Your Turn Solutions

Chapter 1

1.1 $\{1, 2, 3\}$, $\{x : x^3 - 5x^2 + x + 6 = 0\}$, the set of the first three positive integers, $\{x : x \in \mathbb{Z}, 1 \le x \le 3\}$, $\{x : x \in \mathbb{Z}, 0 < x < 4\}$ are some answers.

1.2 $(0, \infty)$, $(0, 1)$, $(1, 4]$, $(-\infty, 3)$.

1.3 24 has factors 1, 2, 3, 4, 6, 8, 12, 24; 15 has 1, 3, 5, 15; 13 has 1, 13. The common factors of 24 and 15 are 1, 3, so $(24, 15) = 3$.

1.4 $1, 0, 1, x^3, -0.5, 25$.

1.5 $t^{-2}/t^{-3} = t^3/t^2 = t$, $y^{5-2} = y^3$, $(4x^{-2})(3x^4) = 12x^{4-2} = 12x^2$.

1.6 $2, -2, 5, -4, 3.1, 4.4$.

1.7 If we replace each of x and y by 2, the equation becomes $6 - 4 = 4$, which is false. The other two suggested solutions lead respectively to $12 - 8 = 4$, which is true, and $9 - 2 = 4$, which is false. So (ii) is a solution, but (i) and (iii) are not.

1.8 First move the term x:

$$3x + 5 - x = 7.$$

Then move the 5:

$$3x - x = 7 - 5.$$

Gather terms:

$$2x = 2.$$

Finally, divide by 2:

$$x = 1.$$

W.D. Wallis, *A Beginner's Guide to Finite Mathematics*,
DOI 10.1007/978-0-8176-8319-1,

1.9 First move $\frac{2x}{3}$ to the left and -5 to the right:

$$\frac{3x}{2} - \frac{2x}{3} = 5.$$

Next, multiply by the common denominator 6:

$$6 \times \frac{3x}{2} - 6 \times \frac{2x}{3} = 6 \times 5,$$
$$9x - 4x = 30.$$

Finally, divide by 5:

$$x = 6.$$

1.10 We move x to the right-hand side:

$$-2y = 2y - 8 - x^2.$$

All y terms belong on the left:

$$-4y = -8 - x^2.$$

Multiply both sides by $-\frac{1}{4}$:

$$y = 2 + \tfrac{1}{4}x^2.$$

1.11 We rewrite the inequality as:

$$2x - x \leq 6 - 3$$

or

$$x \leq 3.$$

Solution set $(-\infty, 3]$.

1.12 We first rewrite the inequality as:

$$2x - 4x \leq 5 - 3$$

or

$$-2x \leq 2.$$

Dividing both sides by -2, we obtain $x \geq -1$, solution set $[-1, \infty)$.

1.14 $\sum_{i=3}^{5} i(i-1) = 3 \cdot 2 + 4 \cdot 3 + 5 \cdot 4 = 6 + 12 + 20 = 38$; $\sum_{i=2}^{6} i = 2 + 3 + 4 + 5 + 6 = 20$.

1.15 $1 + 3 + 5 + 7 + 9 = \sum_{i=1}^{5} 2i - 1$; $8 + 27 + 64 + 125 = \sum_{i=2}^{5} i^3$.

1.17 $3 + 7 + \cdots + 43 = \sum_{i=1}^{11} 4i - 1 = 4 \cdot \sum_{i=1}^{11} i - 11 = 4 \cdot \frac{12 \cdot 11}{2} - 11 = 264 - 11 = 253$.

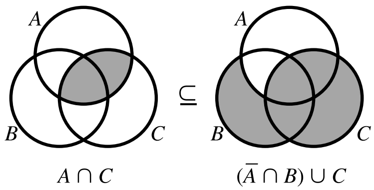
1.19 $\mathbb{Z}\backslash\mathbb{Z}^*$ consists of all the negative integers. $\mathbb{N} \subseteq \mathbb{Z}$, so $\mathbb{Z} \cap \mathbb{N} = \mathbb{N}$. $(\mathbb{N}\backslash\mathbb{E})$ contains just the odd positive integers, and all members of Π other than 2 are odd, so $(\mathbb{N}\backslash\mathbb{E}) \cup \Pi$ contains all the odd positive integers and 2. In symbols,

$$\begin{aligned}\mathbb{Z}\backslash\mathbb{Z}^* &= \{-1, -2, -3, -4, -5, -6, \ldots\},\\ \mathbb{Z} \cap \mathbb{N} &= \mathbb{N},\\ (\mathbb{N}\backslash\mathbb{E}) \cup \Pi &= \{1, 2, 3, 5, 7, 9, 11, \ldots\}.\end{aligned}$$

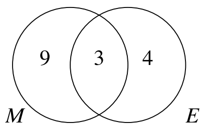
1.20 (i) The sets are not disjoint; their common element is 2. (ii) These sets are disjoint.

1.22 $\{(1, 1), (1, 4), (1, 5), (2, 1), (2, 4), (2, 5)\}$.

1.23

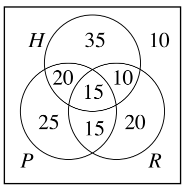


1.24 The figures are represented by the following diagram. As only readers were surveyed, there is no need for any "outside area" outside $M \cup E$.



From the diagram we see that 16 readers were surveyed.

1.26



20 like horror and police procedural movies, but do not like romances. 20 like romances only. 10 like none of these three types.

1.27 U has sum 49, so the mean is $49/7 = 7$. The mode is 7, the only reading to appear twice. To find the median, first write the readings in ascending order: 2, 3, 6, 7, 7, 9, 15. The "center" reading is 7, so the median is 7 also.

V has sum 117, so its mean is 19.5. Its median is 20 (after the readings are reordered as 14, 17, 19, 21, 22, 24, the two "center" values are 19 and 21, and we take the average of the two). There is no mode.

1.28 We define Y by the rule "$y_i = x_i - 830$". Then $Y = \{3, 4, 6, 7, 11, 11\}$; $m_Y = 42/6 = 7$, so $m_X = 7 + 830 = 837$.

1.29 The sum of the terms $x_i f_i$ is

$$2 \times 1 + 3 \times 3 + 4 \times 7 + 5 \times 11 + 6 \times 12 + 7 \times 4 + 8 \times 1 = 202$$

while the sum of the frequencies is

$$1 + 3 + 7 + 11 + 12 + 4 + 1 = 39$$

so the mean is $202/39 = 5.179\ldots$ or about 5.18. We would say the mean is "a little greater than 5".

1.31 The data add to 17, so the mean is $\frac{17}{7} = 2.\overline{428571}$. Using the formula, we find

$$\begin{aligned} 7s_X^2 &= \left[1^2 + 1^2 + 2^2 + 2^2 + 3^2 + 4^2 + 4^2\right] - 7\left(\frac{17}{7}\right)^2 \\ &= [1 + 1 + 4 + 4 + 9 + 16 + 16] - \frac{289}{7} \\ &= 51 - 41.2857 = 9.7143 \end{aligned}$$

so $s_X^2 = 1.38776$, and $s_X = 1.18$ (correct to two decimal places).

Chapter 2

2.2 Equation (2.1) yields

$$|S \cup T| = |S| + |T| - |S \cap T| = 42 + 32 - 22 = 52.$$

From (2.2) we get

$$|S \backslash T| = |S| - |S \cap T| = 42 - 22 = 20.$$

2.3 Each digit can be chosen in 10 ways. So there are 10^4 possibilities.

2.4 For the boys we now get $10 \times 9 \times 8 = 720$. For the girls we get $13 \times 12 \times 11 = 1716$. So the total is $720 \times 1716 = 1235520$.

2.5 $6! = 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$.

2.6 There are $9 \times 8 \times 7 = 504$ committees.

2.7 The boys can be ordered in $5! = 120$ ways. The girls can be ordered in $4! = 24$ ways. So there are $120 \times 24 = 2880$ arrangements.

2.8 $P(6, 4) = 6 \times 5 \times 4 \times 3 = 360$.

2.9 $P(12, 3)$, or 1320.

2.10 The boys can be ordered in $3! = 6$ ways, and the girls can be ordered in $4! = 24$ ways. As the table is circular, it doesn't matter whether the boys are to the left or to the right of the girls. So there are $3! \times 4! = 6 \times 24 = 144$ arrangements.

2.13 There are three A's, two N's and one A, for a total of six letters. So the number of orderings is $6!/(3! \times 2!) = 60$.

2.14 If unlimited numbers of each color were available, there would be $3^4 = 81$ solutions. It is necessary to exclude the solution with four blue marbles, $BBBB$. So the answer is $81 - 1 = 80$.

2.15 $C(9, 5) = \frac{9\times8\times7\times6}{4\times3\times2\times1} = 126$, $\binom{6}{0} = \frac{6!}{6!\times0!} = 1$.

2.16 She must choose 4 of the last 7 questions, so $\binom{7}{4} = 35$ ways.

2.17 There are $C(8, 4)$ choices of where to place the 1's, so the answer is $C(8, 4)$, or 70.

2.18 You can choose the mysteries in $\binom{5}{2}$ ways and the westerns in $\binom{7}{3}$ ways. So you can choose in $\binom{5}{2} \times \binom{7}{3} = 10 \times 35 = 350$ ways.

2.19 The three consonants can be chosen in $\binom{5}{3} = 10$ ways, and the vowels in $\binom{3}{2} = 3$ ways. After the choice is made, the letters can be arranged in $5! = 120$ ways. So there are $10 \times 3 \times 120 = 3600$ "words."

