

where $u_j, y_j, j = 1, 2$ are the input and output of the elementary blocks. The above observations allow to write for the resulting system:

$$\eta(0, t_1) = \sum_{t=0}^{t_1} y^T(t)u(t)$$
$$= \sum_{t=0}^{t_1} y_1^T(t)u_1(t) + \sum_{t=0}^{t_1} y_2^T(t)u_2(t) \geq -\gamma_1^2 - \gamma_2^2; \quad \gamma_1, \gamma_2 < \infty \qquad \square$$

Definition C.5 A symmetric square matrix P is termed positive (semidefinite) definite if:

$$x^T Px(\geq) > 0; \quad \forall x \neq 0; \ x \in R^n$$

A positive definite matrix P can be expressed as: $P = LL^T$ with L a nonsingular matrix.

Definition C.6 A matrix function $H(z)$ of the complex variable z is a *Hermitian matrix* (or simply *Hermitian*) if:

$$H(z) = H^T(z^*)$$

(where $*$ means *conjugate*).

Hermitian matrices feature several properties, including:

1. A *Hermitian* matrix is a square matrix and the diagonal terms are real.
2. The eigenvalues of a *Hermitian* matrix are always real.
3. If $H(z)$ is a *Hermitian* matrix and x is a vector with *complex* components, the quadratic form $x^T Hx^*$ is always real.

Definition C.7 A Hermitian matrix is positive (semidefinite) definite if:

$$x^T H x^* (\geq) > 0; \quad \forall x \neq 0; \; x \in C^n$$

C.3 Discrete Linear Time-Invariant Passive Systems

Consider the linear time invariant discrete time system:

$$x(t+1) = Ax(t) + Bu(t) \tag{C.1}$$

$$y(t) = Cx(t) + Du(t) \tag{C.2}$$

where x is a n-dimensional state vector, u and y are m-dimensional vectors representing the input and the output respectively and A, B, C, D are matrices of appropriate dimension. One assumes that $[A, B, C, D]$ is a minimal realization of the system. The system of (C.1) and (C.2) is also characterized by the square transfer matrix:

$$H(z) = D + C(zI - A)^{-1} B \tag{C.3}$$

Definition C.8 A $m \times m$ discrete matrix $H(z)$ of real rational functions is *positive real* if and only if:

1. All elements of $H(z)$ are analytic on $|z| > 1$ (i.e., they do not have poles in $|z| > 1$).
2. The eventual poles of any element of $H(z)$ on the unit circle $|z| = 1$ are simple and the associated residue matrix is a positive semidefinite Hermitian.
3. The matrix $H(e^{j\omega}) + H^T(e^{-j\omega})$ is a positive semidefinite Hermitian for all real values of ω which are not a pole of any element of $H(e^{j\omega})$.

In the case of a scalar transfer function $H(z)$ condition 3 is replaced by:

$$\operatorname{Re} H(z) \geq 0; \quad \forall |z| = 1$$

Definition C.9 The system (C.1)–(C.2) is termed *hyperstable* if:

$$\eta(0, t_1) \geq \beta_1 \|x(t_1 + 1)\|^2 - \beta_0 \|x(0)\|^2; \quad \beta_1, \beta_0 > 0; \; \forall t_1 \geq 0$$

and for all $u(t)$ and $x(t)$ that satisfy (C.1).

The inequality considered in Definition C.9 is a generalization of the passivity condition from physics. With an appropriate choice of the state vector, the stored energy at $t_1 + 1$ may be expressed as $\beta \|x(t_1 + 1)\|^2$ with $\beta > 0$ and $\eta(0, t_1)$ is the energy supplied to the system's terminals over $(0, t_1)$. Passivity requires that at every moment the stored energy be equal to or less than the initial energy plus the energy supplied to the system.

Hyperstability implies passivity in the sense of Definition C.1 ($\gamma = \beta_0 \|x(0)\|^2$). The converse is also true under the minimality assumption on the system (C.1)–(C.2).

Lemma C.3 (Positive real lemma) *The following propositions concerning the system of* (C.1) *and* (C.2) *are equivalent to each other*:

1. $H(z)$ *given by* (C.3) *is a positive real discrete time transfer matrix.*
2. *The system* (C.1)–(C.2) *is* passive.
3. *There is a positive definite matrix* P, *a positive semidefinite matrix* Q *and matrices* S *and* R *such that*:

$$A^T PA - P = -Q \tag{C.4}$$

$$C - B^T PA = S^T \tag{C.5}$$

$$D + D^T - B^T PB = R \tag{C.6}$$

$$M = \begin{bmatrix} Q & S \\ S^T & R \end{bmatrix} \geq 0 \tag{C.7}$$

4. *There is a positive definite matrix* P *and matrices* K *and* L *such that*:

$$A^T PA - P = -LL^T \tag{C.8}$$

$$C - B^T PA = K^T L^T \tag{C.9}$$

$$D + D^T - B^T PB = K^T K \tag{C.10}$$

5. *Every solution* $x(t)$, $(x(0), u(t))$ *of* (C.1), (C.2) *satisfies the following equality*:

$$\sum_{t=0}^{t_1} y^T(t)u(t) = \frac{1}{2}x^T(t_1+1)Px(t_1+1) - \frac{1}{2}x^T(0)Px(0) + \frac{1}{2}\sum_{t=0}^{t_1}[x^T(t), u^T(t)]M\begin{bmatrix} x(t) \\ u(t) \end{bmatrix} \tag{C.11}$$

6. *The system* (C.1)–(C.2) *is* hyperstable.

Proof Detailed proof can be found in Hitz and Anderson (1969), Popov (1973). Property (2) results immediately from property (5) ($\gamma^2 = -\frac{1}{2}x^T(0)Px(0)$). Equivalence between (3) and (4) results immediately from (C.7), which taking into account the properties of positive semidefinite matrices can be expressed as:

$$\begin{bmatrix} Q & S \\ S^T & R \end{bmatrix} = NN^T = \begin{bmatrix} L \\ K^T \end{bmatrix}[L^T \; K] = \begin{bmatrix} LL^T & LK \\ K^T L^T & K^T K \end{bmatrix} \geq 0$$

where L is a $(n \times q)$-dimensional matrix and K^T is a $(m \times q)$-dimensional matrix. Replacing Q by LL^T, S^T by $K^T L^T$ and R by $K^T K$ in (C.4) through (C.6), one gets (C.8) through (C.10). We will limit ourselves to show that (3) $\rightarrow$ (5), since this will be used subsequently for other proofs.

Using (C.4) and (C.1), one has:

$$\begin{aligned}x^T(t)Qx(t) &= -x^T(t+1)Px(t+1) + x^T(t)Px(t)\\&\quad + u^T(t)B^T Px(t+1) + x^T(t+1)PBu(t) - u^T(t)B^T PBu(t)\\&= -x^T(t+1)Px(t+1) + x^T(t)Px(t) + u^T(t)B^T P[Ax(t) + Bu(t)]\\&\quad + [x^T(t)A^T + u^T(t)B^T]PBu(t) - u^T(t)B^T PBu(t)\end{aligned} \tag{C.12}$$

Adding the term $2u^T(t)S^T x(t) + u^T(t)Ru(t)$ in both sides of (C.12) and taking into account (C.5) and (C.6), one gets:

$$\begin{aligned}&x^T(t)Qx(t) + 2u^T(t)S^T x(t) + u^T(t)Ru(t)\\&\quad = -x^T(t+1)Px(t+1) + x^T(t)Px(t)\\&\qquad + 2u^T(t)Cx(t) + u^T(t)[D + D^T]u(t)\\&\quad = -x^T(t+1)Px(t+1)\\&\qquad + x^T(t)Px(t) + 2y^T(t)u(t)\end{aligned} \tag{C.13}$$

Summing up from 0 to t_1, one gets (C.11). □

Definition C.10 A $m \times m$ discrete transfer matrix $H(z)$ of real rational functions is *strictly positive real* if and only if:

1. All the elements $H(z)$ are analytic in $|z| \geq 1$.
2. The matrix

$$H(e^{j\omega}) + H^T(e^{-j\omega})$$

is a positive definite Hermitian for all real ω.

Definition C.11 The system (C.1)–(C.2) is termed *state strictly passive* if:

$$\eta(0, t_1) \geq \beta_1 \|x(t_1 + 1)\|^2 - \beta_0 \|x(0)\|^2 + \mu \|x\|^2_{2T}; \quad \beta_1, \beta_0, \mu > 0;\ \forall t_1 \geq 0$$

and for all $u(t)$ and $x(t)$ that satisfy (C.1).

Lemma C.4 (Generalized Positive Real Lemma) *Consider the system* (C.1)–(C.2) *characterized by the transfer matrix $H(z)$ given in* (C.3). *Assume that there is a symmetric positive matrix P, a semi-positive definite matrix Q, symmetric matrices R, Δ, K, M_o and a matrix S of appropriate dimensions such that*:

$$A^T PA - P = -Q - C^T \Delta C - M_0 \tag{C.14}$$

$$C^T - B^T PA = S^T + D^T \Delta C \tag{C.15}$$

$$D + D^T - B^T PB = R + D^T \Delta D + K \tag{C.16}$$

$$M = \begin{bmatrix} Q & S \\ S^T & R \end{bmatrix} \geq 0 \tag{C.17}$$

Then:

1. *Every solution $x(t, x(0), u(t))$ of* (C.1)–(C.2) *satisfies the following equality*:

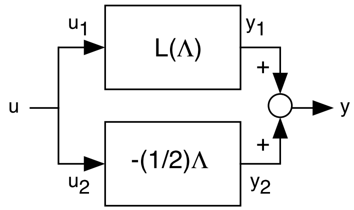
$$\sum_{t=0}^{t_1} y^T(t)u(t) = \frac{1}{2}x^T(t_1+1)Px(t_1+1) - \frac{1}{2}x^T(0)Px(0)$$
$$+ \frac{1}{2}\sum_{t=0}^{t_1}[x^T(t)u^T(t)]M\begin{bmatrix} x(t) \\ u(t) \end{bmatrix} + \frac{1}{2}\sum_{t=0}^{t_1} x^T(t)M_0x(t)$$
$$+ \frac{1}{2}\sum_{t=0}^{t_1} u^T(t)Ku(t) + \frac{1}{2}\sum_{t=0}^{t_1} y^T(t)\Delta y(t) \tag{C.18}$$

2. *For* $M_0 = 0$, $\Delta = 0$, $K = 0$
   - *The system* (C.1)–(C.2) *is passive*
   - $H(z)$ *is positive real*
3. *For* $M_0 \geq 0$, $\Delta \geq 0$, $K > 0$
   - *The system* (C.1)–(C.2) *is input strictly passive*
   - $H(z)$ *is positive real*
4. *For* $M_0 \geq 0$, $\Delta > 0$, $K \geq 0$
   - *The system* (C.1)–(C.2) *is output strictly passive*
   - $H(z)$ *is positive real*
5. *For* $M_0 > 0$, $\Delta \geq 0$, $K > 0$
   - *The system* (C.1)–(C.2) *is input strictly passive*
   - *The system* (C.1)–(C.2) *is state strictly passive*
   - $H(z)$ *is strictly positive real*
6. *For* $M_0 > 0$, $\Delta > 0$, $K \geq 0$
   - *The system* (C.1)–(C.2) *is output strictly passive*
   - *The system* (C.1)–(C.2) *is state strictly passive*
   - $H(z)$ *is strictly positive real*
7. *For* $M_0 > 0$, $\Delta > 0$, $K > 0$
   - *The system* (C.1)–(C.2) *is very strictly passive*
   - *The system* (C.1)–(C.2) *is state strictly passive*
   - $H(z)$ *is strictly positive real*

Proof The properties (2) through (7) result from the property (1) bearing in mind the various definitions of passivity and the properties of positive and strictly positive real transfer matrices. We will show next that (C.14) through (C.17) implies property (C.18). Denoting:

$$\bar{Q} = Q + C^T\Delta C + M_0 \tag{C.19}$$
$$\bar{S} = S + C^T\Delta D \tag{C.20}$$
$$\bar{R} = R + D^T\Delta D + K \tag{C.21}$$
$$\bar{M} = \begin{bmatrix} \bar{Q} & \bar{S} \\ \bar{S}^T & \bar{R} \end{bmatrix} \tag{C.22}$$

