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## Revision of the Simplex Method

The Simplex method [129] is the algorithm most used for solving linear programming problems, such as the ones appearing when using a 1-norm MPC. The Simplex algorithm finds successive and better feasible solutions at the extreme points of the feasible region until it stops, in a finite number of steps, either at the optimum or finds that the optimal solution is not bound by the feasible region. This appendix is dedicated to revising the basic ideas behind the Simplex method.

### A.1 Equality Constraints

The problem of minimizing a linear function subject to equality constraints will be considered first

$$\begin{aligned} & \text{Minimize } c^t x \\ & \text{subject to } Ax = b \\ & \quad \quad \quad x \geq 0 \end{aligned} \tag{A.1}$$

where  $A$  is a  $q \times p$  real matrix with  $q < p$  and full rank.

If the equality constraint equation is multiplied by a matrix  $T$  and the columns of  $A$  and corresponding components of  $x$  are interchanged in such a way that  $TA = [I_{q \times q} \ N]$  and  $Tb = \bar{b}$ , the point  $x = [x_b^t \ x_n^t]^t = [\bar{b}^t \ 0]$  is a basic feasible solution. The components  $x_b$  are called *basic variables* whereas the remaining components (corresponding to  $N$ ) are called *nonbasic variables*. Note that this can be done by applying elementary row transformations to matrix  $A$  and interchanging the columns of  $A$  (and the corresponding  $x$  variables) to take matrix  $A$  to the form  $[I \ N]$ . If the same transformations are applied to  $I$  and  $b$ , matrix  $T$  and vector  $\bar{b}$  are obtained.

The objective function takes the value  $z_0 = c_b^t x_b + c_n^t x_n = c_b^t x_b$ . The basic variables can be expressed as a function of the nonbasic variables from the transformed constraint equation:

$$x_b = \bar{b} - Nx_n$$

by substituting in the cost function

$$z = c^t x = c_b^t(\bar{b} - Nx_n) + c_n^t x_n = z_0 + (c_n^t - c_b^t N)x_n$$

As  $x_n \geq 0$ , the objective function decreases if any component of  $(c_n^t - c_b^t N)_i$  is negative and the corresponding nonbasic variable  $x_{n_i}$  increases. This gives an indication of how to obtain a more feasible solution and is the basic idea behind the algorithm. The problem is determining which of the nonbasic variables should be increased (become basic) and which of the basic variables should leave the basis.

This is done as follows:

1. Find an initial basic solution.

2. Form the following tableau:

	$x_b^t$	$x_n^t$	
$x_b$	$I$	$N$	$\bar{b}$
$J$	$0$	$c_n - c_b^t N$	$c_b^t \bar{b}$

3. If  $c_n - c_b^t N \geq 0$  then STOP, the actual basic solution is optimal.
4. Choose one of the negative elements (say, the  $j^{th}$ ) of row  $c_n - c_b^t N$  (the most negative is usually chosen).
5. Choose  $i$  such that the ratio  $\bar{b}_i/N_{ij}$  is the minimum for all  $N_{ij} > 0$ . If there are no nonnegative elements in that column of the tableau, the problem is not bounded.
6. Make  $x_{n_j}$  a basic variable and  $x_{b_i}$  a non basic variable by *pivoting*:
  - a) Divide the  $i^{th}$  row of the tableau by  $N_{ij}$ .
  - b) Make zero the remaining elements of the  $j^{th}$  column of the nonbasic variable block by multiplying the resulting the  $i^{th}$  row by  $N_{kj}$  and subtracting it from the  $k^{th}$  row.
7. Go to step 2.

## A.2 Finding an Initial Solution

The Simplex method starts from an initial feasible extreme point. An initial point can be found by applying elementary row transformations to matrix  $A$  and vector  $b$  and interchanging the columns of  $A$  (and the corresponding  $x$  variables) to take matrix  $A$  to the form  $[I \ N]$ .

A solution can be obtained by using the Simplex algorithms in different ways. One way, known as the two-phase method, consists of solving the following augmented system:

$$\begin{aligned} &\text{minimize } 1^t x_a \\ &\text{subject to } x_a + Ax = b \\ &\quad x \geq 0, \quad x_a \geq 0 \end{aligned} \tag{A.2}$$

Note that the constraint matrix is now  $[IA]$  and that the obvious solution  $x = 0$  and  $x_a = b$  is an extreme point of the augmented problem. The

variables  $x_a$  are called *artificial* variables and are all in the basis for the initial solution. If the algorithm does not find a solution with  $x_a = 0$ , the problem is not feasible. Otherwise, the solution constitutes an initial solution to the original problem and the second phase of the algorithm can be started for the original problem with this solution.

Another way of dealing with initial conditions, known as the big-M method [15], solves the whole problem in only one phase. Artificial variables are also introduced as in the two-phase method but a term is added to the objective function penalizing the artificial variables with high weighting factors in order to force artificial variables out of the basis. The problem is now stated as:

$$\begin{aligned} & \text{minimize } c^t x + m^t x_a \\ & \text{subject to } x_a + Ax = b \\ & \quad x \geq 0, \quad x_a \geq 0 \end{aligned} \tag{A.3}$$

If all the artificial variables are out of the basis at termination, a solution to the optimization problem has been found. Otherwise, if the variable entering the basis is the one with the most positive cost coefficient, we can conclude that the problem has no feasible solution.