2.20 The committee contains one man or no men. With one man, the number of choices is $3 \times \binom{7}{2} = 63$ (3 ways to choose the man, $\binom{7}{2}$ to choose the women). With no men, there are $\binom{7}{3} = 35$ possibilities. So there are $63 + 35 = 98$ possibilities.

2.22 $\binom{81}{79} = \binom{81}{2} = \frac{81\times80}{1\times2} = 3240$.

2.23 $(x - y)^6 = \binom{6}{0}x^6 - \binom{6}{1}x^5y + \binom{6}{2}x^4y^2 - \binom{6}{3}x^3y^3 + \binom{6}{4}x^2y^4 - \binom{6}{5}xy^5 + \binom{6}{6}y^6 = 1 - 6x^5y + 15x^4y^2 - 20x^3y^3 + 15x^2y^4 - 6xy^5 + y^6$.

2.24 The term involving y^2 is $\binom{4}{2}(3x)^2(2y)^2$, so the numerical coefficient is

$$\binom{4}{2}3^22^2 = 6 \times 9 \times 4 = 216$$

and the coefficient of y^2 is $216x^2$.

2.25 We write $0.99^8 = (1 - 10^{-2})^8$. Then it equals

$$1 - 8 \times 10^{-2} + 28 \times 10^{-4} + 56 \times 10^{-6} + \cdots.$$

Every subsequent term in at most one-tenth of the one before it. So the approximate value is

$$1 - 0.08 + 0.0028 - 0.000056 + \cdots = 0.923 \text{ approx.}$$

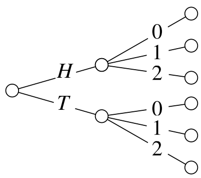
2.26 There are 2^{10} possible ways to choose a subset of the ten books. However, the subsets with 8, 9 or 10 elements are not allowed. So the number is

$$2^{10} - \binom{10}{10} - \binom{10}{9} - \binom{10}{8} = 1024 - 1 - 10 - 45 = 964.$$

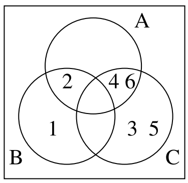
Chapter 3

3.1 $\{3, 4, 5, 6\}$.

3.3 Write H and T for heads and tails on the quarter, and 0, 1, 2 for the number of heads on the pennies. The sample space is $\{H0, H1, H2, T0, T1, T2\}$. The experiment consists of two parts. In the first, the outcomes are H and T; in the second, there are three outcomes, 1, 2, 3. (The two pennies are not flipped separately, so in the case of one head we don't need to worry about which penny got the head and which the tail. Only the number of heads was recorded.) The tree diagram is



3.4



3.5 $A \cap C = \{HHT, HTH\}$, $B \cup C = \{HHH, HHT, HTH, HTT, THT, TTH\}$, and $\overline{A} = \{HHH, HTT, THT, TTH\} = B$.

3.7 E consists of two possible rolls, so $P(E) = \frac{1}{3}$. similarly $P(F) = \frac{1}{2}$. G consists of the rolls 3 and 5, so $P(G) = \frac{1}{3}$.

3.8 Write HT to mean a head on the quarter and a tail on the nickel, and so on—the result for the quarter is shown first. Then there are four equally likely outcomes, HH, HT, TH, and TT, and two of them (HT and TH) are in the event. So

$$P(E) = \frac{|E|}{|S|} = \frac{2}{4} = \frac{1}{2}.$$

3.9 12 of the 52 cards are picture cards, so

$$P(E) = \frac{|E|}{|S|} = \frac{12}{52} = \frac{3}{13}.$$

3.10 The sample space is the set of five outcomes S_2, S_3, S_4, S_5, S_6, where S_j means that the sum is j. In terms of the slightly different experiment, in which we distinguish the two dice and record ab to mean a on die 1 and b on die 2, we have

$$\begin{aligned} S_2 &= \{11\}, \\ S_3 &= \{12, 21\}, \\ S_4 &= \{13, 22, 31\}, \\ S_5 &= \{23, 32\}, \\ S_6 &= \{33\}, \end{aligned}$$

and the new experiment has nine equally likely outcomes, so

$$P(S_2) = P(S_6) = \frac{1}{9}, \qquad P(S_3) = P(S_5) = \frac{2}{9}, \qquad P(S_4) = \frac{3}{9} = \frac{1}{3}.$$

3.11 Four of the marbles are blue and five are not blue, so

$$P(\text{not blue}) = \frac{|E|}{|S|} = \frac{5}{9}.$$

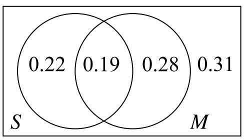
3.12

$$\begin{aligned} P(E \cup F) &= P(E) + P(F) - P(E \cap F) \\ &= 0.7 + 0.3 - 0.1 \\ &= 0.9, \\ P(\overline{E}) &= 1 - P(E) \\ &= 1 - 0.7 \\ &= 0.3. \end{aligned}$$

$E \cup \overline{F} = E \cup (\overline{E} \cap \overline{F}) = E \cup \overline{(E \cup F)}$, using De Morgan's laws. These two events are disjoint, so

$$\begin{aligned} P(E \cup \overline{F}) &= P(E) + P\big(\overline{(E \cup F)}\big) \\ &= P(E) + \big[1 - P(E \cup F)\big] \\ &= 0.7 + [1 - 0.9] \\ &= 0.8. \end{aligned}$$

3.13 The data are represented by



Since the percentages add to 100%, we can interpret "female" as "not male" (not always true in the insect kingdom) and the answers are (a) $P(S) = 41\%$; (b) $P(\overline{M}) = 53\%$; (c) $P(S \cup \overline{M}) = 72\%$.

3.14 There are three ways in which exactly one success can occur: the sequences SFF, FSF, and FFS. Now

$$P(SFF) = \frac{1}{3} \times \frac{2}{3} \times \frac{2}{3} = \frac{4}{27},$$
$$P(FSF) = \frac{2}{3} \times \frac{1}{3} \times \frac{2}{3} = \frac{4}{27},$$
$$P(FFS) = \frac{2}{3} \times \frac{2}{3} \times \frac{1}{3} = \frac{4}{27}.$$

So the probability of exactly two successes is

$$\frac{4}{27} + \frac{4}{27} + \frac{4}{27} = \frac{12}{27} = \frac{4}{9}.$$

3.15 In this case, each call is a Bernoulli trial with $p = \frac{1}{2}$, so the five calls in a day can be thought of as a binomial experiment with $p = \frac{1}{2}, n = 5$. So

$$P(3 \text{ successes}) = C(5,3)\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^2 = 10 \times \frac{1}{32} = \frac{10}{32},$$
$$P(4 \text{ successes}) = C(5,4)\left(\frac{1}{2}\right)^4\left(\frac{1}{2}\right)^1 = 5 \times \frac{1}{32} = \frac{5}{32},$$
$$P(5 \text{ successes}) = C(5,5)\left(\frac{1}{2}\right)^5\left(\frac{1}{2}\right)^0 = 1 \times \frac{1}{32} = \frac{1}{32},$$

and the probability of at least two successes is the sum of these:

$$\frac{10+5+1}{32} = \frac{16}{32} = \frac{1}{2},$$

or 50%.

3.16 In this case, the salesman wants

$$\frac{3^{n+1}(3+n)}{4^n} < \frac{3}{10}.$$

We have already seen that eight calls are not enough. But:

$$n = 9 \frac{3^{n-1}(3+n)}{4^n} = \frac{6561 \times 12}{262144} = 0.3003\ldots$$

which is very slightly more than 0.30. Strictly speaking 10 calls are needed, but the salesman would probably be satisfied with nine calls.

3.17 There are 11 marbles, so the number of ways of selecting three is $C(11, 3)$. So $|S| = C(11, 3) = 165$. The number of ways of selecting one red marble is 5, the number of ways of selecting one blue is 4, and the number of ways of selecting one green is 2. So, by the multiplication principle, $|E| = 5 \times 4 \times 2 = 40$. So

$$P(E) = \frac{40}{C(11, 3)} = \frac{8}{33}.$$

3.18 There are $C(10, 2) = 45$ ways to choose two balls. In $C(4, 2) = 6$ of these cases, there are no white balls—both are blue—so there are 39 selections with at least one white, so

$$P(E) = \frac{|E|}{|S|} = \frac{39}{45} = \frac{13}{15}.$$

3.19 Again, there are $C(52, 5)$ different hands possible. To count all the possible flushes, observe that four suits are possible. If one is chosen—Spades, say—there are $C(13, 5)$ possible flushes. So there ate $4 \times C(13, 5)$ flushes in total. So the probability is

$$P(E) = \frac{|E|}{|S|} = \frac{4 \times C(13, 5)}{C(52, 5)} = \frac{33}{16660},$$

or about 0.2%.

3.20 There are 18 class members, so the number of possible committees is $C(18, 3)$. The number of committees with all Math majors is $C(6, 3)$; there are $C(5, 3)$ with all Economics majors; and $C(7, 3)$ have all computer Science majors. The number of committees with all members in the same major is $C(6, 3) + C(5, 3) + C(7, 3)$, and this is $|E|$. So

$$P(E) = \frac{|E|}{|S|} = \frac{C(6, 3) + C(5, 3) + C(7, 3)}{C(18, 3)} = \frac{6 \cdot 5 \cdot 4 + 5 \cdot 4 \cdot 3 + 7 \cdot 6 \cdot 5}{18 \cdot 17 \cdot 16},$$

and this comes to $65/816$, or about 8%.