Fig. C.2 The class $L(\Lambda)$



one can use the result (C.11) of Lemma C.3, which yields:

$$\sum_{t=0}^{t_1} y^T(t)u(t) = \frac{1}{2}x^T(t_1+1)Px(t_1+1) - \frac{1}{2}x^T(0)Px(0) + \frac{1}{2}\sum_{t=0}^{t_1}[x^T(t), u^T(t)]\bar{M}\begin{bmatrix} x(t) \\ u(t) \end{bmatrix} \tag{C.23}$$

Taking into account the particular form of $\bar{M}$, one obtains:

$$\begin{aligned}\sum_{t=0}^{t_1} y^T(t)u(t) &= \frac{1}{2}x^T(t_1+1)Px(t_1+1) - \frac{1}{2}x^T(0)Px(0) \\ &\quad + \frac{1}{2}\sum_{t=0}^{t_1}[x^T(t)u^T(t)]M\begin{bmatrix} x(t) \\ u(t) \end{bmatrix} \\ &\quad + \frac{1}{2}\sum_{t=0}^{t_1}x^T(t)M_0x(t) + \frac{1}{2}\sum_{t=0}^{t_1}u^T(t)Ku(t) \\ &\quad + \frac{1}{2}\sum_{t=0}^{t_1}[Cx(t)+Du(t)]^T\Delta[Cx(t)+Du(t)]\end{aligned} \tag{C.24}$$

and using (C.2), one gets (C.18). □

Definition C.12 (Landau and Silveira 1979) The linear time invariant system described by (C.1) and (C.2) is said to belong to the class $L(\Lambda)$ if and only if for a given positive definite matrix Λ of appropriate dimension:

$$H'(z) = C(zI - A)^{-1}B + D - \frac{1}{2}\Lambda = H(z) - \frac{1}{2}\Lambda \tag{C.25}$$

is a strictly positive real transfer matrix.

The interpretation of this definition is given in Fig. C.2.

The linear time invariant system (C.1)–(C.2) characterized by the discrete time transfer function $H(z)$ belongs to the class $L(\Lambda)$ if the parallel combination of this block with a block having the gain $-\frac{1}{2}\Lambda$ is a system characterized by a strictly positive real transfer matrix.

Lemma C.5 *If the system* (C.1)–(C.2) *belongs to the class* $L(\Lambda)$, *one has*:

$$\begin{aligned}\sum_{t=0}^{t_1} y^T(t)u(t) &= \frac{1}{2}x^T(t_1+1)Px(t_1+1) - \frac{1}{2}x^T(0)Px(0) \\ &\quad + \frac{1}{2}\sum_{t=0}^{t_1}[x^T(t)u^T(t)]M\begin{bmatrix} x(t) \\ u(t)\end{bmatrix} + \frac{1}{2}\sum_{t=0}^{t_1} x^T(t)M_0x(t) \\ &\quad + \frac{1}{2}\sum_{t=0}^{t_1} u^T(t)\Lambda u(t) + \frac{1}{2}\sum_{t=0}^{t_1} u^T(t)Ku(t) \\ \Lambda > 0;\ M_0 > 0;\ K > 0 \end{aligned} \tag{C.26}$$

In fact, as it results from (C.26) a system belonging to $L(\Lambda)$ is input strictly passive and the excess of passivity is defined by Λ (see Lemma C.4).

C.4 Discrete Linear Time-Varying Passive Systems

Consider the discrete linear time-varying system:

$$\bar{x}(t+1) = A(t)\bar{x}(t) + B(t)\bar{u}(t) \tag{C.27}$$

$$\bar{y}(t) = C(t)\bar{x}(t) + D(t)\bar{u}(t) \tag{C.28}$$

where $\bar{x}$, $\bar{u}$, $\bar{y}$ are the state, the input and the output vectors respectively ($\bar{u}$ and $\bar{y}$ are of the same dimension), and $A(t)$, $B(t)$, $C(t)$, $D(t)$ are sequences of time-varying matrices of appropriate dimensions defined for all $t \geq 0$.

Lemma C.6 *The system* (C.27)–(C.28) *is* passive *if one of the following equivalent propositions holds*:

1. *There are sequences of time-varying positive semidefinite matrices* $\bar{P}(t)$, *of positive semidefinite matrices* $Q(t)$ *and* $R(t)$ *and a matrix sequence* $S(t)$ *such that*:

$$A^T(t)\bar{P}(t+1)A(t) - \bar{P}(t) = -Q(t) \tag{C.29}$$

$$C(t) - B^T(t)\bar{P}(t+1)A(t) = S(t) \tag{C.30}$$

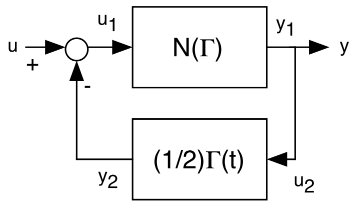
$$D(t) + D^T(t) - B^T(t)P(t+1)B(t) = R(t) \tag{C.31}$$

$$\bar{M}(t) = \begin{bmatrix} Q(t) & S(t) \\ S^T(t) & R(t)\end{bmatrix} \geq 0 \tag{C.32}$$

   with $\bar{P}(0)$ *bounded.*

2. *Every solution* $x(t)(x(0), u(t), t)$ *of* (C.27)–(C.28) *satisfies the following equality*:

Fig. C.3 The class $N(\Gamma)$



$$\sum_{t=0}^{t_1} \bar{y}^T(t)\bar{u}(t) = \frac{1}{2}x^T(t_1+1)\bar{P}(t+1)x(t_1+1) - \frac{1}{2}x^T(0)\bar{P}(0)x(0) + \frac{1}{2}\sum_{t=0}^{t_1}[\bar{x}^T(t), \bar{u}^T(t)]\bar{M}(t)\begin{bmatrix}\bar{x}(t)\\ \bar{u}(t)\end{bmatrix} \quad \text{(C.33)}$$

Proof The passivity property results from (C.33). To obtain (C.33) from (C.32), one follows exactly the same steps as for the proof of Lemma C.3. □

For the study of the stability of interconnected systems, it is useful to consider the class of linear time-varying discrete time systems defined next.

Definition C.13 (Landau and Silveira 1979) The linear time-varying system (C.27)–(C.28) is said to belong to the class $N(\Gamma)$ if for a given sequence of symmetric matrices $\Gamma(t) \geq 0$ of appropriate dimension, one has:

$$\sum_{t=0}^{t_1} \bar{y}^T(t)\bar{u}(t) = \frac{1}{2}\bar{x}^T(t_1+1)\bar{P}(t_1+1)\bar{x}(t_1+1) - \frac{1}{2}\bar{x}^T(0)\bar{P}(0)\bar{x}(0) + \frac{1}{2}\sum_{t=0}^{t_1}[\bar{x}^T(t), \bar{u}^T(t)]\bar{M}(t)\begin{bmatrix}\bar{x}(t)\\ \bar{u}(t)\end{bmatrix} - \frac{1}{2}\sum_{t=0}^{t_1}\bar{y}^T(t)\Gamma(t)\bar{y}(t) \quad \text{(C.34)}$$

with $\bar{P}(t) \geq 0$, $\bar{M}(t) \geq 0$, $\forall t \geq 0$ and $\bar{P}(0)$ bounded.

The interpretation of this definition is given in Fig. C.3.

Equation (C.34) indicates that the block resulting from the negative feedback connection of the linear time-varying system of (C.27) and (C.28) belonging to the class $N(\Gamma)$ with a block having a gain $\frac{1}{2}\Gamma(t)$ is passive. The resulting block has the input $\bar{u}^R(t) = \bar{u}(t) + \frac{1}{2}\Gamma(t)\bar{y}(t)$ and the output $\bar{y}^R(t) = y(t)$. In fact, the system belonging to the class $N(\Gamma)$ has a lack of passivity which is expressed by the last term of (C.34).

Algebraic conditions upon $A(t), B(t), C(t), D(t)$ in order to satisfy (C.34) are given next:

Lemma C.7 *The system of* (C.27) *and* (C.28) *belongs to the class* $N(\Gamma)$ *if for a given sequence of symmetric matrices* $\Gamma(t) \geq 0$ *there exist three sequences of non-negative definite matrices* $\bar{P}(t)$, $R(t)$, $Q(t)$ *and a matrix sequence* $S(t)$ *such that*:

$$A^T(t)\bar{P}(t+1)A(t) - \bar{P}(t) = -Q(t) + C^T(t)\Gamma(t)C(t) \quad \text{(C.35)}$$

$$C^T(t) - B^T(t)\bar{P}(t+1)A(t) = S^T(t) - D^T(t)\Gamma(t)C(t) \quad \text{(C.36)}$$

$$D(t) + D^T(t) - B^T(t)\bar{P}(t+1)B(t) = R(t) - D^T(t)\Gamma(t)D(t) \quad \text{(C.37)}$$

and:

$$\bar{M}(t) = \begin{bmatrix} Q(t) & S(t) \\ S^T(t) & R(t) \end{bmatrix} \geq 0 \quad \text{(C.38)}$$

with $\bar{P}(0)$ *bounded.*

Proof A detailed proof can be found in Landau and Silveira (1979). The proof is similar to the one for Lemma C.4 by replacing constant matrices Q, S, R, P by the corresponding time-varying matrices $Q(t)$, $R(t)$, $S(t)$, $\bar{P}(t)$, replacing Δ by $-\Gamma(t)$ and taking $K = 0$, $M_0 = 0$. □

C.5 Stability of Feedback Interconnected Systems

Theorem C.1 (Landau 1980) *Consider a linear time invariant block described by* (C.1) *and* (C.2) *belonging to the class* $L(\Lambda)$ *in feedback connection with a discrete linear time-varying block described by* (C.27) *and* (C.28) *belonging to the class* $N(\Gamma)$. *If*:

$$\Lambda - \Gamma(t) \geq 0 \quad \text{(C.39)}$$

the following properties hold:

(P1)

$$\lim_{t\to\infty} x(t) = 0; \qquad \lim_{t\to\infty} y(t) = 0; \qquad \lim_{t\to\infty} u(t) = -\lim_{t\to\infty} \bar{y}(t) = 0 \quad \text{(C.40)}$$

with $\bar{P}(t)$, $Q(t)$, $S(t)$, $R(t)$ *bounded or unbounded for* $t > 0$.

(P2)

$$\lim_{t\to\infty} [\bar{x}(t)^T, \bar{u}(t)^T] \begin{bmatrix} Q(t) & S(t) \\ S(t)^T & R(t) \end{bmatrix} \begin{bmatrix} \bar{x}(t) \\ \bar{u}(t) \end{bmatrix} = 0 \quad \text{(C.41)}$$

with $Q(t)$, $S(t)$, $R(t)$ *bounded or unbounded for* $t > 0$.

(P3)

$$\bar{x}(t)^T P(t)\bar{x}(t) \leq C_1 < \infty; \quad \forall t > 0 \quad \text{(C.42)}$$

with $P(t)$ *bounded or unbounded for* $t > 0$.

(P4)

$$\lim_{t\to\infty} \bar{x}(t)^T P(t)\bar{x}(t) = \text{const} \tag{C.43}$$

with $P(t)$ bounded or unbounded for $t > 0$.