### A.3 Inequality Constraints

Consider the problem:

$$\begin{aligned} & \text{minimize } c^t x \\ & \text{subject to } Ax \leq b \\ & \quad x \geq 0 \end{aligned} \tag{A.4}$$

The Simplex method can be used to solve inequality constraint problems by transforming the problem into the standard format by introducing a vector of variables,  $x_s \geq 0$ , called *slack* variables, such that the inequality constraint  $Ax \leq b$  is transformed into the equality constraint  $Ax + x_s = b$ . The problem can now be stated in the standard form as:

$$\begin{aligned} & \text{minimize } c^t x \\ & \text{subject to } [A \ I] \begin{bmatrix} x \\ x_s \end{bmatrix} = b \\ & \quad \begin{bmatrix} x \\ x_s \end{bmatrix} \geq 0 \end{aligned} \tag{A.5}$$

The number of variables is now  $q + p$  and the number of constraints is  $q$ . Notice that the form of the equality constraint matrix is  $[A \ I]$  and the point  $[x^t \ x_s^t] = [0 \ b^t]$  is an initial basic solution to this problem.

### Duality

The number of *pivoting* operations needed to solve a standard linear programming problem is on the order of  $q$  [129], while the number of floating-point operations needed for each pivoting operation is on the order of  $q \times p$ .

The number of floating-point operations needed to solve a standard LP problem is therefore on the order of  $(q^2 + q \times p)$ . For the inequality constraint problem, the number of variables is increased by the number of slack variables which is equal to the number of inequality constraints. In the linear programming problems resulting from robust MPC, all of the constraints are inequality constraints, so the number of operations needed is on the order of  $q(q \times (q + p)) = q^3 + q^2p$ . That is, it is linear in the number of variables and cubic in the number of inequality constraints. The problem can be transformed into an LP problem with a different and more convenient structure using duality.

Given the problem (*primal*)

$$\begin{aligned} & \text{minimize } c^t x \\ & \text{subject to } Ax \leq b \\ & \quad x \geq 0 \end{aligned} \tag{A.6}$$

the *dual* problem is defined as [129]:

$$\begin{aligned} & \text{minimize } -b^t \lambda \\ & \text{subject to } -A^t \lambda \leq -c \\ & \quad \lambda \geq \mathbf{0} \end{aligned} \tag{A.7}$$

The number of operations required by the *dual* problem is on the order of  $p(p \times (p + q)) = p^3 + p^2q$ . That is, it is cubic in the number of variables and linear in the number of inequality constraints. So, for problems with more inequality constraints than variables, solving the *dual* problem will require less computation than the primal.

The solutions to both problems are obviously related (Bazaraa and Shetty [15], theorem 6.6.1.). If  $x_o$  and  $\lambda_o$  are the optimal of the *primal* and *dual* problem then,  $c^t x_o = b^t \lambda_o$  (the cost is identical) and  $(c^t - \lambda_o^t A)x_o = 0$ . If the primal problem has a feasible solution and has an unbounded objective value the *dual* problem is unfeasible. If the *dual* problem has a feasible solution and an unbounded objective value the *primal* problem is unfeasible.

## B

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# Dynamic Programming and Linear Quadratic Optimal Control

Model Predictive Control is closely related to Linear Quadratic (LQ) Optimal Control. This appendix shows the main characteristics of LQ and its relation to MPC.

Dynamic Programming can be used to find the solution of the LQ problem, since it provides an efficient means for sequential decision making. It is based on Bellman's principle of optimality [16], which states that an optimal policy has the property that whatever the initial state and the initial decision are, the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decision. Notice that a *decision* is the control action at a particular time instant while *policy* is equivalent to the control sequence.

If the goal is to move the process from an initial state to a final state with minimum cost, it is clear that the optimal solution can be obtained calculating the cost associated to every possible route and choosing the route with minimum cost. This implies an evaluation of all possible alternatives. However, the problem can be solved by defining the cost associated to a particular state as the sum of two terms: the part attributable to the current decision and the part representing the minimum value of all future costs, starting with the state which results from the first decision.

The principle of optimality replaces a choice among all alternatives by a sequence of decisions among fewer alternatives. Dynamic Programming allows one to concentrate on a sequence of current decisions rather than being concerned about all decisions simultaneously.

## B.1 Linear Quadratic Problem

When the cost is quadratic and the system is linear, the problem can be solved analytically and the controller results in a linear state feedback.

The process is modelled by

$$x(t+1) = Ax(t) + Bu(t) \quad (\text{B.1})$$

with  $x(0)$  known, and the objective is to find the control sequence  $u(0), u(1), \dots, u(N-1)$  that drives the process from the initial to the final state minimizing the cost given by

$$J = x(N)^T Q_N x(N) + \sum_{k=0}^{N-1} x(k)^T Q_k x(k) + u(k)^T R_k u(k)$$

where  $Q_k$  are symmetric positive semidefinite matrices and  $R_k > 0$ .