3.21 Suppose you took a pencil and labeled the faces with a 1 on die B as $1a$, $1b$, $1c$ and those with a 6 as $6a$, $6b$, $6c$. If Xy means die X was rolled and the face showing was a y, then there are 12 equally likely outcomes, $A1$, $A2$, $A3$, $A4$, $A5$, $A6$, $B1a$, $B1b$, $B1c$, $B6a$, $B6b$, $B6c$. Each has probability $\frac{1}{12}$. Four rolls, namely $A6$, $B6a$, $B6b$, $B6c$, result in a 6, so the probability of a 6 is $P(6) = \frac{4}{12} = \frac{1}{3}$, while 3 results from roll $A3$ only, and $P(3) = \frac{1}{12}$.

3.22 Given that the first card is a Spade, the second card is a random selection from 51 equally likely possibilities. Twelve of these outcomes are Spades and thirteen are Hearts. So

(i) $P(S \mid S) = \frac{|E|}{|S|} = \frac{12}{51}$;

(ii) $P(H \mid S) = \frac{|E|}{|S|} = \frac{13}{51}$.

3.23 We write S and N for “a spade” and “a card other than a spade”. Then

$$P(S \mid S) = \frac{12}{51},$$
$$P(K \mid N) = \frac{13}{51}.$$

Given that the first card is a spade, the second card is either a spade or not, so

$$P(S \mid S) + P(N \mid S) = 1,$$

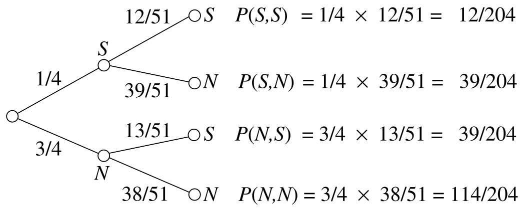
therefore

$$P(N \mid S) = \frac{39}{51},$$

and similarly

$$P(N \mid N) = \frac{38}{51}.$$

So the diagram is



(where, for example, “S, N” means “spade first, non-spade second”).

3.24 Write D for the event that at least one committee member is a Democrat, and B for the event that both are Democrats. There are $C(8, 2) = 28$ possible committees, and the number containing two Democrats is $C(3, 2) = 3$. So

$$P(B) = \frac{|B|}{|S|} = \frac{3}{28}.$$

Next, observe that there are $C(5, 2) = 10$ committees with both Republicans, so there are 15 possible committees with one member from each party. We can now proceed in two ways. Using the conditional probability formula, we observe that $P(D) = \frac{18}{28}$ and $P(B \cap D) = P(B) = \frac{3}{28}$ (if you think for a moment, you will realize that $B \cap D$ and B are the same event), so

$$P(B \mid D) = \frac{P(B \cap D)}{P(D)} = \frac{3}{18}.$$

Alternatively we can restrict ourselves to the new experiment “choose a committee of two at random from those committees with at least one Democrat”, so now $|S| = 18$, $|B| = 3$, and

$$P(B) = \frac{|B|}{|E|} = \frac{3}{18}.$$

3.25 In the obvious notation, $P(D) = \frac{2}{5}$ and $P(R \mid D) = \frac{1}{3}$, so

$$P(R \cap D) = P(R \mid D) \times P(D) = \frac{1}{3} \times \frac{2}{5} = \frac{2}{15}.$$

3.26 $P(A) = \frac{1}{6}$ and $P(B) = \frac{1}{2}$. We expect these events to be independent because the two rolls are unconnected, and they are: when considered as one experiment, there are 36 outcomes; there are three outcomes in $A \cap B$, namely 6 on the red and 2, 4 and 6 on the green, so $P(A \cap B) = \frac{3}{36} = \frac{1}{12}$, and this equals $P(A) \times P(B)$.

3.27 From the data,

$$\begin{aligned} P(A) &= \frac{3}{8}, \\ P(B) &= \frac{1}{8}, \\ P(C) &= \frac{1}{2}, \\ P(A \cap B) &= 0, \\ P(A \cap C) &= \frac{2}{8}, \\ P(B \cap C) &= 0. \end{aligned}$$

No two are independent.

3.28 We write F for faulty, and X and Y to indicate the suppliers. Then $P(X) = 0.4$, $P(F \mid X) = 0.04$, $P(Y) = 0.6$, $P(F \mid Y) = 0.02$, and

$$\begin{aligned} P(F) = P(F \cap X) + P(F \cap Y) &= P(F \mid X)P(X) + P(F \mid Y)P(Y) \\ &= 0.04 \times 0.4 + 0.02 \times 0.6 \\ &= 0.016 + 0.012 = 0.028. \end{aligned}$$

3.29 Use B, F, H, T for *biased, fair, heads, tails.* Then

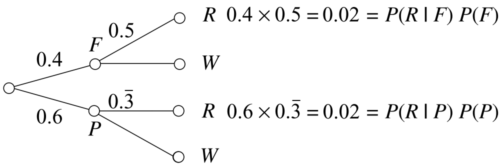
$$\begin{aligned} P(H) &= P(H \mid F)P(F) + P(H \mid B)P(B) \\ &= \left(\frac{1}{2}\right)\left(\frac{2}{3}\right) + \left(\frac{2}{3}\right)\left(\frac{1}{3}\right) \\ &= \frac{1}{3} + \frac{2}{9} = \frac{5}{9}; \\ P(B \cap H) &= P(H \mid B)P(B) = \frac{2}{9}. \end{aligned}$$

So $P(B \mid H) = \frac{2}{9}/\frac{5}{9} = \frac{2}{5}$.

3.30 Write E for the event that a blue pen is chosen. We know that $P(A) = P(B) = 0.5$. From Bayes' formula,

$$\begin{aligned} P(E)P(A \mid E) &= P(A)P(E \mid A), \\ 0.5 \times P(A \mid E) &= 0.5 \times 0.4, \\ P(A \mid E) &= 0.4. \end{aligned}$$

3.31



From the diagram, the denominator is 0.04. So

$$P(F \mid R) = \frac{P(R \mid F)P(F)}{0.04} = \frac{0.02}{0.04} = 0.5.$$

3.32 We want $P(D \mid V)$. From the diagram,

$$P(D \mid V) = \frac{0.112}{0.653} = 0.17.$$

3.33

	H	T	
B	$\frac{2}{9}$	$\frac{1}{9}$	$\frac{1}{3}$
U	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{2}{3}$
	$\frac{5}{9}$	$\frac{4}{9}$	

3.34 We use the abbreviations T (tests positive), N (tests negative), D (has the disease) and H (is healthy). We want to find $P(D \mid T)$. From the data, we know

$$\begin{aligned} P(T \mid D) &= 0.95, & P(N \mid D) &= 0.05, \\ P(T \mid H) &= 0.05, & P(N \mid H) &= 0.95, \\ P(D) &= 0.05, & P(H) &= 0.95. \end{aligned}$$

So

$$\begin{aligned} P(D \mid T) &= \frac{P(T \mid D)P(D)}{P(T \mid H)P(H) + P(T \mid D)P(D)} \\ &= \frac{(0.95)(0.05)}{(0.05)(0.95) + (0.95)(0.05)} \\ &= \frac{0.0475}{0.0475 + 0.0475} = \frac{0.0475}{0.095} = 0.5. \end{aligned}$$

This example shows that, in some cases, the test gives no useful information.

3.35 Write E_i for the event that i boys are chosen and define the value of E_i to be i. What is required is the expected value of this experiment. If S is the sample space, then

$$|S| = C(8, 3) = 56.$$

If a committee contains i boys, it must contain $3 - i$ girls. There are $C(3, i)$ ways of choosing i boys from the three, and $C(5, 3 - i)$ ways of selecting $3 - i$ girls, so

$$|E_i| = C(4, i)C(6, 3 - i).$$

Therefore,

$$\begin{aligned}
|E_0| &= C(3, 0) \times C(5, 3) = 10,\\
|E_1| &= C(3, 1) \times C(5, 2) = 30,\\
|E_2| &= C(3, 2) \times C(5, 1) = 15,\\
|E_3| &= C(3, 3) \times C(5, 0) = 1,
\end{aligned}$$

so

$$p_0 = \frac{10}{56}, \qquad p_1 = \frac{30}{56}, \qquad p_2 = \frac{15}{56}, \qquad p_3 = \frac{1}{56},$$

and

$$\begin{aligned}
\mu &= \frac{10 \times 0 + 30 \times 1 + 15 \times 2 + 1 \times 3}{56}\\
&= \frac{63}{56} = 1.125.
\end{aligned}$$

So the expected number is 1.125.

3.36 Let E_1 denote the event that total 3 or 9 is rolled, E_2 denote the event that an even total (2, 4, 6, 8, 10 or 12) is rolled, and E_0 the event that some other total is rolled. We write x_i for the payoff to gambler B when E_i occurs, so $x_1 = 5$, $x_2 = -2$, and $x_0 = 0$. Out of every 36 rolls of the dice, we expect total 3 twice, 9 four times, and an even total 18 times. We have

$$\begin{aligned}
&E_0, \text{ probability } \frac{1}{3}, \text{ value } x_0 = 0,\\
&E_1, \text{ probability } \frac{1}{6}, \text{ value } x_1 = 5,\\
&E_2, \text{ probability } \frac{1}{2}, \text{ value } x_2 = -2.
\end{aligned}$$

So the expected value of the payout is

$$\frac{1}{3} \times 0 + \frac{1}{6} \times 5 - \frac{1}{2} \times 2 = -\frac{1}{6}.$$

So A expects to win in the long run, at an average of $\$\frac{1}{6}$ per play.