Proof The feedback connection corresponds to:

$$u(t) = -\bar{y}(t) \tag{C.44}$$

$$y(t) = \bar{u}(t) \tag{C.45}$$

which implies:

$$\sum_{t=0}^{t_1} y(t)^T u(t) = -\sum_{t=0}^{t_1} \bar{y}(t)^T \bar{u}(t) \tag{C.46}$$

1. Evaluating the left hand side term of (C.46) from (C.26), and the right hand side term of (C.46) from (C.34), one obtains after rearranging the various terms (and taking into account (C.44) and (C.45)):

$$\begin{aligned} x^T(t_1+1)Px(t_1+1) &+ \sum_{t=0}^{t_1} x^T(t)M_0x(t) + \sum_{t=0}^{t_1} u^T(t)Ku(t) \\ &+ \bar{x}^T(t_1+1)\bar{P}(t_1+1)\bar{x}(t_1+1) + \sum_{t=0}^{t_1}[\bar{x}^T(t) y^T(t)]\bar{M}(t)\begin{bmatrix} c\bar{x}(t) \\ y(t) \end{bmatrix} \\ &+ \sum_{t=0}^{t_1} \bar{y}^T(t)[\Lambda - \Gamma(t)]\bar{y}(t) + \sum_{t=0}^{t_1}[x^T(t), -\bar{y}^T(t)]M\begin{bmatrix} x(t) \\ -\bar{y}(t) \end{bmatrix} \\ &= \bar{x}^T(0)\bar{P}(0)\bar{x}(0) + x^T(0)Px(0) \end{aligned} \tag{C.47}$$

   Since $\Lambda - \Gamma(t) \geq 0$, one obtains:

$$\begin{aligned} &\lim_{t_1\to\infty}\left[x^T(t_1+1)Px(t_1+1) + \sum_{t=0}^{t_1} x^T(t)M_0x(t) + \sum_{t=0}^{t_1} u^T(t)Ku(t)\right] \\ &\leq x(0)Px(0) + \bar{x}^T(0)\bar{P}(0)\bar{x}(0) \end{aligned} \tag{C.48}$$

   with $\bar{P}(t)$, $Q(t)$, $S(t)$, $R(t)$ bounded or unbounded for $t > 0$. Since $P > 0$, $M_0 > 0$, $K > 0$, this implies that:

$$\lim_{t\to\infty} x(t) = 0; \qquad \lim_{t\to\infty} u(t) = 0; \quad \forall x(0),\ \forall \bar{x}(0) < \infty \tag{C.49}$$

   and from (C.2), it also results that:

$$\lim_{t\to\infty} y(t) = 0 \tag{C.50}$$

   Bearing in mind that $u(t) = -\bar{y}(t)$, one has:

$$\lim_{t\to\infty} \bar{y}(t) = 0 \tag{C.51}$$

2. From (C.47), taking into account (C.45), one obtains:

$$\lim_{t\to\infty}\left\{\bar{x}^T(t_1+1)\bar{P}(t_1+1)\bar{x}(t_1+1)\right.$$
$$\left.+\sum_{t\to\infty}^{t_1}[\bar{x}^T(t)\bar{u}^T(t)]\begin{bmatrix} Q(t) & S(t) \\ S^T(t) & R(t)\end{bmatrix}\begin{bmatrix}\bar{x}(t)\\ \bar{u}(t)\end{bmatrix}\right\}$$
$$\leq x^T(0)Px(0)+\bar{x}^T(0)\bar{P}(0)\bar{x}(0) \tag{C.52}$$

with $\bar{P}(t)$, $S(t)$, $Q(t)$, $R(t)$ bounded or unbounded for $t>0$. This implies that:

$$\lim_{t\to\infty}[\bar{x}^T(t),\bar{u}^T(t)]\begin{bmatrix} Q(t) & S(t) \\ S^T(t) & R(t)\end{bmatrix}\begin{bmatrix}\bar{x}(t)\\ \bar{u}(t)\end{bmatrix}=0 \tag{C.53}$$

3. From (C.52) one also has:

$$\bar{x}^T(t)\bar{P}(t)\bar{x}(t)\leq C_1<\infty;\quad \forall t>0 \tag{C.54}$$

4. Denote

$$V(t)=x^T(t)Px(t)+\bar{x}^T(t)\bar{P}(t)\bar{x}(t) \tag{C.55}$$

Again from (C.47), one obtains:

$$V(t+1)-V(t)=-\bar{y}^T(t)[\Lambda-\Gamma(t)]\bar{y}(t)-x^T(t)M_0(t)x(t)$$
$$-u^T(t)Ku(t)-[x^T(t),u^T(t)]M\begin{bmatrix}x(t)\\ u(t)\end{bmatrix}$$
$$-[\bar{x}^T(t),y^T(t)]\bar{M}(t)\begin{bmatrix}\bar{x}(t)\\ y(t)\end{bmatrix}\leq 0 \tag{C.56}$$

Since $V(t)>0;\ \forall x(t)\neq 0;\ \forall\bar{x}(t)\neq 0$ and $V(t+1)-V(t)\leq 0$, $V(t)$ will converge to a fixed value. One has proven already that $x(t)\to 0$. One concludes that:

$$\lim_{t\to\infty}\bar{x}^T(t)\bar{P}(t)\bar{x}(t)=\text{const} \tag{C.57}$$

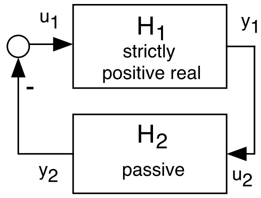
□

Theorem C.1 can be viewed as an extension of the asymptotic hyperstability theorem (see next paragraph) which corresponds to $\Lambda=0;\ \Gamma(t)\equiv 0$. In fact, in the asymptotic hyperstability theorem, there is no specific structure attached to the feedback block (which can be linear, nonlinear, time-varying). The only condition is that the feedback block is *passive*. By considering a state space structure for the feedback block, it is possible to get certain properties for the state of the feedback block. This is needed in adaptive control.

C.6 Hyperstability and Small Gain

Theorem C.2 (Asymptotic Hyperstability) *Consider the feedback connection (Fig. C.4) of a linear time-invariant block H_1 (state: $x(t)$), characterized by a*

Fig. C.4 Feedback connection (hyperstability)



strictly positive real transfer function (*which implies that* H_1 *is input strictly passive*) *with a block* H_2 (*linear or nonlinear, time invariant or time-varying*) *characterized by*:

$$\eta_2(0, t_1) = \sum_{t=0}^{t_1} y_2^T(t) u_2(t) \geq -\gamma_2^2; \quad \gamma_2^2 < \infty; \ \forall t_1 \geq 0 \tag{C.58}$$

Then:

$$\lim_{t\to\infty} x(t) = 0; \qquad \lim_{t\to\infty} u_1(t) = \lim_{t\to\infty} y_1(t) = 0; \quad \forall x(0) \tag{C.59}$$

The proof is similar to that of Theorem C.1 and it is omitted.

Definition C.14 Consider a system S with input u and output y, the infinity norm of the system S denoted $\|S\|_\infty$ is such that:

$$\|y\|_2^2 \leq \|S\|_\infty^2 \|u\|_2^2$$

Definition C.15 Given a transfer function $H(z)$, the infinity norm $\|H\|_\infty$ is:

$$\|H\|_\infty = \max_\omega |H(e^{j\omega})|; \quad 0 \leq \omega \leq 2\pi$$

Lemma C.8 (Small Gain Lemma) *The following propositions concerning the system of* (C.1)–(C.2) *are equivalent to each other*:

1. $H(z)$ *given by* (C.3) *satisfies*:

$$\|H(z^{-1})\|_\infty \leq \gamma; \quad 0 < \gamma < \infty \tag{C.60}$$

2. *There exist a positive definite matrix* P, *a positive semidefinite matrix* Q *and matrices* S *and* R *such that*:

$$A^T P A - P = -Q - C^T C \tag{C.61}$$

$$-B^T P A = S^T + D^T C \tag{C.62}$$

$$B^T P B = R + D^T D - \gamma^2 I \tag{C.63}$$

$$M = \begin{bmatrix} Q & S \\ S^T & R \end{bmatrix} \geq 0 \tag{C.64}$$

3. *There is a positive definite matrix* P *and matrices* K *and* L *such that*:

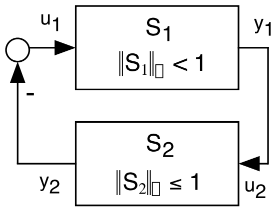


Fig. C.5 Feedback connection (small gain)

$$A^T PA - P + C^T C = -LL^T \tag{C.65}$$

$$-B^T PA - D^T C = K^T L^T \tag{C.66}$$

$$B^T PB - D^T D + \gamma^2 I = K^T K \tag{C.67}$$

4. *Every solution $x(t)(x(0), u(t))$ of* (C.1)–(C.2) *satisfies the following equality*:

$$\sum_{t=0}^{t_1} y^T(t)y(t) = \gamma^2 \sum_{t=0}^{t_1} u^T(t)u(t) + x^T(0)Px(0) - x^T(t_1+1)Px(t_1+1) - \sum_{t=0}^{t_1} [x^T(t), u^T(t)]M \begin{bmatrix} x(t) \\ u(t) \end{bmatrix} \tag{C.68}$$

Remark From (C.68) one gets for $x(0) = 0$:

$$\|y(t)\|_2^2 \le \gamma^2 \|u\|_2^2 \le \|H\|_\infty^2 \|u\|^2 \tag{C.69}$$

Proof We will only show that (C.61) through (C.64) implies (C.68). From (C.61), one gets:

$$\begin{aligned} x^T(t)Qx(t) &= -x^T(t)C^T Cx(t) + x^T(t)Px(t) - x^T(t)A^T PAx(t) \\ &= -x^T(t)C^T Cx(t) + x^T(t)Px(t) - x^T(t+1)Px(t+1) \\ &\quad + 2u^T(t)B^T PAx(t) - u^T(t)B^T PBu(t) \end{aligned} \tag{C.70}$$

Using (C.62) and (C.63) and adding the terms $2u^T(t)S^T x(t) + u^T(t)Ru(t)$ in the both sides of (C.58), one obtains:

$$[x^T(t), u^T(t)]M \begin{bmatrix} x(t) \\ u(t) \end{bmatrix} = -y^T(t)y(t) + \gamma^2 u^T(t)u(t) + x^T(t)Px(t) - x^T(t+1)Px(t+1) \tag{C.71}$$

Summing up now from 0 to t_1, one gets (C.65). □

Theorem C.3 (Small Gain) *Consider the feedback connection* (*Fig*. C.5) *between a linear time invariant block S_1* (*state*: x) *characterized by $\|S_1\|_\infty < 1$ and a block S_2 characterized by $\|S_2\|_\infty \le 1$. Then*:

$$\lim_{t\to\infty} x(t) = 0; \qquad \lim_{t\to\infty} u_1(t) = \lim_{t\to\infty} y_1(t) = 0 \tag{C.72}$$

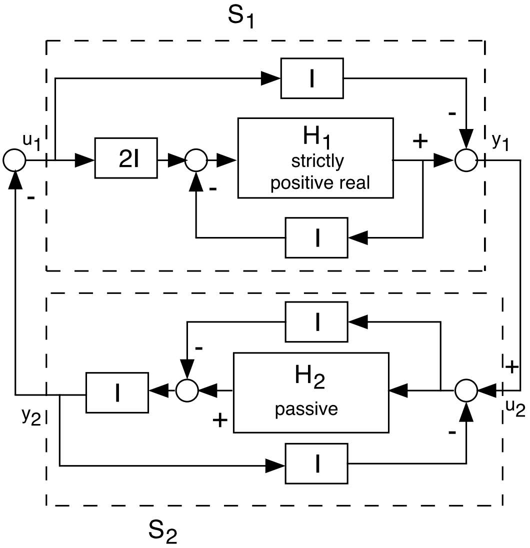


Fig. C.6 Equivalent representation of the system shown in Fig. C.4

Proof The small gain theorem (Theorem C.3) can be obtained from Theorem C.2, using a series of equivalent loop transformation on the scheme of Fig. C.4 leading to the scheme shown in Fig. C.6, where the input-output operator S_1 ($y_1 = S_1 u_1$) is given by:

$$S_1 = (H_1 - I)(H_1 + I)^{-1} \tag{C.73}$$

and the operator $S_2(y_2 = S_2 u_2)$ is given by:

$$S_2 = (H_2 - I)(H_2 + I)^{-1} \tag{C.74}$$

It can be shown (Desoer and Vidyasagar 1975) that under the hypotheses of Theorem C.2, $\|S_1\|_\infty < 1$ and $\|S_2\|_\infty \leq 1$. □

Appendix D
Martingales

This appendix gives a number of results related to the properties of feedback systems in the presence of stochastic disturbances represented as a martingale sequence (for the definition of a martingale sequence see Appendix A). The properties of passive (hyperstable) systems are extensively used in relation to some basic results concerning the convergence of non-negative random variables. The results of this appendix are useful for the proofs of Theorems 4.2 and 4.3, as well as for the convergence analysis of various recursive identification and adaptive control schemes.