The procedure to obtain the control sequence is based on solving the problem in reverse sense, that is, start computing  $u(N-1)$  and finishing with  $u(0)$ . Let us define  $I_1^*$  as the optimal cost of the last stage (from state  $x(N-1)$  to the end), which is expressed as

$$I_1^*(x(N-1)) = \min_{u(N-1)} x(N)^T Q_N x(N) + u(N-1)^T R_{N-1} u(N-1)$$

and can be computed analytically deriving with respect to  $u(N-1)$ , giving:

$$u(N-1) = -(B^T Q_N B + R)^{-1} B^T Q_N A x(N-1) = K_{N-1} x(N-1) \quad (\text{B.2})$$

Therefore, the control action is a linear feedback of the state vector. The last stage cost is then given by:

$$I_1^* = (Ax + BK_{N-1}x)^T Q_N (Ax + BK_{N-1}x) + x^T K_{N-1}^T R_{N-1} K_{N-1} x$$

Defining

$$P_{N-1} = (A + BK_{N-1})^T Q_N (A + BK_{N-1}) + K_{N-1}^T R_{N-1} K_{N-1}$$

this cost can be written as a quadratic form of the state:

$$I_1^* = x(N-1)^T P_{N-1} x(N-1)$$

At this point Bellman's optimality principle is used to calculate the next cost:

$$\begin{aligned} I_2^* &= \min_{u(N-2)} x(N-1)^T Q_{N-1} x(N-1) + u(N-2)^T R_{N-2} u(N-2) + I_1^* \\ &= \min_{u(N-2)} x(N-1)^T (Q_{N-1} + P_{N-1}) x(N-1) + u(N-2)^T R_{N-2} u(N-2) \end{aligned}$$

As  $x(N-1)$  can be expressed as a function of  $x(N-2)$  and  $u(N-2)$  using (B.1),  $I_2^*$  depends on these last values and the optimal control action can be obtained analytically in the same fashion as (B.2); that is,

$$u(N-2) = K_{N-2} x(N-2)$$

This procedure can be extended to all the states leading to the general expression of the control law

$$u(k) = K_k x(k) \text{ with } K_k = -(B^T P_{k+1} B + R)^{-1} B^T P_{k+1} A \quad (\text{B.3})$$

and the symmetric semidefinite matrix  $P_k$  is given by:

$$P_k = (A + BK_k)^T P_{k+1} (A + BK_k) + K_k^T R_k K_k + Q_k$$

which after a few manipulations is transformed into:

$$P_k = A^T P_{k+1} A + A^T P_{k+1} B K_k + Q_k \quad (\text{B.4})$$

This is called the *discrete-time Riccati equation* and can be used to compute the value of  $P_k$  recursively starting with  $P_N = Q_N$ .

The problem is solved backwards, starting at time  $N$  and calculating  $u(k)$  using (B.3) and matrix  $P$  from (B.4).

As the controller is a linear feedback of the state, the use of a state estimator or observer is required to compute the control action. If the observer is a Kalman filter, then it gives rise to the well-known control strategy called Linear Quadratic Gaussian (LQG). A detailed study of the relationship between MPC (especially GPC) and LQG can be found in [27].

## B.2 Infinite Horizon

In some situations it is justifiable to assume that the terminal time is infinitely far in the future. This so-called infinite horizon case leads to a constant feedback gain matrix, which can be calculated from (B.4) considering that the weighting matrices are constant. Then  $P_k \rightarrow P_\infty \geq 0$  and is calculated using the *algebraic Riccati equation*:

$$P_\infty = A^T P_\infty A + A^T P_\infty B K_\infty + Q$$

Now the control action becomes the constant state feedback law:

$$u(k) = K_\infty x(k) \text{ with } K_\infty = -(B^T P_\infty B + R)^{-1} B^T P_\infty A$$

It can be proved that this control law is stabilizing by defining the Lyapunov function as  $V(x(k)) = x(k)^T P_\infty x(k)$ .

The close relationship between LQ and MPC can be used to derive stability properties of MPC based on the well-known LQ properties, as shown in [27]. However, there are some differences between the two methods, the main one being that LQ does not take constraints into account. Also the cost function, although similar, is not exactly the same, since MPC uses increments in the control actions and LQ weights the control actions. The predictive control problem can be put in the standard LQ framework using the incremental state space model of Equation (2.8) with  $\bar{x}(t) = [x(t) \ u(t-1)]^T$ . In this case, the new error weighting matrix is:

$$\bar{Q} = \begin{bmatrix} Q & 0 \\ 0 & 0 \end{bmatrix}$$

Notice that the control weight  $R_k$  remains unchanged but now it has the meaning of weights on the control increments. The concept of control horizon does not exist in LQ but can easily be added setting  $R_k = \infty$  for  $k \geq N_u$ .

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