3.37 With no information to the contrary, we assume that whether or not she bowls a strike in one frame does not affect her performance in other frames. So we can model her performance as a binomial experiment, in which each frame is a Bernoulli trial and success equates with a strike. We have $n = 40$, $p = \frac{1}{5}$, and $\mu = 40\frac{1}{5} = 8$.

Chapter 4

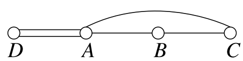
4.1 $s \leq t$ corresponds to $\{(1,1), (1,2), (1,3), (1,4), (2,2), (2,3), (2,4), (3,3), (3,4)\}$. $s \geq t$ corresponds to $\{(1,1), (2,1), (2,2), (3,1), (3,2), (3,3)\}$.

4.2 Both $=$ and $\geq$ are reflexive. $=$ is symmetric while $\geq$ antisymmetric. Both are transitive. So $=$ is an equivalence relation, but $\geq$ is not.

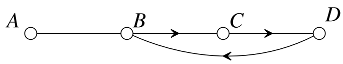
4.3 $f_1(f_1(x)) = f_1(x)$ so $f_1(f_1) = f_1$, $f_2(f_4(x)) = f_2(1/(1-x)) = 1/[1/(1-x)] = 1-x = f_3(x)$ so $f_2(f_4) = f_3$, $f_5(f_6(x)) = f_5((x-1)/x) = ((x-1)/x)/[((x-1)/x) - 1] = ((x-1)/x)/(-1/x) = (x-1)/(-1) = 1-x = f_3(x)$ so $f_5(f_6) = f_3$ also.

4.4 $f_3(x) = 1-x$. If $f_3(x) = y$ then $y = 1-x$ so $x = 1-y$ and $f_3^{-1}(y) = 1-y$. So $f_3^{-1} = f_3$.

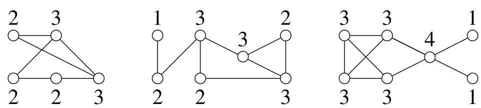
4.5



4.6



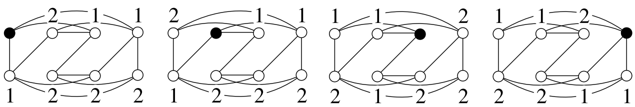
4.8



4.9 Suppose such a graph existed. Let x and y be the vertices of degrees 5 and 4, respectively. Then x is adjacent to all five other vertices, y being one of them. Therefore, y is adjacent to three vertices other than x. Of the remaining four vertices, at least three are adjacent both to x and to y, so they have degree at least 2. This is impossible.

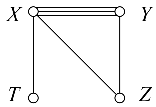
4.11 In a city's road network, it represents an intersection such that all traffic between two areas has to pass through that intersection. As such, it might get a traffic officer assigned in peak hours, or special priority might be given to servicing its traffic lights.

4.12 There are four different-looking vertices, shown in black. The distance of every other vertex from the black one is shown in each case. (The top left and second from left vertices are actually equivalent, but this is not easy to see.)



4.13 Look at the above diagram. The graph has $D = R = 2$.

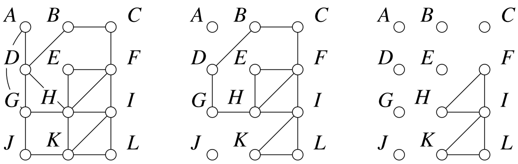
4.14 The walk must start (or finish) at T because there is only one bridge to that island. With a little experimentation we find the solution $TXYZXYX$.



4.15 The original network is shown in Figure (i). We start from A, and randomly choose the walk $AGJKHDA$. After these edges are deleted, Figure (ii) remains.

We now start at D (alphabetically, the first vertex remaining that was in the first walk and is not yet isolated). One walk is $DBCFEHGD$, and its deletion leaves Figure (iii).

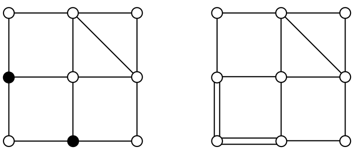
Finally, walk $FILKIHF$ uses up the remaining edges.



(i) (ii) (iii)

Putting these together, we get $AGJKHDBCFILKIHFEHGDA$.

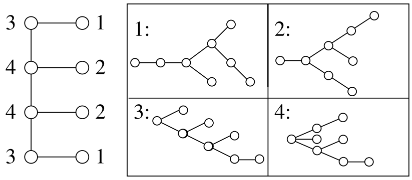
4.16 The original is shown on the left. The two black vertices need another edge. They are not joined, so one edge will not suffice. So $eu(H) > 1$. The right-hand shows an Eulerization that requires just two edges. So $eu(H) = 2$, and the right-hand picture is a good Eulerization.



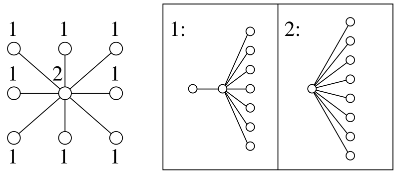
4.18 Graph (a) is not a tree because it is not connected; it has an isolated vertex. Graph (b) is a tree. Graph (c) contains a cycle, so it is not a tree.

4.19 There are four different-looking diagrams for the second tree and two for the third. We show which trees correspond to the different starting vertices using the same notation as in the sample problem.

B:



C:



4.20 If a tree has four vertices, then the largest possible degree is 3. Moreover, there are three edges (by Theorem 13), so from Theorem 10 the sum of the four degrees is 6. As there are no vertices of degree 0 and at least two vertices of degree 2, the list of degrees must be one of 3, 1, 1, 1 or 2, 2, 1, 1. In the first case, the only solution is the star $K_{1,3}$. The only case with the second degree list is the path P_4. So there are two trees.

4.22 Candidates (i) and (iv) are Hamiltonian. Candidate (ii) is not Hamiltonian because it contains a repeated vertex, b. Candidate (iii) is not Hamiltonian because the graph contains no edge cd.

4.23 To traverse vertex a, a Hamiltonian cycle in G must contain one of the paths bad, bae or ead. If bad is included, the other edge through d might be dc or de; in the former case, neither bc nor de can be edges, and the only cycle is $badcfe$, while the latter case bars be, and the only cycle is $badefc$. If bae is included, ad is not an edge, so cd and de are edges, so we have the path $baedc$, and the cycle is $baedcf$. If ead is included, de is not an edge, so dc is an edge, and there are two possibilities, $adcbfe$ and $adcfbe$. So there are five Hamiltonian cycles.

Any Hamiltonian cycle in H must contain edges ab and ad, because a has degree 2. This means bd is not an edge (it would form a triangle), so de must be in the cycle. There are two ways to finish a cycle: $bcfe$ or $bfce$. So there are two cycles: $dabcfe$ and $dabfce$.

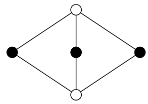
4.24 The nearest neighbor algorithm, starting from Evansville, begins with EM, because it has the least cost of the three edges incident with E. The next edge must have M as an endpoint, and ME is not allowed (one cannot return to E, it has already been used), so the cheaper of the remaining edges is chosen, namely MN. The only available edge from N is NS, as E and M have already been visited, and the route is EMNSE, with cost \$430.

Starting at Nashville, the first edge selected is NE, with cost \$90. The next choice is EM, then MS, then SN, and the resulting cycle NEMSN costs \$410. If you start at St. Louis, the first stop will be Memphis (\$110 is the cheapest flight from St. Louis), then Evansville, then Nashville, costing \$410. From Memphis, the cheapest leg is to Evansville, then Nashville, and finally St. Louis, for \$410. So both St. Louis and Memphis yield the same cycle as the Nashville case (with different starting points, and in reverse).

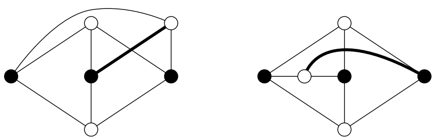
To apply the sorted edges algorithm, first sort the edges in order of increasing cost: EM(\$80), EN(\$90), NM(\$100), MS(\$110), ES (\$120), NS(\$130). Edge EM is included, and so is EN. The next choice would be MN, but this is not allowed because its inclusion would complete a cycle of length 3 (too short), so the only other choices are MS and NS, forming route EMSNE (or ENSME) at a cost of \$410.

In this example, the route ENMSE, with cost \$420, does not arise from either algorithm.

4.25 The graph $K_{2,3}$ can be drawn without crossings, as the illustration shows, and any representation of $K_{2,3}$ without crossings will look like this picture. Now $K_{3,3}$ can be constructed from $K_{2,3}$ by adding one vertex adjacent to the three black vertices. In the representation, this new vertex could be placed outside the diagram or inside one of the enclosed areas.

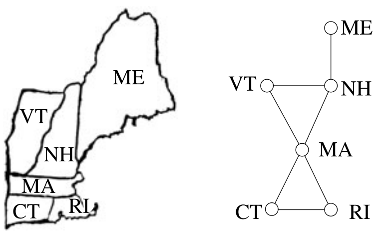


If it is placed outside, then the edge joining it to the central vertex must cross another edge. If it is placed inside one enclosed area, then the edge joining it to the vertex not on that enclosed area must cross an edge. That is, one of the following cases must occur (the thick line represents the offending edge). In either case, there is a crossing, so $\nu(K_{3,3}) > 0$.



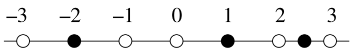
These representations show $K_{3,3}$ with one crossing, so $\nu(K_{3,3}) = 1$.