Theorem D.1 (Neveu 1975) *If $T(t)$ and $\alpha(t+1)$ are non-negative random variables measurable with respect to an increasing sequence of σ-algebras $\mathcal{F}_t$ and satisfy*:

$$(1) \quad \mathbf{E}\{T(t+1)|\mathcal{F}_t\} \leq T(t) + \alpha(t+1) \tag{D.1}$$

$$(2) \quad \sum_{t=1}^{\infty} \alpha(t) < \infty; \quad a.s. \tag{D.2}$$

then:

$$\lim_{t\to\infty} T(t) = T \quad a.s. \tag{D.3}$$

where T is a finite non-negative random variable.

Consider the stochastic feedback system shown in Fig. D.1 and described by:

$$x(t+1) = Ax(t) + Bu(t+1) \tag{D.4}$$

$$y(t+1) = Cx(t) + Du(t+1) \tag{D.5}$$

$$\bar{u}(t+1) = y(t+1) + \omega(t+1) \tag{D.6}$$

$$\bar{x}(t+1) = A(t)\bar{x}(t) + B(t)\bar{u}(t+1) \tag{D.7}$$

$$\bar{y}(t+1) = -u(t+1) = C(t)\bar{x}(t) + D(t)\bar{u}(t+1) \tag{D.8}$$

Note that the output of the linear time-invariant feedforward block described by (D.4) and (D.5) is disturbed by the sequence $\{\omega(t+1)\}$. The following assumptions are made upon the system of (D.4)–(D.8).

I.D. Landau et al., *Adaptive Control*, Communications and Control Engineering, DOI 10.1007/978-0-85729-664-1, © Springer-Verlag London Limited 2011

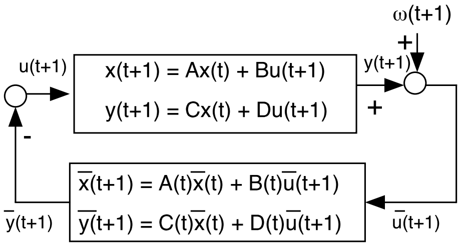


Fig. D.1 Stochastic feedback system associated with (D.4) through (D.8)

D.1 There exists a symmetric matrix Λ such that the linear time-invariant block described by (D.4) and (D.5) belongs to the class $L(\Lambda)$ (see Definition C.3).

D.2 There exists a sequence of matrices $\Gamma(t)$ such that the feedback linear time-varying block described by (D.7) and (D.8) belongs to the class $N(\Gamma)$ (see Definition C.8).

D.3 $\{\omega(t)\}$ is a vectorial martingale difference sequence defined on a probability space $(\Omega, \mathcal{A}, \mathcal{P})$ adapted to the sequence of increasing σ-algebras $\mathcal{F}_t$ generated by the observations up to and including time t. The sequence $\{\omega(t+1)\}$ satisfies:

$$\mathbf{E}\{\omega(t+1)|\mathcal{F}_t\} = 0 \quad \text{a.s.} \tag{D.9}$$

$$\mathbf{E}\{\omega(t+1)\omega^T(t+1)|\mathcal{F}_t\} = \text{diag}[\sigma_i^2] \quad \text{a.s.} \tag{D.10}$$

$$\lim_{N\to\infty} \sup \frac{1}{N}\sum_{t=1}^{N} \omega^T(t+1)\omega(t+1) < \infty \quad \text{a.s.} \tag{D.11}$$

Theorem D.2 (Landau 1982b) *Let Assumptions* D.1, D.2 *and* D.3 *hold for the system of* (D.4)–(D.8). *If*:

$$(1) \quad \alpha(t+1) = \frac{1}{t+1}\mathbf{E}\{\bar{y}^T(t+1)\omega(t+1)|\mathcal{F}_t\} \geq 0; \quad \forall t \geq 0 \tag{D.12}$$

$$(2) \quad \sum_{t=1}^{\infty} \alpha(t) < \infty \quad a.s. \tag{D.13}$$

$$(3) \quad \Lambda - \Gamma(t) \geq 0; \quad \forall t \geq 0 \tag{D.14}$$

then:

$$\text{(a)} \quad \lim_{t\to\infty} \frac{1}{t}\sum_{i=1}^{t} \|x(i)\|^2 = \lim_{t\to\infty} \frac{1}{t}\sum_{i=1}^{t} \|u(i)\|^2 = \lim_{t\to\infty} \frac{1}{t}\sum_{i=1}^{t} \|y(i)\|^2 = 0 \quad a.s. \tag{D.15}$$

$$\text{(b)} \quad \lim_{t\to\infty} \bar{x}^T(t)\left(\frac{\bar{P}(t)}{t}\right)\bar{x}(t) = 0 \quad a.s. \tag{D.16}$$

$$\text{(c)} \quad \textit{If, in addition,} \ \lim_{t\to\infty} \frac{1}{t}\bar{P}(t) > 0 \ a.s., \ \textit{then}: \ \lim_{t\to\infty} \bar{x}(t) = 0 \ a.s. \tag{D.17}$$

Proof The proof is based on the use of the near supermartingale convergence theorem (Theorem D.1). The appropriate near supermartingale is:

$$T(t) = \frac{V(t)}{t} + \sum_{i=1}^{t-1} \frac{1}{i+1} \frac{V(i)}{i} \tag{D.18}$$

where:

$$\begin{aligned} V(t) &= x^T(t)Px(t) + \bar{x}^T(t)\bar{P}(t)\bar{x}(t) + \sum_{i=0}^{t-1} x^T(i)M_0x(i) \\ &\quad + \sum_{i=0}^{t-1} u^T(i+1)Ku(i+1) \end{aligned} \tag{D.19}$$

Taking into account (D.6) and (D.8), one has:

$$\begin{aligned} \sum_{i=0}^{t} y^T(i+1)u(i+1) = &-\sum_{i=0}^{t} \bar{y}^T(i+1)\bar{u}(i+1) \\ &+ \sum_{i=0}^{t} \bar{y}^T(i+1)\omega(i+1) \end{aligned} \tag{D.20}$$

The expressions for $\sum_{i=0}^{t} y^T(i+1)u(i+1)$ and $\sum_{i=0}^{t} \bar{y}^T(i+1)\bar{u}(i+1)$ can be obtained from Lemmas C.5 and C.7, since the two blocks involved belong to the classes $L(\Lambda)$ and $N(\Gamma)$ respectively. Combining (C.26) and (C.34) through the use of (D.20) and rearranging the various terms taking into account (D.19), one obtains:

$$\begin{aligned} V(t+1) &= x^T(t+1)Px(t+1) + \bar{x}^T(t+1)\bar{P}(t+1)\bar{x}(t+1) \\ &\quad + \sum_{i=0}^{t} x^T(i)M_0x(i) + \sum_{i=0}^{t} u^T(i+1)Ku(i+1) \\ &= x^T(0)Px(0) + \bar{x}^T(0)\bar{P}(0)\bar{x}(0) - \sum_{i=0}^{t}[x^T(i), u^T(i+1)]M \begin{bmatrix} x(i) \\ u(i+1) \end{bmatrix} \\ &\quad - \sum_{i=0}^{t}[\bar{x}^T(i), \bar{u}^T(i+1)]\bar{M}(t) \begin{bmatrix} \bar{x}(i) \\ \bar{u}(i+1) \end{bmatrix} \\ &\quad - \sum_{i=0}^{t} \bar{y}^T(i+1)[\Lambda - \Gamma(i)]\bar{y}(i+1) \\ &\quad + 2\sum_{i=0}^{t} \bar{y}^T(i+1)\omega(i+1) \end{aligned} \tag{D.21}$$

Since $\Lambda - \Gamma(t) \geq 0$ (from (D.14)) and $\bar{M}(t) \geq 0$ (from (C.34)), one concludes from (D.21) that:

$$V(t+1) \leq V(t) + 2\bar{y}(t+1)\omega(t+1) \tag{D.22}$$

and:

$$\frac{V(t+1)}{t+1} \leq \frac{V(t)}{t} - \frac{1}{t+1}\frac{V(t)}{t} + \frac{2}{t+1}\bar{y}(t+1)\omega(t+1) \tag{D.23}$$

Adding in both sides of (D.23) the term $\sum_{i=1}^{t}(\frac{1}{i+1})\frac{V(i)}{i}$, one concludes that $T(t)$ given by (D.18) satisfies:

$$T(t+1) \leq T(t) + \frac{2}{t+1}\bar{y}^T(t+1)\omega(t+1) \tag{D.24}$$

and:

$$\mathbf{E}\{T(t+1)|\mathcal{F}_t\} \leq T(t) + \frac{2}{t+1}\mathbf{E}\{\bar{y}^T(t+1)\omega(t+1)\} \tag{D.25}$$

Conditions (D.12) and (D.13) together with (D.25) lead to the satisfaction of Theorem D.1 and therefore:

$$\lim_{t\to\infty}\left[\frac{V(t)}{t} + \sum_{i=1}^{t-1}\frac{1}{i+1}\frac{V(i)}{i}\right] < \infty \quad \text{a.s.} \tag{D.26}$$

But since $V(t) \geq 0$, it results in order to avoid contradiction that:

$$\text{Prob}\left\{\lim_{t\to\infty}\frac{V(t)}{t} = 0\right\} = 1 \tag{D.27}$$

Bearing in mind that the matrix M_0 in (C.26) is positive definite and $K > 0$, it results from (D.27) that:

$$\lim_{t\to\infty}\frac{1}{t}\sum_{i=1}^{t}x^T(i)M_0x(i) = \lim_{t\to\infty}\frac{1}{t}\sum_{i=1}^{t}\|x(i)\|^2 = 0 \quad \text{a.s.} \tag{D.28}$$

and:

$$\lim_{t\to\infty}\frac{1}{t}\sum_{i=1}^{t}\|u(i+1)\|^2 = 0 \quad \text{a.s.} \tag{D.29}$$

From (D.5), one has:

$$\|y(i+1)\|^2 \leq \alpha\|x(i)\|^2 + \beta\|u(i+1)\|^2; \quad 0 < \alpha,\ \beta < \infty \tag{D.30}$$

and therefore:

$$\lim_{t\to\infty}\frac{1}{t}\sum_{i=1}^{t}\|y(i+1)\|^2 = 0 \quad \text{a.s.} \tag{D.31}$$

The results (D.16) and (D.17) are directly obtained from (D.27) and (D.19). □

Theorem D.3 *Let Assumptions* D.1, D.2 *and* D.3 *hold for the system* (D.4)–(D.8). *If*:

$$(1) \quad r(t+1) \geq r(t), \quad r(t) > 0, \; \forall t \geq 0 \tag{D.32}$$

$$(2) \quad \alpha(t+1) = E\left\{ \frac{\bar{y}^T(t+1)\omega(t+1)}{r(t+1)} | \mathcal{F}_t \right\} \geq 0, \quad \forall t \geq 0 \tag{D.33}$$

$$(3) \quad \sum_{t=1}^{\infty} \alpha(t) < \infty \quad a.s. \tag{D.34}$$

$$(4) \quad \Lambda - \Gamma(t) \geq 0; \quad \forall t \geq 0 \tag{D.35}$$

then:

$$\text{(a)} \quad \lim_{t\to\infty} \sum_{i=0}^{t} \frac{\|x(i)\|^2}{r(i+1)} < \infty \quad a.s. \tag{D.36}$$

$$\text{(b)} \quad \lim_{t\to\infty} \sum_{i=0}^{t} \frac{\|u(i+1)\|^2}{r(i+1)} < \infty \quad a.s. \tag{D.37}$$

$$\text{(c)} \quad \lim_{t\to\infty} \sum_{i=0}^{t} \frac{\|y(i+1)\|^2}{r(i+1)} < \infty \quad a.s. \tag{D.38}$$

$$\text{(d)} \quad \lim_{t\to\infty} \sum_{i=0}^{t} \frac{[\bar{x}^T(i), \bar{u}^T(i+1)]\bar{M}(i)\left[\begin{smallmatrix} \bar{x}(i) \\ \bar{u}(i+1) \end{smallmatrix}\right]}{r(i+1)} < \infty \quad a.s. \tag{D.39}$$

Remark This theorem is useful when one cannot conclude that, for a given scheme, that (D.13) is satisfied. One can then choose eventually a sequence $r(t)$ to satisfy the conditions of (D.35) and (D.34). The results of Theorem D.3 are weaker than those of Theorem D.2, but from the specific properties of the sequence $r(t)$ is then possible in certain cases to conclude upon the convergence of certain schemes (Goodwin and Sin 1984; Goodwin et al. 1980c). Note that if in addition of (D.34) one assumes $\lim_{t\to\infty} r(t) = \infty$, using Kronecker's lemma one obtains from (D.36), (D.37), (D.38) and (D.40) results of the form of Theorem D.2.

Proof One considers the following near supermartingale:

$$\begin{aligned} T(t) = {} & \frac{1}{r(t)}[x^T(t)Px(t) + \bar{x}^T(t)\bar{P}(t)\bar{x}(t)] \\ & + \sum_{i=0}^{t-1} \frac{[x^T(i), u^T(i+1)]M\left[\begin{smallmatrix} x(i) \\ u(i+1) \end{smallmatrix}\right]}{r(i+1)} + \sum_{i=0}^{t-1} \frac{x^T(i)M_0 x(i)}{r(i+1)} \\ & + \sum_{i=0}^{t-1} \frac{[\bar{x}(i), \bar{u}(i+1)]\bar{M}(t)\left[\begin{smallmatrix} \bar{x}(i) \\ \bar{u}(i+1) \end{smallmatrix}\right]}{r(i+1)} \\ & + \sum_{i=0}^{t-1} \frac{u^T(i+1)Ku(i+1)}{r(i+1)} \end{aligned} \tag{D.40}$$

where P, $\bar{P}(t)$, M, M_0, $\bar{M}(t)$, K result from Lemmas C.5 and C.7 respectively. Denote:

$$V(t) = x^T(t)Px(t) + \bar{x}^T(t)\bar{P}(t)\bar{x}(t) \tag{D.41}$$

Using Lemmas C.5 and C.7 and taking also into account (D.6) and (D.8), one has:

$$\begin{aligned} V(t+1) &= V(t) - [x^T(t), u^T(t+1)]M\begin{bmatrix} x(t) \\ u(t+1) \end{bmatrix} - x^T(t)M_0x(t) \\ &\quad - [\bar{x}^T(t), \bar{u}^T(t+1)]\bar{M}(t)\begin{bmatrix} \bar{x}(t) \\ \bar{u}(t+1) \end{bmatrix} - \bar{y}(t+1)[\Lambda - \Gamma(t)]\bar{y}(t+1) \\ &\quad - u^T(t+1)Ku(t+1) + 2\bar{y}^T(t+1)\omega(t+1) \end{aligned} \tag{D.42}$$

From (D.42), taking into account (D.32), one gets:

$$\begin{aligned} \frac{V(t+1)}{r(t+1)} &\le \frac{V(t)}{r(t)} - \frac{1}{r(t+1)}\Bigg\{[x^T(t), u^T(t+1)]M\begin{bmatrix} x(t) \\ u(t+1) \end{bmatrix} + x^T(t)M_0x(t) \\ &\quad + [\bar{x}^T(t), \bar{u}^T(t+1)]\bar{M}(t)\begin{bmatrix} \bar{x}(t) \\ \bar{u}(t+1) \end{bmatrix} + \kappa\|u(t+1)\|^2\Bigg\} \\ &\quad + 2\frac{\bar{y}(t+1)^T\omega(t+1)}{r(t+1)} \end{aligned} \tag{D.43}$$

Adding in both sides of (D.43):

$$\begin{aligned} &\sum_{i=0}^{t}\frac{[x^T(i), u^T(i+1)]M\left[\begin{smallmatrix} x(i) \\ u(i+1) \end{smallmatrix}\right]}{r(i+1)} + \sum_{i=0}^{t}\frac{x^T(t)M_0x(t)}{r(i+1)} \\ &\quad + \sum_{i=0}^{t}\frac{[\bar{x}^T(i), \bar{u}^T(i+1)]\bar{M}^T(t)\left[\begin{smallmatrix} \bar{x}(i) \\ \bar{u}(i+1) \end{smallmatrix}\right]}{r(i+1)} + \kappa\sum_{i=0}^{t}\frac{\|u(i+1)\|^2}{r(i+1)} \end{aligned} \tag{D.44}$$

and taking the conditional expectation one gets using (D.40):

$$\mathbf{E}\{T(t+1)|\mathcal{F}_t\} \le T(t) + 2\mathbf{E}\left\{\frac{\bar{y}^T(t+1)\omega(t+1)}{r(t+1)}|\mathcal{F}_t\right\} \tag{D.45}$$

Under the hypotheses (D.33) and (D.34), it results using Theorem D.1:

$$\text{Prob}\Big\{\lim_{t\to\infty} T(t) < \infty\Big\} = 1 \tag{D.46}$$

Taking into account the structure of $T(t)$ given in (D.40), it results immediately:

$$\lim_{t\to\infty}\sum_{i=0}^{t}\frac{x^T(i)M_0x(i)}{r(i+1)} < \infty; \qquad M_0 > 0 \text{ a.s.} \tag{D.47}$$

and:

$$\lim_{t\to\infty}\kappa\sum_{i=0}^{t}\frac{\|u(i+1)\|^2}{r(i+1)} < \infty; \qquad \kappa > 0 \text{ a.s.} \tag{D.48}$$

which implies (D.36) and (D.37). From (D.5), one has:

$$\|y(i+1)\|^2 \leq \alpha\|x(i)\|^2 + \beta\|u(i+1)\|^2; \quad \alpha, \beta > 0 \tag{D.49}$$

and therefore (D.38) holds. (D.39) results also from the structure of $T(t)$. □

Lemma D.1 (Kronecker Lemma): (Goodwin and Sin 1984) *Assume that* $\{x(t)\}$ *and* $\{b_t\}$ *are sequences of reals such that*:

(i) $\lim_{N\to\infty} \sum_{t=1}^{N} x(t) < \infty$ (ii) $\{b_N\} : b_N \geq b_{N-1}$ (iii) $\lim_{N\to\infty} b_N = \infty$

Then:

$$\lim_{N\to\infty} \frac{1}{b_N} \sum_{t=1}^{N} b_t x(t) = 0 \tag{D.50}$$

References

Adaptech (1988) WimPim + (includes, WinPIM, WinREG and WinTRAC) system identification and control software. User's manual. 4, rue du Tour de l'Eau, 38400 St. Martin-d'Hères, France
Amara FB, Kabamba PT, Ulsoy AG (1999a) Adaptive sinusoidal disturbance rejection in linear discrete-time systems—Part I: Theory. J Dyn Syst Meas Control 121:648–654
Amara FB, Kabamba PT, Ulsoy AG (1999b) Adaptive sinusoidal disturbance rejection in linear discrete-time systems—Part II: Experiments. J Dyn Syst Meas Control 121:655–659
Anderson BDO (1967) A system theory criterion real matrices. SIAM J Control 5(2):171–182, 196
Anderson BDO (1998) From Youla-Kucera to identification, adaptive and nonlinear control. Automatica 34:1485–1506
Anderson BDO, Johnson CR (1982) Exponential convergence of adaptive identification and control algorithms. Automatica 18(1):1–13
Anderson BDO, Johnstone RM (1985) Global adaptive pole positioning. IEEE Trans Autom Control AC-30(4):11–12
Anderson BDO, Landau ID (1994) Least squares identification and the robust strict positive real property. IEEE Trans Circuits Syst 41(9)
Anderson BDO, Moore J (1971) Linear optimal control. Prentice Hall, Englewood Cliffs
Anderson BDO, Vongpanitlerd S (1972) Network analysis and synthesis. A modern systems approach. Prentice Hall, Englewood Cliffs
Anderson BDO, Bitmead RR, Johnson CR, Kokotovic PV, Kosut R, Mareels I, Praly L, Riedle BD (1986) Stability of adaptive systems—passivity and averaging analysis. MIT Press, Cambridge
Anderson BDO, Brinsmead TS, Bruyne FD, Hespanha J, Liberzon D, Morse AS (2000) Multiple model adaptive control. Part 1: Finite controller coverings. Int J Robust Nonlinear Control 10(11–12):909–929
Arimoto S, Miyazaki F (1984) Stability and robustness of PID feedback control of robot manipulators sensor capability. In: Brady M, Paul R (eds) Robotics research. MIT Press, Cambridge, pp 783–799
Armstrong B, Amin B (1996) PID control in the presence of static friction: a comparison of algebraic and describing function analysis. Automatica 32(5):679–692
Åström KJ (1970) Introduction to stochastic control theory. Academic Press, New York
Åström KJ (1980) Direct methods for non minimum phase systems. In: Proc IEEE—CDC 80, pp 611–615
Åström KJ (1993) Matching criteria for control and identification. In: Proc European control conference, Groningen, The Netherlands
Åström KJ, Wittenmark B (1973) On self-tuning regulators. Automatica 9:185–199
Åström KJ, Wittenmark B (1984) Computer controlled systems, theory and design. Prentice-Hall, Englewood Cliffs
Åström KJ, Wittenmark B (1995) Adaptive control, 2nd edn. Addison Wesley, Boston

I.D. Landau et al., *Adaptive Control*, Communications and Control Engineering,
DOI 10.1007/978-0-85729-664-1,