4.26

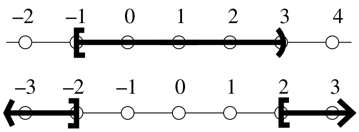


Chapter 5

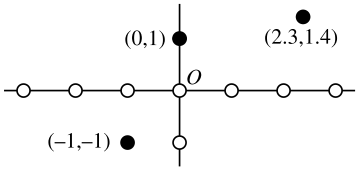
5.1



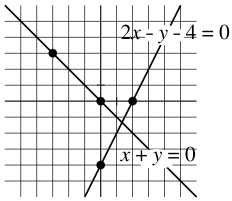
5.2



5.3



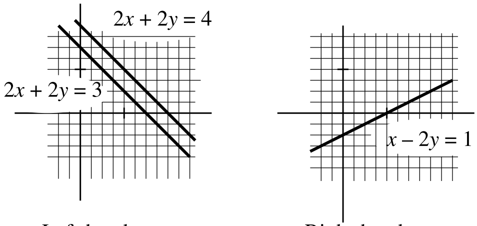
5.5



5.6 The line $2x + y - 4 = 0$ has slope -2. Its slope-intercept form is $y = -2x + 4$. The line $x + y = 0$ has slope -1 and slope-intercept form $y = -x$.

5.7 From $3x - y = 1$ we get $y = 3x - 1$. Substituting, $2x + (3x - 1) = 4$, or $5x = 5$, so $x = 1$. Therefore, $y = 3(1) - 1 = 2$. The solution is $x = 1$, $y = 2$; the point of intersection is $(1, 2)$.

5.8 In the first case, $2x+2y$ cannot equal both 3 and 4, so there are no solutions, and the solution set is empty. In the second system, whenever the first equation is true, the second will be true also: each side of the second equation is 2 times the corresponding side of the first one. The solution set could be written $\{(x, y) \mid x - 2y = 1\}$ or $\{(2y + 1, y) \mid y \in \mathbb{R}\}$. The graphs are



5.9 We would like to eliminate y from the first equation. The coefficient of y in the second equation is -1, so we multiply both sides of the second equation by 3 and proceed to solve the equations

$$4x + 3y = 11,$$
$$3x - 3y = 3.$$

Adding the equations, we get

$$7x = 14,$$

so $x = 2$. Substituting this back into the second equation,

$$2 - y = 1,$$

so $y = 1$, and the solution is $(x, y) = (2, 1)$.

5.13 We eliminate x from the first and third equations. We add $-3\times$ the second equation to the first equation and $-2\times$ the second equation to the third equation; those two equations become

$$-y - z = -2,$$
$$-3y - 3z = -6,$$

so $y + z = 2$. The equations are equivalent to

$$y + z = 2,$$
$$x + y = 3.$$

The solution is $x = 3 - y$, $z = 2 - y$, for any real number y, or

$$(x, y, z) \in \{(3 - y, y, 2 - y) \mid y \in \mathbb{R}\}.$$

5.14 We eliminate z from the second equation by adding $-1\times$ the first equation to the second equation, obtaining

$$x - 5y = -2,$$

or $x = 5y - 2$.The first equation yields $z = x + 2y - 5 = 7y - 7$, and the solution is

$$(x, y, z) \in \{(5y - 2, y, 7y - 7) \mid y \in \mathbb{R}\}.$$

5.15 The augmented matrix is

$$\left[\begin{array}{ccc|c} 4 & 3 & -2 & 1 \\ 3 & -2 & 4 & 6 \\ 2 & -3 & 2 & 8 \end{array}\right].$$

Its (2, 2) element equals −2.

5.18

$$\left[\begin{array}{cccc|c} 2 & -3 & -1 & -2 & 1 \\ 1 & 2 & -2 & 1 & 1 \\ 1 & -1 & 1 & 1 & 4 \end{array}\right],$$

$$\left[\begin{array}{cccc|c} 0 & -7 & 3 & -4 & -1 \\ 1 & 2 & -2 & 1 & 1 \\ 0 & -3 & 3 & 0 & 3 \end{array}\right] \quad \begin{array}{l} R1 \leftarrow R1 - 2R2 \quad \text{(using E3)}, \\ \\ R3 \leftarrow R3 + R2 \quad \text{(using E3)}, \end{array}$$

$$\left[\begin{array}{cccc|c} 1 & 2 & -2 & 1 & 1 \\ 0 & 1 & -1 & 0 & -1 \\ 0 & -7 & 3 & -4 & -1 \end{array}\right] \quad \begin{array}{l} R1 \leftarrow R2 \quad \text{(using E1)}, \\ R2 \leftarrow -\frac{1}{3}R3 \quad \text{(using E1, E2)}, \\ R3 \leftarrow R1 \quad \text{(using E1)}, \end{array}$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 1 & 3 \\ 0 & 1 & -1 & 0 & -1 \\ 0 & 0 & -4 & -4 & -8 \end{array}\right] \quad \begin{array}{l} R1 \leftarrow R1 - 2R2 \quad \text{(using E3)}, \\ \\ R3 \leftarrow R3 + 7R2 \quad \text{(using E3)}, \end{array}$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 1 & 3 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 2 \end{array}\right] \quad \begin{array}{l} R2 \leftarrow R2 - \frac{1}{4}R3 \quad \text{(using E3)}, \\ R3 \leftarrow -\frac{1}{4}R3 \quad \text{(using E2)}. \end{array}$$

So the solution (in terms of t) is

$$\begin{aligned} x &= 3 - t, \\ y &= 1 - t, \\ z &= 2 - t, \end{aligned}$$

with t any real number.

To solve in terms of x, pivot on position $(1, 4)$:

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 1 & 3 \\ -1 & 1 & 0 & 1 & -2 \\ -1 & 0 & 1 & 0 & -1 \end{array}\right] \quad \begin{aligned} y &= -2 + x, \\ z &= -1 + x, \\ t &= 3 - x. \end{aligned}$$

To solve in terms of y, pivot on position $(2, 4)$:

$$\left[\begin{array}{cccc|c} 1 & -1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & -1 & 1 & 0 & 1 \end{array}\right] \quad \begin{aligned} x &= 2 + y, \\ z &= 1 + y, \\ t &= 1 - y. \end{aligned}$$

To solve in terms of z, pivot on position (3, 4):

$$\left[\begin{array}{cccc|c} 1 & 0 & -1 & 0 & 1 \\ 0 & 1 & -1 & 0 & -1 \\ 0 & 0 & 1 & 1 & 2 \end{array}\right] \quad \begin{aligned} x &= 1 + z, \\ y &= -1 + z, \\ t &= 2 - z. \end{aligned}$$

5.19 Write S for sedans and P for pickups.

	S	P
Apr	32	16
May	44	12

5.20 The matrix has shape 4×3. Its third row is

$$\boxed{\begin{matrix} -1 & 4 & 6 \end{matrix}}$$

and its first column is

$$\boxed{\begin{matrix} 1 \\ 2 \\ -1 \\ 1 \end{matrix}}.$$

5.21 We have the two equations $2x = y$ and $2 = x$. The second gives $x = 2$, so the first gives $y = 2 \cdot 2 = 4$.

5.22 $-A = \left[\begin{smallmatrix} -1 & -3 \\ 1 & -2 \end{smallmatrix}\right]$, $3A - B = \left[\begin{smallmatrix} 5 & 9 \\ -4 & 2 \end{smallmatrix}\right]$, $B + C$ is not defined, as B and C are of different sizes.

5.24

$$B^T = \begin{bmatrix} 2 & 2 \\ 3 & 1 \end{bmatrix}.$$

5.25 $4(2, 0, -1) = (8, 0, -4)$, so $4(2, 0, -1) + (1, 4, -3) = (8, 0, -4) + (1, 4, -3) = (9, 4, -7)$.

5.27 $\boldsymbol{u} \cdot \boldsymbol{t} = (-1, 3, 0) \cdot (1, 2, 3) = (-1 \cdot 1) + (3 \cdot 2) + (0 \cdot 3) = (-1) + 6 + 0 = 5$; $(2\boldsymbol{u} - 3\boldsymbol{v}) \cdot \boldsymbol{t} = [(-2, 6, 0) + (-6, 6, -6)] \cdot (1, 2, 3) = (-8, 12, -6) \cdot (1, 2, 3) = (-8 \cdot 1) + (24 \cdot 2) + (-6 \cdot 3) = -8 + 12 - 18 = -2$.

5.29

$$\begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \end{bmatrix}; \qquad \begin{bmatrix} 1 & -2 \\ 2 & -4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}.$$

5.30

$$\begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} w_1 & b_1 & p_1 \\ w_2 & b_2 & p_2 \\ w_3 & b_3 & p_3 \\ w_4 & b_4 & p_4 \end{bmatrix} \begin{bmatrix} 15 \\ 18 \\ 20 \end{bmatrix}.$$

5.31

$$CD = DC = \begin{bmatrix} 5 & 3 \\ -3 & -1 \end{bmatrix}.$$

5.32 Suppose C has inverse

$$E = \begin{bmatrix} x & y \\ z & t \end{bmatrix}.$$

Then $CE = I$, so

$$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x & y \\ z & t \end{bmatrix} = \begin{bmatrix} x & y \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

The (2, 2) entries of the two matrices cannot be equal ($0 \neq 1$), so no such E exists. Similarly, if D has inverse

$$\begin{bmatrix} x & y \\ z & t \end{bmatrix},$$

then

$$\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x & y \\ z & t \end{bmatrix} = \begin{bmatrix} z & t \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},$$

and again we have the impossible equation $0 = 1$.