Åström KJ, Hagander P, Sternby J (1984) Zeros of sampled systems. Automatica 20:31–38
Athans M, Chang CB (1976) Adaptive estimation and parameter identification using multiple model estimation algorithms. Technical report 28, MIT Lincoln Laboratory
Athans M, Castanon D, Dunn K, Greene C, Lee W, Sandell N Jr, Willsky A (2002) The stochastic control of the F-8C aircraft using a multiple model adaptive control (MMAC) method—Part I: Equilibrium flight. IEEE Trans Autom Control 22(5):768–780
Banon G, Aguilar-Martin J (1972) Estimation linéaire récurrente des paramètres des processus dynamiques soumis à des perturbations aléatoires. Rev CETHEDEC 9:38–86
Barmish R, Ortega R (1991) On the radius of stabilizability of LTI systems: application to projection implementation in indirect adaptive control. Int J Adapt Control Signal Process 5:251–258
Bengtsson G (1977) Output regulation and internal models—a frequency domain approach. Automatica 13:333–345
Besançon A (1997) Estimation of relay-type non-linearity on relay systems analysis. In: Proc 97 IFAC symp "System, structure and control", Bucarest, Romania
Bethoux G (1976) Approche unitaire des méthodes d'identification et de commande adaptative des procédés dynamiques. Thèse 3ème cycle, Institut National Polytechnique de Grenoble
Bethoux G, Courtiol B (1973) A hyperstable discrete model reference adaptive control systems. In: Proc 3rd IFAC symp on "Sensitivity, adaptivity and optimality". ISA, Ischia, Italy, pp 282–289
Bierman GJ (1977) Factorization methods for discrete sequential estimation. Academic Press, New York
Bitmead RR (1993) Iterative control design approaches. In: Prepr 12th IFAC World congress, vol 9, Sydney, Australia, pp 381–384
Bitmead RR, Gevers M, Wertz V (1990) Adaptive optimal control. Prentice-Hall, New York
Bodson M (2005) Rejection of periodic disturbances of unknown and time-varying frequency. Int J Adapt Control Signal Process 19:67–88
Bodson M, Douglas SC (1997) Adaptive algorithms for the rejection of sinusoidal disturbances with unknown frequency. Automatica 33:2213–2221
Böling JM, Seborg DE, Hespanha JP (2007) Multi-model adaptive control of a simulated PH neutralization process. Control Eng Pract 15(6):663–672
Boyd S, Ghaoui LE, Feron E, Balakrishnan V (1994) Linear matrix inequalities in system and control theory. SIAM, Philadelphia
Brogliato B, Lozano R (1994) Adaptive control of systems of the form $\dot{x} = \theta_1^t f(x) + \theta_2^t g(x)u$ with reduced knowledge of the plant parameters. IEEE Trans Autom Control AC-39(8):1764–1768
Brogliato B, Lozano R, Landau ID (1993) New relationships between Lyapunov functions and the passivity theorem. Int J Adapt Control Signal Process 7(5):353–366
Butchart RL, Shakcloth B (1966) Synthesis of model reference adaptive control systems by Lyapunov's second method. In: Proc 2nd IFAC symp on theory of self-adaptive control systems. Plenum, Teddington, pp 145–152
Canudas C, Olsson H, Åström KJ, Lichinsky P (1995) A new model for control of systems with friction. IEEE Trans Autom Control AC-40(3):419–425
Chang CB, Athans M (1978) State estimation for discrete systems with switching parameters. IEEE Trans Aerosp Electron Syst 14(3):418–425
Chaoui F, Giri F, Dion JM, M'Saad M, Dugard L (1996a) Direct adaptive control subject to input amplitude constraints. In: Proc IEEE—CDC, Kobe, Japan
Chaoui F, Giri F, Dion JM, M'Saad M, Dugard L (1996b) Indirect adaptive control in the presence of input saturation constraints. In: Proc IEEE—CDC, Kobe, Japan
Chen HF, Guo L (1991) Identification and stochastic adaptive control. Birkhauser, Boston
Clarke D, Gawthrop PJ (1975) A self-tuning controller. Proc IEEE 122:929–934
Clarke D, Gawthrop PJ (1979) Self-tuning control. Proc IEEE 126(6):633–640
Clarke D, Mohtadi C (1989) Properties of generalized predictive control. Automatica 25:859–876
Clarke DW, Scatollini R (1991) Constrained receding horizon predictive control. In: Proc IEE-D, vol 138, pp 347–354
Clarke D, Tuffs P, Mohtadi C (1987) Generalized predictive control. Automatica 23:137–160
Clary JP, Franklin GF (1985) A variable dimension self-tuning controller. In: Proc ACC conf, Boston, USA

Cluett WR, Shah SL, Fisher DG (1987) Robust design of adaptive control systems using conic sector theory. Automatica 23:221–224

Constantinescu A (2001) Commande robuste et adaptative d'une suspension active. Thèse de doctorat, Institut National Polytechnique de Grenoble

Constantinescu A, Landau ID (2003) Direct controller order reduction by identification in closed loop applied to a benchmark problem. Eur J Control 9(1)

Dahhou B, Najim K, M'Saad M, Youlal B (1983) Model reference adaptive control of an industrial phosphate drying furnace. In: Proc IFAC workshop "Adaptive systems in control and signal processing". Pergamon, San Fransisco, pp 315–321

Datta A (1998) Adaptive internal model control. Springer, London

de Larminat P (1980) Unconditional stabilizers for non minimum phase systems. In: Proc int symp on adaptive systems, Ruhr-University, Bochum

de Larminat P (1984) On the stabilization condition in indirect adaptive control. Automatica 20:793–795

de Larminat P (1986) Une solution robuste au problème de la stabilité dans la commande adaptative indirecte passive. In: Landau ID, Dugard L (eds) Commande adaptative: aspects pratiques et théoriques. Masson, Paris

de Larminat P, Raynaud HF (1988) A robust solution to the admissibility problem in indirect adaptive control without persistency excitation. Int J Adapt Control Signal Process 2:95–110

de Mathelin M, Bodson M (1995) Multivariable model reference adaptive control without constraints on the high frequency gain matrix. Automatica 31:597–604

De Nicolao G, Scatollini R (1994) Stability and output terminal constraints in predictive control. In: Clarke DW (ed) Advances in model based predictive control. Oxford University Press, Oxford

Desoer CA, Vidyasagar M (1975) Feedback systems: input-output properties. Academic Press, New York

Ding Z (2003) Global stabilization and disturbance suppression of a class of nonlinear systems with uncertain internal model. Automatica 39:471–479

Dion JM, Dugard L, Carrillo J (1988) Interactor and multivariable adaptive control. IEEE Trans Autom Control AC-33:399–401

Doyle JC, Francis BA, Tannenbaum AR (1992) Feedback control theory. MacMillan, New York

Dugard L, Dion JM (1985) Direct multivariable adaptive control. Int J Control 42(6):1251–1281

Dugard L, Goodwin GC (1985) Global convergence of Landau's "output error with adjustable compensator" adaptive algorithm. IEEE Trans Autom Control AC-30:593–595

Dugard L, Landau ID (1980) Recursive output error identification algorithms, theory and evaluation. Automatica 16:443–462

Dugard L, Egardt B, Landau ID (1982) Design and convergence analysis of stochastic model reference adaptive controllers. Int J Control 35:755–773

Dumont G (1992) Fifteen years in the life of an adaptive controller. In: Proc IFAC-ACASP symp. Pergamon, Oxford, pp 261–272

Duong HG, Landau ID (1994) On statistical properties of a test for model structure selection using the extended instrumental variable approach. IEEE Trans Autom Control AC-39:211–215

Duong HG, Landau ID (1996) An IV based criterion for model order selection. Automatica 32:909–914

Egardt B (1979) Stability of adaptive controllers. Lectures notes in control and information sciences. Springer, Heidelberg

Eliott H (1980) Direct adaptive pole placement with application to non-minimum phase systems. IEEE Trans Autom Control 27:720–722

Eliott H (1985) Global stability of adaptive pole placement algorithm. IEEE Trans Autom Control 30:348–356

Elliott SJ, Nelson PA (1994) Active noise control. Noise/news international, pp 75–98

Elliott SJ, Sutton TJ (1996) Performance of feedforward and feedback systems for active control. IEEE Trans Speech Audio Process 4(3):214–223

Espinoza-Perez G, Ortega R (1995) An output feedback globally stable controller for induction motors. IEEE Trans Autom Control AC-40(1):138–143

Faurre P, Clerget M, Germain F (1979) Opérateurs rationnels positifs: Application à l'hyperstabilité et aux processus aléatoires. Bordas, Paris
Fekri S, Athans M, Pascoal A (2006) Issues, progress and new results in robust adaptive control. Int J Adapt Control Signal Process 20:519–579
Feldbaum A (1965) Optimal control theory. Academic Press, New York
Feng G, Zhang C, Palaniswami M (1994) Stability of input amplitude constrained adaptive pole placement control systems. Automatica 30:1065–1070
Fenot C, Rolland F, Vigneron G, Landau ID (1993) Open loop adaptive digital control in hot-dip galvanizing. Control Eng Pract 1(5):779–790
Ficocelli M, Amara FB (2009) Adaptive regulation of MIMO linear systems against unknown sinusoidal exogenous inputs. Int J Adapt Control Signal Process 23(6):581–603
Francis BA, Wonham WM (1976) The internal model principle of control theory. Automatica 12:457–465
Franklin GF, Powell JD, Workman M (1990) Digital control of dynamic systems, 2nd edn. Addison Wesley, Reading
Fu M, Barmish BR (1986) Adaptive stabilization of linear systems via switching control. IEEE Trans Autom Control 31:1097–1103
Fuchs JJ (1982) Indirect stochastic adaptive control the general delay-colored noise case. IEEE Trans Autom Control AC-27:470–472
Garrido-Moctezuma R, Lozano R, Suarez DA (1993) MRAC with unknown relative degree. Int J Adapt Control Signal Process 7(5):457–474
Geromel JC, Colaneri P (2006a) Stability and stabilization of continuous-time switched linear systems. SIAM J Control Optim 45(5):1915–1930
Geromel JC, Colaneri P (2006b) Stability and stabilization of discrete time switched systems. Int J Control 45(5):719–728
Gevers M (1993) Towards a joint design of identification and control. In: Trentelman HL, Willems JC (eds) Essays on control: perspectives in the theory and its applications. Birkhäuser, Boston, pp 111–152
Gilbart JW, Winston GC (1974) Adaptive compensation for an optical tracking telescope. Automatica 10:125–131
Giri F, M'Saad M, Dion JM, Dugard L (1989) A globally convergent pole placement indirect adaptive controller. IEEE Trans Autom Control AC-34:353–356
Giri F, M'Saad M, Dion JM, Dugard L (1990) A general lemma for the stability analysis of discrete-time adaptive control. Int J Control 51:283–288
Giri F, M'Saad M, Dion JM, Dugard L (1991) On the robustness of discrete-time indirect adaptive (linear) controllers. Automatica 27:153–160
Glower JS (1996) MRAC for systems with sinusoidal parameters. Int J Adapt Control Signal Process 10:85–92
Goodwin GC, Sin KS (1984) Adaptive filtering prediction and control. Prentice Hall, New York
Goodwin GC, Teoh EK (1985) Persistency of excitation in presence of possibly unbounded signals. IEEE Trans Autom Control AC-30:595–597
Goodwin GC, Ramadge PJ, Caines PE (1980a) Discrete-time multivariable adaptive control. IEEE Trans Autom Control AC-25:44
Goodwin GC, Ramadge PJ, Caines PE (1980b) Stochastic adaptive control. SIAM J Control 18
Goodwin GC, Sin KS, Soluja KK (1980c) Stochastic adaptive control and prediction: the general delay-coloured noise case. IEEE Trans on Automatic Control AC-25
Gouraud T, Gugliemi M, Auger F (1997) Design of robust and frequency adaptive controllers for harmonic disturbance rejection in a single-phase power network. In: Proc of the European control conference, Bruxelles
Green M, Limebeer DJN (1995) Linear robust control. Prentice Hall, New York
Guo L (1993) The Åström-Wittenmark self-tuning regulator revisited and ELS-based adaptive trackers. IEEE Trans Autom Control AC-40
Guo L (1996) Self-convergence of weighted least squares with applications to stochastic adaptive control. IEEE Trans Autom Control AC-41(1):79–89

Hespanha J, Liberzon D, Morse AS, Anderson BDO, Brinsmead TS, Bruyne FD (2001) Multiple model adaptive control. Part 2: Switching. Int J Robust Nonlinear Control 11(5):479–496
Hill DJ, Moylan PJ (1980) Dissipative dynamical systems: basic input-output and state properties. J Franklin Inst 309(5)
Hillerstrom G, Sternby J (1994) Rejection of periodic disturbances with unknown period—a frequency domain approach. In: Proc of American control conference, Baltimore, pp 1626–1631
Hitz L, Anderson BDO (1969) Discrete positive real functions and their application to system stability. In: Proc IEEE, vol 111, pp 153–155
Hu J, Linn JF (2000) Feedforward active noise controller design in ducts without independent noise source measurements. IEEE Trans Control Syst Technol 8(3):443–455
Ioannou PA, Datta A (1989) Robust adaptive control: a unified approach. Proc IEEE 79:1736–1768
Ioannou PA, Kokotovic PV (1983) Adaptive systems with reduced order models. Lecture notes in control and information sciences, vol 47. Springer, Heidelberg
Ioannou PA, Sun J (1996) Robust adaptive control. Prentice Hall, Englewood Cliffs
Ionescu T, Monopoli R (1977) Discrete model reference adaptive control with an augmented error signal. Automatica 13:507–518
Irving E, Falinower CM, Fonte C (1986) Adaptive generalized predictive control with multiple reference models. In: Proc 2nd IFAC workshop on adaptive systems in control and signal processing, Lund, Sweden
Jacobson CA, Johnson CR, Cormick DCM, Sethares WA (2001) Stability of active noise control algorithms. IEEE Signal Process Lett 8(3):74–76
Johansson R (1995) Supermartingale analysis of minimum variance adaptive control. Control Theory Adv Technol, Part 2 10:993–1013
Johnson CD (1976) Theory of disturbance-accomodating controllers. In: Leondes CT (ed) Control and dynamical systems, vol 12, pp 387–489
Kailath T (1980) Linear systems. Prentice-Hall, New York
Kalman RE (1958) Design of self-optimizing control systems. Trans ASME, J Basic Eng 80:468–478
Kalman RE, Bertram JE (1960) Control system analysis and design via the second method of Lyapunov. Part i and ii. Trans ASME, J Basic Eng 82:371–400
Karimi A (1997) Conception des régulateurs numériques robustes et adaptatifs. PhD thesis, Institut National Polytechnique de Grenoble, LAG, Grenoble, France
Karimi A (2002) Design and optimization of restricted complexity controllers—benchmark. http://lawww.epfl.ch/page11534.html
Karimi A, Landau ID (1998) Comparison of the closed loop identification methods in terms of the bias distribution. Syst Control Lett 34(4):159–167
Karimi A, Landau ID (2000) Robust adaptive control of a flexible transmission system using multiple models. IEEE Trans Control Syst Technol 8(2):321–331
Karimi A, Landau ID, Motee N (2001) Effects of the design parameters of multimodel adaptive control on the performance of a flexible transmission system. Int J Adapt Control Signal Process 15(3):335–352
Kidron O, Yaniv O (1995) Robust control of uncertain resonant systems. Eur J Control 1(2):104–112
Knopp K (1956) Infinite sequence series. Dover, New York
Kreisselmeier G (1986) A robust indirect adaptive control approach. Int J Control 43:161–175
Kreisselmeier G, Anderson BDO (1986) Robust model reference adaptive control. IEEE Trans Autom Control AC-31:127–132
Kreisselmeier G, Smith MC (1986) Stable adaptive regulation of arbitrary nth order plants. IEEE Trans Autom Control AC-31:299–305
Krstic M, Kanellakopoulos I, Kokotovic P (1995) Nonlinear and adaptive control design. Wiley, New York
Kumar R, Moore JB (1982) Convergence of adaptive minimum variance algorithm via weighting coefficient selection. IEEE Trans Autom Control AC-27:146–153
Kuo MS, Morgan DR (1996) Active noise control systems-algorithms and DSP implementation. Wiley, New York

Kuo MS, Morgan DR (1999) Active noise control: a tutorial review. Proc IEEE 87:943–973
Kushner HJ, Clark DS (1978) Stochastic approximation methods for constrained and unconstrained systems. Springer, Heidelberg
Kwakernaak H (1993) Robust control and H_∞-optimization—a tutorial. Automatica 29:255–273
Kwakernaak H (1995) Symmetries in control system design. In: Isidori A (ed) Trends in control. Springer, Heidelberg, pp 17–52
Lam K (1982) Design of stochastic discrete-time linear optimal control. Int J Syst Sci 13(19):979–1011
Lancaster P, Tismenetsky M (1985) The theory of matrices, 2nd edn. Academic Press, New York
Landau ID (1969a) Analyse et synthèse des commandes adaptatives à l'aide d'un modèle par des méthodes d'hyperstabilité. Automatisme 14:301–309
Landau ID (1969b) A'hyperstability criterion for model reference adaptive control systems. IEEE Trans Autom Control AC-14:552–555
Landau ID (1971) Synthesis of discrete model reference adaptive systems. IEEE Trans Autom Control AC-16:507–508
Landau ID (1973) Design of discrete model reference adaptive systems using the positivity concept. In: Proc 3rd IFAC symp on "Sensitivity, adaptivity and optimality". ISA, Ischia, Italy, pp 307–314
Landau ID (1974) A survey of model reference adaptive techniques—theory and applications. Automatica 10:353–379
Landau ID (1979) Adaptive control—the model reference approach. Marcel Dekker, New York
Landau ID (1980) A stability theorem applicable to adaptive control. IEEE Trans Autom Control AC-25(4):814–817
Landau ID (1981) Model reference adaptive controllers and stochastic self-tuning regulators: a unified approach. Trans ASME, J Dyn Syst Meas Control 103:404–414
Landau ID (1982a) Combining model reference adaptive controllers and stochastic self tuning regulators. Automatica 18(1):77–84
Landau ID (1982b) Near supermartingales for convergence analysis of recursive identification and adaptive control schemes. Int J Control 35(2):197–226
Landau ID (1985) Adaptive control techniques for robot manipulators—the status of the art. In: Proc IFAC symp robot control, Barcelona, Spain
Landau ID (1990a) Algorithmes d'adaptation paramétrique. In: Dugard L, Landau ID (eds) Ecole d'Eté d'Automatique, LAG, Grenoble
Landau ID (1990b) System identification and control design. Prentice Hall, Englewood Cliffs
Landau ID (1993a) Evolution of adaptive control. Trans ASME, J Dyn Syst Meas Control 115:381–391
Landau ID (1993b) Identification et Commande des Systèmes, 2nd edn. Série Automatique. Hermès, Paris
Landau ID (1995) Robust digital control of systems with time delay (the Smith predictor revisited). Int J Control 62(2):325–347
Landau ID, Alma M (2010) Adaptive feedforward compensation algorithms for active vibration control. In: Proc of 49th IEEE conf on decision and control, 2010, IEEE-CDC, Atlanta, USA, pp 3626–3631
Landau ID, Horowitz R (1988) Synthesis of adaptive controllers for robot manipulators using a passive feedback systems approach. In: Proc IEEE int conf robotics and automation. Also in Int J Adapt Control Signal Process 3:23–38 (1989)
Landau ID, Karimi A (1996) Robust digital control using the combined pole placement sensitivity function shaping: an analytical solution. In: Proc of WAC, vol 4, Montpellier, France, pp 441–446
Landau ID, Karimi A (1997a) An output error recursive algorithm for unbiased identification in closed loop. Automatica 33(5):933–938
Landau ID, Karimi A (1997b) Recursive algorithms for identification in closed loop: a unified approach and evaluation. Automatica 33(8):1499–1523
Landau ID, Karimi A (1998) Robust digital control using pole placement with sensitivity function shaping method. Int J Robust Nonlinear Control 8:191–210

Landau ID, Karimi A (1999) A recursive algorithm for ARMAX model identification in closed loop. IEEE Trans Autom Control 44(4):840–843
Landau ID, Karimi A (2002) A unified approach to model estimation and controller reduction (duality and coherence). Eur J Control 8(6):561–572
Landau ID, Lozano R (1981) Unification and evaluation of discrete-time explicit model reference adaptive designs. Automatica 17(4):593–611
Landau ID, Silveira HM (1979) A stability theorem with applications to adaptive control. IEEE Trans Autom Control AC-24(2):305–312
Landau ID, Zito G (2005) Digital control systems—design, identification and implementation. Springer, London
Landau ID, M'Sirdi N, M'Saad M (1986) Techniques de modélisation récursive pour l'analyse spectrale paramétrique adaptative. Rev Trait Signal 3:183–204
Landau ID, Normand-Cyrot D, Montano A (1987) Adaptive control of a class of nonlinear discrete-time systems: application to a heat exchanger. In: Proc IEEE CDC, Los Angeles, USA, pp 1990–1995
Landau ID, Rey D, Karimi A, Voda-Besançon A, Franco A (1995a) A flexible transmission system as a benchmark for robust digital control. Eur J Control 1(2):77–96
Landau ID, Karimi A, Voda-Besançon A, Rey D (1995b) Robust digital control of flexible transmissions using the combined pole placement/sensitivity function shaping method. Eur J Control 1(2):122–133
Landau ID, Constantinescu A, Loubat P, Rey D, Franco A (2001a) A methodology for the design of feedback active vibration control systems. In: Proc European control conference 2001, Porto, Portugal
Landau ID, Karimi A, Constantinescu A (2001b) Direct controller order reduction by identification in closed loop. Automatica 37(11):1689–1702
Landau ID, Constantinescu A, Rey D (2005) Adaptive narrow band disturbance rejection applied to an active suspension—an internal model principle approach. Automatica 41(4):563–574
Landau ID, Alma M, Martinez JJ, Buche G (2010, to appear) Adaptive suppression of multiple time varying unknown vibrations using an inertial actuator. IEEE Trans Control Syst Technol
Landau ID, Alma M, Martinez JJ, Buche G (2011) Adaptive suppression of multiple time-varying unknown vibrations using an inertial actuator. IEEE Trans Control Syst Technol doi:10.1109/TCST.2010.2091641
Langer J, Constantinescu A (1999) Pole placement design using convex optimisation criteria for the flexible transmission benchmark: robust control benchmark: new results. Eur J Control 5(2–4):193–207
Langer J, Landau ID (1996) Improvement of robust digital control by identification in closed loop: application to a 360° flexible arm. Control Eng Pract 4(8):1079–1088
Langer J, Landau ID (1999) Combined pole placement/sensitivity function shaping method using convex optimization criteria. Automatica 35(6):1111–1120
Li XR, Bar-Shalom Y (2002) Design of an interacting multiple model algorithm for air traffic control tracking. IEEE Trans Control Syst Technol 1(3):186–194
Liberzon D (2003) Switching in systems and control. Birkhauser, Basel
Ljung L (1977a) Analysis of recursive stochastic algorithms. IEEE Trans Autom Control AC-22:551–575
Ljung L (1977b) On positive real transfer functions and the convergence of some recursions. IEEE Trans Autom Control AC-22:539–551
Ljung L (1999) System identification: theory for the user, 2nd edn. Prentice-Hall, Englewood Cliffs
Ljung L, Söderström T (1983) Theory and practice of recursive identification. MIT Press, Cambridge
Lozano R (1989) Robust adaptive regulation without persistent excitation. IEEE Trans Autom Control AC-34:1260–1267
Lozano R (1992) Singularity-free adaptive pole placement without resorting to persistency of excitation detailed analysis for first order systems. Automatica 28:27–33

Lozano R, Brogliato B (1992a) Adaptive control of a simple nonlinear system without a-priori information on the plant parameters. IEEE Trans Autom Control AC-37(1):30–37
Lozano R, Brogliato B (1992b) Adaptive control of robot manipulators with flexible joints. IEEE Trans Autom Control AC-37(2):174–181
Lozano R, Brogliato B (1992c) Adaptive hybrid force-position control for redundant manipulators. IEEE Trans Autom Control AC-37(10):1501–1505
Lozano R, Goodwin GC (1985) A globally convergent pole-placement algorithm without persistency of excitation requirement. IEEE Trans Autom Control AC-30:795–797
Lozano R, Landau ID (1982) Quasi-direct adaptive control for non-minimum phase systems. Trans ASME J Dyn Syst Meas Control 104
Lozano R, Ortega R (1987) Reformulation of the parameter identification problem for systems with bounded disturbances. Automatica 23(2):247–251
Lozano R, Zhao X (1994) Adaptive pole placement without excitation probing signals. IEEE Trans Autom Control AC-39(1):47–58
Lozano R, Dion JM, Dugard L (1993) Singularity-free adaptive pole placement for second order systems. IEEE Trans Autom Control AC-38:104–108
Mareels I, Polderman JW (1996) Adaptive systems: an introduction. Birkhauser, Basel
Marino R, Tomei P (1995) Nonlinear control design: geometric, adaptive, robust. Prentice-Hall, Englewood Cliffs
Marino R, Peresada S, Tomei P (1996) Adaptive observer-based control of induction motors with unknown rotor resistance. Int J Adapt Control Signal Process 10:345–363
Marino R, Santosuosso GL, Tomei P (2003) Robust adaptive compensation of biased sinusoidal disturbances with unknown frequency. Automatica 39:1755–1761
Martensson B (1986) Adaptive stabilization. PhD thesis, Lund Institute of Technology, Lund, Sweden
Mendel JM (1973) Discrete techniques of parameter estimation—the equation error formulation. Dekker, New York
Middleton RH, Goodwin GC, Hill DJ, Mayne DQ (1988) Design issues in adaptive control. IEEE Trans Autom Control AC-33(1):50–58
Miller DE (1994) Adaptive stabilization using a nonlinear time-varying controller. IEEE Trans Autom Control 39:1347–1359
Miller DE, Davison EJ (1989) An adaptive controller which provides Lyapunov stability. IEEE Trans Autom Control 34:599–609
Morari M, Zafiriou E (1989) Robust process control. Prentice Hall International, Englewood Cliffs
Morse AS (1980) Global stability of parameter adaptive control systems. IEEE Trans Autom Control AC-25:433–440
Morse AS (1995) Control using logic-based switching. In: Isidori A (ed) Trends in control. Springer, Heidelberg, pp 69–114
Mosca E, Zhang J (1992) Stable redesign of predictive control. Automatica 28(6):1229–1234
M'Saad M (1994) Un logiciel pour la commande avancée des procédés industriels. In: Proc 2AO conf, Noisy-le-Grand, France
M'Saad M, Chebassier J (1997) Simart: un progiciel pour l'automatique et ses applications. In: Proc des journées d'études sur les logiciels, Nancy, France
M'Saad M, Hejda I (1994) Partial state reference model (adaptive) control of a benchmark example. Automatica 30(4):605–614
M'Saad M, Sanchez G (1992) Partial state model reference adaptive control of multivariable systems. Automatica 28(6):1189–1194
M'Saad M, Ortega R, Landau ID (1985) Adaptive controllers for discrete-time systems with arbitrary zeros: an overview. Automatica 21(4):413–423
M'Saad M, Duque M, Landau ID (1986) Practical implications of recent results in robustness of adaptive control schemes. In: Proc of IEEE 25th CDC, Athens, Greece
M'Saad M, Duque M, Irving E (1987) Thermal process robust adaptive control: an experimental evaluation. In: Proc of the 10th IFAC World congress, Munich, Germany
M'Saad M, Landau ID, Duque M (1989) Example applications of the partial state reference model adaptive control design technique. Int J Adapt Control Signal Process 3(2):155–165