5.33 Suppose the inverse is

$$C = \begin{bmatrix} x & z \\ y & t \end{bmatrix}.$$

Then $BC = I$ means

$$\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x & z \\ y & t \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},$$

which is equivalent to the four equations

$$\begin{aligned} 2x + y &= 1, \qquad & 2z + t &= 0, \\ x + y &= 0, \qquad & z + t &= 1. \end{aligned}$$

The left-hand pair of equations are easily solved to give $x = 1$ and $y = -1$, while the right-hand pair give $z = -1$ and $t = 2$. So the inverse exists, and is

$$A = \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix}.$$

5.34 For A: $R3 \leftarrow R3 - R1$; $R2 \leftrightarrow R3$; $R1 \leftarrow R1 - 2R2$, $R3 \leftarrow R3 + 2R2$; $R1 \leftarrow R1 + R3$. For B: $R1 \leftrightarrow R3$; $R2 \leftarrow R2 - 2R1$, $R3 \leftarrow R3 - 3R1$; $R3 \leftarrow R3 - 2R2$.

5.35

$$\det(C) = 1 \cdot 4 - 1 \cdot 3 = 1; \qquad C^{-1} = \begin{bmatrix} 4 & -1 \\ -3 & 1 \end{bmatrix},$$

$$\det(D) = 2 \cdot 2 - 4 \cdot 1 = 0; \qquad \text{no inverse.}$$

5.36 As observed in the Sample Problem, the matrix of coefficients has inverse

$$\begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & -1 \\ 2 & 1 & -2 \end{bmatrix}.$$

Now

$$\begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & -1 \\ 2 & 1 & -2 \end{bmatrix} \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \\ 4 \end{bmatrix},$$

so the first system has solution $x = 4$, $y = 1$, $z = 4$.

$$\begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & -1 \\ 2 & 1 & -2 \end{bmatrix} \begin{bmatrix} 4 \\ -1 \\ 3 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix},$$

and the second system has solution $x = 3$, $y = 1$, $z = 1$.

5.38 The technology matrix is

$$T = \begin{bmatrix} 0.3 & 0.2 \\ 0.5 & 0.5 \end{bmatrix}$$

and

$$(I - T) = \begin{bmatrix} 0.7 & -0.2 \\ -0.5 & 0.5 \end{bmatrix}$$

which has determinant $0.35 - 0.10 = 0.25$. So

$$(I - T)^{-1} = 4 \begin{bmatrix} 0.5 & 0.2 \\ 0.5 & 0.7 \end{bmatrix}^{-1} = \begin{bmatrix} 2 & 0.8 \\ 2 & 2.8 \end{bmatrix}$$

so

$$P = \begin{bmatrix} 2 & 0.8 \\ 2 & 2.8 \end{bmatrix} \begin{bmatrix} 25 \\ 50 \end{bmatrix} = \begin{bmatrix} 90 \\ 190 \end{bmatrix}.$$

So the required production is 90 tons of steel and 190 tons of vegetables.

Chapter 6

6.2 The quantity to be maximized is profit. The profit depends only on the number of acres of each crop, potatoes, corn, and beans. So we define variables:

$$x_1 = \text{number of acres of potatoes to be planted,}$$
$$x_2 = \text{number of acres of corn,}$$
$$x_3 = \text{number of acres of beans.}$$

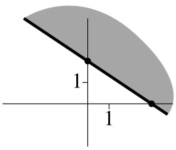
The profit is $\$P$, where $P = 150x_1 + 50x_2 + 80x_3$.
The farmer's outlay will be $\$(500x_1 + 250x_2 + 400x_3)$, and this cannot exceed \$30 000, and the total number of acres $(x_1 + x_2 + x_3)$ cannot exceed 120. If $x_3 > 20$, the excess beans will not be sold. So we have

$$\begin{aligned} \text{Maximize} \quad & P = 150x_1 + 50x_2 + 80x_3 \\ \text{subject to} \quad & 500x_1 + 250x_2 + 400x_3 \le 30000, \\ & x_1 + x_2 + x_3 \le 120, \\ & x_3 \le 20, \\ & x_1 \ge 0, \quad x_2 \ge 0, \quad x_3 \ge 0. \end{aligned}$$

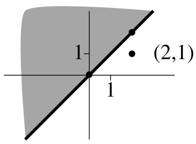
6.5 $(1, 0)$ is feasible, $P = 1$. $(4, 4)$ is not feasible ((A) and (B) are both false), $P = 8$. $(2, 1)$ is feasible, $P = 3$. $(-1, 1)$ is not feasible (x is negative), $P = 0$.

6.6 For $2x + 3y \ge 6$, we test with $(0, 0)$, and find that it does not satisfy the inequality; putting $x = 0$, $y = 0$ results in $0 \ge 6$, which is false. So we take the half-plane not containing the origin.

For $x - y \le 0$, $(0, 0)$ is not a suitable test point, so we try $(2, 1)$. This does not satisfy the inequality. The point $(2, 1)$ is also shown as a dot in the diagram. Both solution-sets are shown in gray.



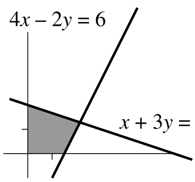
$2x + 3y \ge 6$



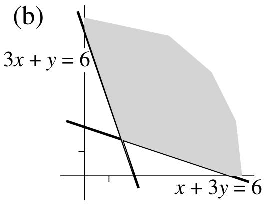
$x - y \le 0$

6.8

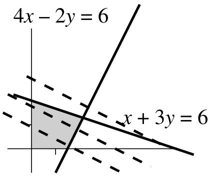
(a)



(b)

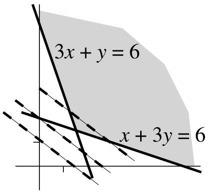


6.10



Some level curves are shown. The maximum clearly occurs at the intersection point, $x = \frac{15}{7}$, $y = \frac{9}{7}$, where $P = \frac{33}{7}$.

6.11



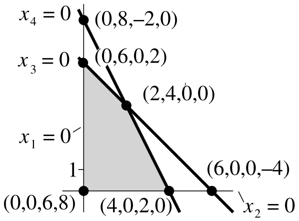
Some level curves are shown. The minimum clearly occurs at the intersection point, $x = \frac{3}{2}$, $y = \frac{3}{2}$, where $C = \frac{21}{2}$.

6.13

$$\begin{aligned} \text{Maximize} \quad & P = 3x_1 + 3x_2 + 0x_3 + 0x_4 \\ \text{subject to} \quad & x_1 + 3x_2 + x_3 + 0x_4 = 2, \\ & 2x_1 - x_2 + 0x_3 - x_4 = 1, \\ & x_1 \geq 0, \quad x_2 \geq 0, \quad x_3 \geq 0, \quad x_4 \geq 0. \end{aligned}$$

6.14

$$\begin{aligned} \text{Maximize} \quad & P = x_1 + 2x_2 + 0x_3 + 0x_4 \\ \text{subject to} \quad & x_1 + x_2 + x_3 + 0x_4 = 6, \\ & 2x_1 + x_2 + 0x_3 + x_4 = 8, \\ & x_1 \geq 0, \quad x_2 \geq 0, \quad x_3 \geq 0, \quad x_4 \geq 0. \end{aligned}$$



6.15

Non-basic variables	Basic variables	Basic solution	Feasible?	P
x_1, x_2	x_3, x_4	(0, 0, 6, 8)	Yes	0
x_1, x_3	x_2, x_4	(0, 6, 0, 2)	Yes	12
x_1, x_4	x_2, x_3	(0, 8, −2, 0)	No	16
x_2, x_3	x_1, x_4	(6, 0, 0, −4)	No	6
x_2, x_4	x_1, x_3	(4, 0, 2, 0)	Yes	4
x_3, x_4	x_1, x_2	(2, 4, 0, 0)	Yes	10

The problem is bounded (from the diagram in Problem 6.14, above), so the optimum value is $P = 12$, attained at $x_1 = 0$, $x_2 = 6$.

6.16 We need a form in which x_2 has coefficient 1 in one equation and 0, in the other, while x_3 plays the same role with the equations reversed. This can be achieved by adding a multiple of the second equation to the first. Adding $-2\times$ equation 2 to equation 1, the resulting equations are

$$-5x_1 + 0x_2 + x_3 - 2x_4 = -2,$$
$$3x_1 + x_2 + 0x_3 + x_4 = 3.$$

6.18 (i) The entry a_{12} is chosen because the ratio $\frac{4}{2} = 2$ is smaller than $\frac{6}{1} = 6$.

(ii) a_{12} is used because the ratio is negative.

6.19 Originally the basic feasible solution (4, 0, 6, 0) was exhibited.

$$\left[\begin{array}{c|cccc|c} \text{BV} & x_1 & x_2 & x_3 & x_4 & b \\ \hline x_1 & 1 & \underline{2} & 0 & 4 & 4 \\ x_3 & 0 & 1 & 1 & 4 & 6 \end{array}\right],$$

$$\left[\begin{array}{c|cccc|c} \text{BV} & x_1 & x_2 & x_3 & x_4 & b \\ \hline x_2 & \frac{1}{2} & 1 & 0 & 2 & 2 \\ x_3 & -\frac{1}{2} & 0 & 1 & 2 & 4 \end{array}\right] \quad \begin{array}{l} R1 \to \frac{1}{2}R1, \\ R2 \to R2 - \frac{1}{2}R1. \end{array}$$

The basic feasible solution (0, 2, 4, 0) is now exhibited.