M'Saad M, Landau ID, Samaan M (1990) Further evaluation of the partial state reference model adaptive control design. Int J Adapt Control Signal Process 4(2):133–146
M'Saad M, Dugard L, Hammad S (1993a) A suitable generalized predictive adaptive controller case study: control of a flexible arm. Automatica 29(3):589–608
M'Saad M, Giri F, Dion JM, Dugard L (1993b) Techniques in discrete time robust adaptive control. In: Leondes CT (ed) Control and dynamical systems, vol 56. Academic Press, San Diego, pp 93–161
Mutoh Y, Ortega R (1993) Interactor structure estimation for adaptive control of discrete-time multivariable systems. Automatica 29(3):635–347
Najim K, Najim M, Youlal H (1982) Self-tuning control of an industrial phosphate dry process. Optim Control Appl Methods 3:435–442
Najim K, Hodouin D, Desbiens A (1994) Adaptive control: state of the art and an application to a grinding circuit. Powder Technol 82:56–68
Narendra KS, Balakrishnan J (1997) Adaptive control using multiple models. IEEE Trans Autom Control AC-42:171–187
Narendra KS, Lin YH, Valavani LS (1980) Stable adaptive controller design—Part ii: Proof of stability. IEEE Trans Autom Control AC-25:440–448
Neveu J (1975) Discrete parameter martingales. North Holland, Amsterdam
Nicosia S, Tomei P (1990) Robot control by using only joint position measurements. IEEE Trans Autom Control AC-35:1058–1061
Ortega R (1993) Adaptive control of discrete time systems: a performance oriented approach. In: Leondes C (ed) Control of dynamic systems: advances in theory and applications, vol 55. Academic Press, New York
Ortega R, Lozano-Leal R (1987) A note on direct adaptive control of systems with bounded disturbances. Automatica 23(2):253–254
Ortega R, Spong M (1989) Adaptive motion control: a tutorial. Automatica 25(6):877–898
Ortega R, Tang Y (1989) Robustness of adaptive controllers—a survey. Automatica 25(5):651–677
Ortega R, Praly L, Landau ID (1985) Robustness of discrete-time direct adaptive controllers. IEEE Trans Autom Control AC-30(12):1179–1187
Ossman KA, Kamen ED (1987) Adaptive regulation of MIMO linear discrete-time systems without requiring a persistent excitation. IEEE Trans Autom Control AC-32:397–404
Pajunen G (1992) Adaptive control of Wiener type nonlinear systems. Automatica 28:781–785
Parks PC (1966) Lyapunov redesign of model reference adaptive control systems. IEEE Trans Autom Control AC-11:362–367
Polderman JW (1989) A state space approach to the problem of adaptive pole assignment. Math Control Signals Syst 2(1):71–94
Popov VM (1960) Nouveaux critères de stabilité pour les systèmes automatiques non linéaires. Rev Electrotechn Energ 5(1)
Popov VM (1963) Solution of a new stability problem for controlled systems. Autom Remote Control 24(1):1–23
Popov VM (1964) Hyperstability of automatic systems with several nonlinear elements. Rev Electrotech Energ 9(4):33–45
Popov VM (1966) Hiperstabilitatea sistemelor automate. Editura Academiei, Bucharest
Popov VM (1973) Hyperstability of automatic control systems. Springer, Heidelberg
Praly L (1983a) Commande adaptative avec modèle de référence: Stabilité et robustesse. In: Landau ID (ed) Outils et Modèles Mathématique pour l'Automatique, l'Analyse de Systèmes et le traitement du signal, vol 3. CNRS, Paris, pp 805–816
Praly L (1983b) Robustness of indirect adaptive control based on pole placement design. In: Proc IFAC workshop "Adaptive systems in control and signal processing". Pergamon, San Fransisco, pp 55–60
Praly L (1983c) Robustness of model reference adaptive control. In: Proc 3rd Yale workshop on adaptive systems, New Haven, Connecticut, USA
Praly L, Bastin G, Pomet JB, Jiang ZP (1991) Adaptive stabilization of nonlinear systems. In: Kokotovic PV (ed) Foundations of adaptive control. Springer, Berlin

Press W, Flamery B, Tenkolsky S, Veterling W (1988) Numerical recipes in C. Cambridge University Press, Cambridge

Procházka H, Landau ID (2003) Pole placement with sensitivity function shaping using 2nd order digital notch filters* 1. Automatica 39(6):1103–1107

Queinnec I, Dahhou B, M'Saad M (1992) An adaptive control of fermentation processes. Int J Adapt Control Signal Process 6:521–536

Raumer T, Dion JM, Dugard L (1993) Adaptive nonlinear speed control of induction motors. Int J Adapt Control Signal Process 7(5):435–455

Ren W, Kumar PR (2002) Stochastic adaptive prediction and model reference control. IEEE Trans Autom Control 39(10):2047–2060

Richalet J, Rault A, Testud JL, Papon J (1978) Model predictive heuristic control: applications to industrial processes. Automatica 14:413–428

Robbins H, Monro S (1951) A stochastic approximation method. Ann Math Stat 22:400–407

Rohrs C, Valavani L, Athans M, Stein G (1981) Analytical verification of undesirable properties of direct model reference adaptive control algorithms. In: Proc IEEE-CDC, San Diego, USA

Rohrs C, Valavani L, Athans M, Stein G (1985) Robustness of continuous time adaptive control algorithms in the presence of unmodeled dynamics. IEEE Trans Autom Control AC-30(9):881–889

S Fekri MA, Pasqual A (2006) Issues, progress and new results in robust adaptive control. Int J Adapt Control Signal Process, 519–579

Samson C (1982) An adaptive LQ controller for non-minimum phase systems. Int J Control 3:389–397

Sastry S, Bodson M (1989) Adaptive control: stability, convergence, and robustness. Prentice Hall, New York

Sastry S, Isidori A (1989) Adaptive control of linearizable systems. IEEE Trans Autom Control AC-34:1123–1131

Seborg DE, Edgar TF, Shah SL (1989) Adaptive control strategies for process control: a survey. AIChE J 32(6):881–913

Serrani A (2006) Rejection of harmonic disturbances at the controller input via hybrid adaptive external models. Automatica 42:1977–1985

Shah S, Franklin GF (1982) On satisfying strict positive real condition for convergence by overparameterization. IEEE Trans Autom Control AC-27(3):715–716

Slotine JJ, Li W (1991) Applied nonlinear control. Prentice-Hall, Englewood Cliffs

Söderström T, Stoica P (1989) System identification. Prentice-Hall, New York

Solo V (1979) The convergence of AML. IEEE Trans Autom Control AC-24:958–963

Stoica P, Söderström T (1981) Analysis of an output error identification algorithm. Automatica 17(6):861–863

Suarez D, Lozano R, Åström KJ, Wittenmark B (1996) Adaptive control linear systems with poles in the closed LHP with constrained inputs. In: Proc 35th IEEE CDC, Kobe, Japan

Sung HK, Hara S (1988) Properties of sensitivity and complementary sensitivity functions in single-input single-output digital systems. Int J Control 48(6):2429–2439

Sussmann H, Sontag ED, Yang Y (1994) A general result on the stabilization of linear systems using bounded controls. IEEE Trans Autom Control AC-39:2411–2425

Tao G, Kokotovic PV (1996) Adaptive control of systems with actuator and sensor nonlinearities. Wiley, New York

Tomizuka M (1982) Parallel MRAS without compensation block. IEEE Trans Autom Control AC-27(2):505–506

Tsypkin YZ (1971) Adaptation and learning in automatic systems. Academic Press, New York

Tsypkin YZ (1993) Robust internal model control. Trans ASME, J Dyn Syst Meas Control 115:419–425

Tsypkin YZ (1997) Stochastic discrete systems with internal models. J Autom Inf Sci 29(4–5):156–161

Valentinotti S (2001) Adaptive rejection of unstable disturbances: application to a fed-batch fermentation. Thèse de doctorat, École Polytechnique Fédérale de Lausanne

Van den Hof P, Schrama R (1995) Identification and control—closed-loop issues. Automatica 31(12):1751–1770
Vandenberghe L, Boyd S (1996) Semidefinite programming. SIAM Rev 38(1):49–95
Visual Solutions (1995) Vissim—User manual, version 2.0. Westford, MA, USA
Voda-Besançon A, Landau ID (1995a) An iterative method for the auto-calibration of the digital controllers. In: Proc ACC 95, Rome, Italy, pp 12,463–12,468
Voda-Besançon A, Landau ID (1995b) The joint closed-loop identification and control design—theoretical issues and applications. In: Proc ACC, Seattle, USA
Whitaker HP, Yamron J, Kezer A (1958) Design of a model-reference adaptive control system for aircraft. Technical report, R-164, MIT, Instrumentation Lab, Cambridge, USA
Widrow B, Stearns SD (1985) Adaptive signal processing. Prentice-Hall, Englewood Cliffs
Willems JC (1971) The analysis of feedback systems. MIT Press, Cambridge
Wolowich WA (1974) Linear multivariable systems. Springer, Heidelberg
Ydstie BE (1989) Stability of discrete model reference adaptive control revisited. Syst Control Lett 13:429–438
Young PC (1969) An instrumental variable method for real time identification of a noisy process. Automatica 66:271–288
Zames G (1966) On the input-output stability of nonlinear time-varying systems, Parts I and II. IEEE Trans Autom Control AC-11:228–238, and 465–477
Zames G (1981) Feedback and optimal sensibility: model reference transformations, multiplicative seminorms and approximate inverses. IEEE Trans Autom Control 26:301–320
Zang Z, Bitmead RR, Gevers M (1995) Iterative weighted least-squares identification and weighted LQG control design. Automatica 31(11):1577–1594
Zeng J, de Callafon RA (2006) Recursive filter estimation for feedforward noise cancellation with acoustic coupling. J Sound Vib 291:1061–1079
Zhang C, Evans RJ (1994) Continuous direct adaptive control with saturation input constraint. IEEE Trans Autom Control AC-39:1718–1722
Zhou K (1998) Essentials of robust control. Prentice Hall, New York

Index

I.D. Landau et al., *Adaptive Control*, Communications and Control Engineering,
DOI 10.1007/978-0-85729-664-1,