6.20

$$\left[\begin{array}{c|ccccc|c} \text{BV} & x_1 & x_2 & x_3 & x_4 & P & b \\ \hline x_3 & 4 & 2 & 1 & 0 & 0 & 6 \\ x_4 & 2 & 4 & 0 & 1 & 0 & 6 \\ \hline & -2 & -3 & 0 & 0 & 1 & 0 \end{array}\right].$$

6.22

$$\left[\begin{array}{c|ccccc|c} \text{BV} & x_1 & x_2 & x_3 & x_4 & P & b \\ \hline x_1 & 1 & \frac{1}{2} & \frac{1}{4} & 0 & 0 & \frac{3}{2} \\ x_4 & 0 & 3 & -\frac{1}{2} & 1 & 0 & 3 \\ \hline & 0 & -2 & \frac{1}{2} & 0 & 1 & 3 \end{array}\right].$$

6.24 We pivot on a_{42}.

$$\left[\begin{array}{c|ccccc|c} \text{BV} & x_1 & x_2 & x_3 & x_4 & P & b \\ \hline x_1 & 1 & 0 & \frac{1}{3} & -\frac{1}{6} & 0 & 1 \\ x_2 & 0 & 1 & -\frac{1}{6} & \frac{1}{3} & 0 & 1 \\ \hline & 0 & 0 & \frac{1}{6} & \frac{2}{3} & 1 & 5 \end{array}\right].$$

The solution is $P = 5$, obtained by putting $x_1 = 1$, $x_2 = 1$.

6.25

$$\begin{aligned} \text{Maximize} \quad & P = -2x_1 + 3x_2 \\ \text{subject to} \quad & -2x_1 + 2x_2 \leq 5, \\ & 3x_1 + x_2 \leq 4, \\ & x_1 \geq 0, \quad x_2 \geq 0. \end{aligned}$$

6.26

Step 1. The problem is a maximization. We insert slack variables, obtaining

$$\begin{aligned} \text{Maximize} \quad & P = 5x_1 + 3x_2 + 0x_3 + 0x_4 \\ \text{subject to} \quad & 2x_1 + 2x_2 + x_3 + 0x_4 = 4, \\ & 3x_1 + x_2 + 0x_3 + x_4 = 7, \\ & x_1 \geq 0, \quad x_2 \geq 0, \quad x_3 \geq 0, \quad x_4 \geq 0. \end{aligned}$$

Step 2. The initial tableau is

$$\left[\begin{array}{c|ccccc|c} \text{BV} & x_1 & x_2 & x_3 & x_4 & P & b \\ \hline x_3 & 2 & 2 & 1 & 0 & 0 & 4 \\ x_4 & 3 & 1 & 0 & 1 & 0 & 7 \\ \hline & -5 & -3 & 0 & 0 & 1 & 0 \end{array}\right].$$

Step 3. Column x_1 is a suitable pivot column.

Step 4. We pivot on entry a_{31}, obtaining

$$\left[\begin{array}{c|ccccc|c} \text{BV} & x_1 & x_2 & x_3 & x_4 & P & b \\ \hline x_1 & 1 & 1 & \frac{1}{2} & 0 & 0 & 2 \\ x_4 & 0 & -2 & -\frac{3}{2} & 1 & 0 & 1 \\ \hline & 0 & 2 & \frac{5}{2} & 0 & 1 & 10 \end{array}\right].$$

There is now no pivot. The maximum value is $P = 10$, obtained by putting $x_1 = 2$, $x_2 = 0$.

6.27

$$2x_1 - 2x_2 + 0x_3 + x_4 + 0x_5 + 0A_1 + 0A_2 = 4,$$
$$x_1 + 4x_2 + 0x_3 + 0x_4 - x_5 + A_1 + 0A_2 = 4,$$
$$x_1 + 2x_2 + x_3 + 0x_4 + 0x_5 + 0A_1 + A_2 = 7,$$
$$x_1 \geq 0, \quad x_2 \geq 0, \quad x_3 \geq 0, \quad x_4 \geq 0, \quad x_5 \geq 0, \quad A_1 \geq 0, \quad A_2 \geq 0.$$

6.29

BV	x_1	x_2	x_3	x_4	x_5	A_1	A_2	P	b
x_4	2	−2	0	1	0	0	0	0	4
A_1	1	4	0	0	−1	1	0	0	4
A_2	1	2	1	0	0	0	1	0	7
	4	−3	0	0	0	M	M	1	0

,

BV	x_1	x_2	x_3	x_4	x_5	A_1	A_2	P	b
x_4	2	−2	0	1	0	0	0	0	4
A_1	1	4	0	0	−1	1	0	0	4
A_2	1	2	1	0	0	0	1	0	7
	4	−3	0	0	0	0	0	1	0
	$-2M$	$-6M$	$-M$	0	M	0	0	0	$-11M$

.

6.30 We pivot on row A_1, column x_2. We add $6M$ times the new row to the lower part of the objective row and 3 times the row to the upper part.

BV	x_1	x_2	x_3	x_4	x_5	A_1	A_2	P	b
x_4	$\frac{5}{2}$	0	0	1	$-\frac{1}{2}$	$\frac{1}{2}$	0	0	6
x_2	$\frac{1}{4}$	1	0	0	$-\frac{1}{4}$	$\frac{1}{4}$	0	0	1
A_2	$\frac{1}{2}$	0	1	0	$\frac{1}{2}$	$-\frac{1}{2}$	1	0	5
	$\frac{19}{4}$	0	0	0	$-\frac{3}{4}$	$\frac{3}{4}$	0	1	3
	$-\frac{1}{2}M$	0	$-M$	0	$-\frac{1}{2}M$	$\frac{3}{2}M$	0	0	$-5M$

.

Next we pivot on row A_2, column x_3:

BV	x_1	x_2	x_3	x_4	x_5	A_1	A_2	P	b
x_4	$\frac{5}{2}$	0	0	1	$-\frac{1}{2}$	$\frac{1}{2}$	0	0	6
x_2	$\frac{1}{4}$	1	0	0	$-\frac{1}{4}$	$\frac{1}{4}$	0	0	1
x_3	$\frac{1}{2}$	0	1	0	$\frac{1}{2}$	$-\frac{1}{2}$	1	0	5
	$\frac{19}{4}$	0	0	0	$-\frac{3}{4}$	$\frac{3}{4}$	0	1	3
	0	0	0	0	0	M	M	0	0

.

6.31 The new tableau is

BV	x_1	x_2	x_3	x_4	x_5	P	b
x_4	$\frac{5}{2}$	0	0	1	$-\frac{1}{2}$	0	6
x_2	$\frac{1}{4}$	1	0	0	$-\frac{1}{4}$	0	1
x_3	$\frac{1}{2}$	0	1	0	$\frac{1}{2}$	0	5
	$\frac{19}{4}$	0	0	0	$-\frac{3}{4}$	1	3

We pivot on row x_3, column x_5, obtaining

BV	x_1	x_2	x_3	x_4	x_5	P	b
x_4	3	0	1	1	0	0	11
x_2	$\frac{1}{2}$	1	$\frac{1}{2}$	0	0	0	$\frac{7}{2}$
x_3	1	0	2	0	1	0	10
	$\frac{11}{2}$	0	$\frac{3}{2}$	0	0	1	$\frac{21}{2}$

We have $P = \frac{21}{2}$, so $C = -\frac{21}{2}$, at $x_1 = 0$, $x_2 = \frac{7}{2}$.

Chapter 7

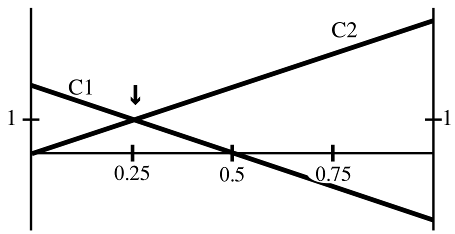
7.2 Game (i) has no dominant row or column. In (ii), column 2 is dominant—remember, a *smaller* result is better for the column player. The column player's strategy is (0, 1) and the row player's strategy is (1, 0).

7.3 The array is

				MIN
	2	2	3	∗2∗
	3	1	2	1
	−2	0	1	−2
MAX	3	∗2∗	3	

so there is a saddle point R1C2 with value 2.

7.4 The diagram is



The lines meet at $(\frac{1}{4}, 1)$. So the value is 1 and the row player's strategy is $(\frac{3}{4}, \frac{1}{4})$. For the column player, $(1 - z) + 0z = 1$ so $z = \frac{1}{2}$ and the column strategy is $(\frac{1}{2}, \frac{1}{2})$.

7.5 For this game $a = 2, b = -1, c = -2, d = 1$. So $a + d - b - c = 2 + 1 + 1 + 2 = 6$, $d - c = 3$, $a - b = 3$, $d - b = 2$, $a - c = 4$, $ad - bc = 0$. So the row strategy is

$$\left(\frac{d-c}{a+d-b-c}, \frac{a-b}{a+d-b-c}\right) = \left(\frac{3}{6}, \frac{3}{6}\right) = \left(\frac{1}{2}, \frac{1}{2}\right).$$

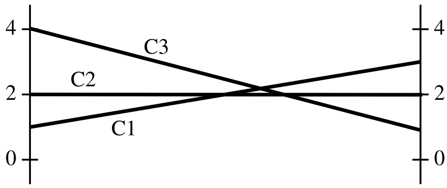
The column strategy is

$$\left(\frac{d-b}{a+d-b-c}, \frac{a-c}{a+d-b-c}\right) = \left(\frac{2}{6}, \frac{4}{6}\right) = \left(\frac{1}{3}, \frac{2}{3}\right).$$

The value is

$$v = \frac{ad - bc}{a+d-b-c} = \frac{0}{6} = 0.$$

7.7 From the diagram



it is clear that column 2 can be deleted. The solution to

1	4
3	1

is found from the formula to be row strategy $(\frac{2}{5}, \frac{3}{5})$, column strategy $(\frac{3}{5}, \frac{2}{5})$, value $(\frac{11}{5})$, so the original game has solution

$$\text{row strategy}\left(\frac{2}{5}, \frac{3}{5}\right), \qquad \text{column strategy}\left(\frac{3}{5}, 0, \frac{2}{5}\right), \qquad \text{value } \frac{11}{5}.$$

7.8 We first solve

2	−3	−2
−2	0	−1

This game was solved in Sample Problem 7.7; it has solution

$$\text{row strategy}\left(\frac{1}{5}, \frac{4}{5}\right), \qquad \text{column strategy}\left(\frac{1}{5}, 0, \frac{4}{5}\right), \qquad \text{value } -\frac{6}{5}.$$

So the original has solution

$$\text{row strategy } \left(\frac{1}{5}, 0, \frac{4}{5}\right), \qquad \text{column strategy } \left(\frac{1}{5}, \frac{4}{5}\right), \qquad \text{value } \frac{6}{5}.$$

7.10 We add 3; the revised game is

$$\begin{array}{|ccc|}\hline 4 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 4 \\ \hline\end{array}\,.$$

For this, we start with the tableau

$$\left[\begin{array}{c|ccccccc|c} \text{BV} & y_1 & y_2 & y_3 & y_4 & y_5 & y_6 & P & b \\ \hline y_4 & 4 & 2 & 1 & 1 & 0 & 0 & 0 & 1 \\ y_5 & 2 & 4 & 2 & 0 & 1 & 0 & 0 & 1 \\ y_6 & 1 & 2 & 4 & 0 & 0 & 1 & 0 & 1 \\ \hline & -1 & -1 & -1 & 0 & 0 & 0 & 1 & 0 \end{array}\right].$$

We pivot on row y_5, column y_2:

$$\left[\begin{array}{c|ccccccc|c} \text{BV} & y_1 & y_2 & y_3 & y_4 & y_5 & y_6 & P & b \\ \hline y_4 & 3 & 0 & 0 & 1 & -\frac{1}{2} & 0 & 0 & \frac{1}{2} \\ y_2 & \frac{1}{2} & 1 & \frac{1}{2} & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{4} \\ y_6 & 0 & 0 & 3 & 0 & -\frac{1}{2} & 1 & 0 & \frac{1}{2} \\ \hline & -\frac{1}{2} & 0 & -\frac{1}{2} & 0 & \frac{1}{4} & 0 & 1 & \frac{1}{4} \end{array}\right].$$

The next pivot is the (y_4, y_1) position:

$$\left[\begin{array}{c|ccccccc|c} \text{BV} & y_1 & y_2 & y_3 & y_4 & y_5 & y_6 & P & b \\ \hline y_1 & 1 & 0 & 0 & \frac{1}{3} & -\frac{1}{6} & 0 & 0 & \frac{1}{6} \\ y_2 & 0 & 1 & \frac{1}{2} & -\frac{1}{6} & \frac{1}{3} & 0 & 0 & \frac{1}{12} \\ y_6 & 0 & 0 & 3 & 0 & -\frac{1}{2} & 1 & 0 & \frac{1}{2} \\ \hline & 0 & 0 & -\frac{1}{2} & \frac{1}{6} & \frac{1}{6} & 0 & 1 & \frac{1}{3} \end{array}\right].$$

The final pivot is (y_6, y_3):

$$\left[\begin{array}{c|ccccccc|c} \text{BV} & y_1 & y_2 & y_3 & y_4 & y_5 & y_6 & P & b \\ \hline y_1 & 1 & 0 & 0 & \frac{1}{3} & -\frac{1}{6} & 0 & 0 & \frac{1}{6} \\ y_2 & 0 & 1 & 0 & -\frac{1}{6} & \frac{5}{12} & -\frac{1}{6} & 0 & \frac{1}{12} \\ y_3 & 0 & 0 & 1 & 0 & -\frac{1}{6} & \frac{1}{3} & 0 & \frac{1}{6} \\ \hline & 0 & 0 & 0 & \frac{1}{6} & \frac{1}{12} & \frac{1}{6} & 0 & \frac{5}{12} \end{array}\right].$$

So the revised game has value $\frac{12}{5}$, and the original game has value $-\frac{3}{5}$. The column player's strategy is $\frac{12}{5}(\frac{1}{6}, \frac{1}{12}, \frac{1}{6}) = (\frac{2}{5}, \frac{1}{5}, \frac{2}{5})$, and the row player's strategy is also $(\frac{2}{5}, \frac{1}{5}, \frac{2}{5})$.

Chapter 8

8.1 Use the formula $A = P(1 + nR/100)$ with $P = 1200$ and $R = 10$. In the first case, n is $\frac{1}{4}$, so

$$P = 1200 \times (1 + 0.10/4) = 1200 \times (1.025) = 1230$$

and you repay \$1230. In the second case, n is 3, so

$$P = 1200 \times (1 + 0.10 \times 3) = 1200 \times (1.3) = 1560,$$

and you repay \$1560.

8.2 From $A = P(1 + nR/100)$ with $A = 1428$, $P = 1400$ and $n = \frac{1}{3}$, we get

$$1428 = 1400 \times \left(1 + \frac{R}{300}\right) = 1400 + \frac{14}{3} \times R,$$

and therefore $14 \times R = 28 \times 3$, $R = 6$. So the interest rate was 6%.

8.4 We have $P = 2200$, $A = 2530$ and $n = 2$. So

$$2530 = 2200\left(1 + 2 \times \frac{R}{100}\right) = 2200 + 44R; \quad R = \frac{330}{44} = 7.5.$$

The rate was 7.5%.

8.5 Simple interest is 3% per month, or 9% for the whole period. This comes to \$54, so the total to be paid is \$654. There are three monthly payments. Therefore, the monthly payment will be \$654/3, or \$218.

8.8 For arithmetic growth,

$$1200\left(1 + \frac{10 \times 4}{100}\right) = 1200 \times 1.4 = 1680$$

so the interest is \$480. Under geometric growth, the amount is

$$1200\left(1 + \frac{10}{100}\right)^4 = 1200 \times 1.1^4 = 1200 \times 1.4641 = 1756.92$$

and the interest is \$556.92.

8.10 In this case $P = 40000$, $R = 12$, $t = 4$ and $N = 15$. So

$$P\left(1 + \frac{R}{t \times 100}\right)^{tN}$$

becomes

$$40000 \times \left(1 + \frac{12}{400}\right)^{60} = 40000 \times (1.03)^{60} = 0 \times 5.8916$$

which comes to 235664.12 . . . , and you owe \$235664.

8.11 The amount owing from a \$100 loan at the end of one year is

$$\begin{aligned} A &= 100\left(1 + \frac{12}{12 \times 100}\right)^{12} \\ &= 100 \times 1.01^{12} \\ &= 112.68, \end{aligned}$$

so the APY is 12.68%.

8.12 Again, 6% annual interest is 0.5% per month, so $m = 0.005$, $D = 100$, but in this case $n = 12$, so you get

$$\begin{aligned} &\$100(1.005^{10} - 1.005)/0.005 \\ &\quad = \$100(1.05114 - 1.005) \times 200 \\ &\quad = \$20000(0.04614) \\ &\quad = \$922.80. \end{aligned}$$

8.13 We use the formula

$$P(1+m)^n = \frac{D \times [(1+m)^n - 1]}{m}$$

with $P = 12000$, $n = 60$ and $M = \frac{2}{3}$, $(1+m) = \frac{151}{150}$. Now

$$\left(\frac{151}{150}\right)^{60} = 1.4858457\ldots,$$

so

$$12000 \times \left(\frac{151}{150}\right)^{60} = \frac{D \times [(\frac{151}{150})^{60} - 1]}{\frac{1}{150}}$$

becomes

$$12000 \times 1.4858457 = 150 \times D \times 0.4858457;$$

that is,

$$D = 12000 \times 1.4858457/(150 \times 0.4858457) = 244.676,$$

and your payment is \$244.68.

8.14 The interest rate is $1/150$ per month. So the accumulation after 24 years at 8% is

$$\$100000 \times (151/150)^{288}$$

or \$677763.55. If you deposit \$$D$ per month, your accumulated payments would be

$$\frac{\$D \times [(151/150)^{288} - 1)]}{1/150} = \$D \times 5.77763554 \times 150 = \$D \times 866.645.$$

So your monthly payment is $D = 782.05$.

8.15 $m = 7.5/12\% = 5/8\% = 1/160$, $D = 200$, $n = 60$. So

$$mA(1+m)^n = D \times [(1+m)n - 1]$$

becomes

$$A(161/160)^{60}/160 = 200[(161/160)^{60} - 1].$$

So $A \times 1.45329 = 160 \times 200 \times 0.45329$, $A = 9980.99$, and you can afford about \$9981.

8.16 In this case $P = 40000$, $R = 12$, $N = 15$ and $r = 0.12$. So

$$Pe^{rN} = 40000 \times e^{1.8} = 241985.89\ldots,$$

and you owe \$241986. Compare this with the result of Your Turn problem 8.10, where compounding was quarterly and the answer was \$235664.

8.17 Suppose the rate is r. Then $40000e^{7r} = 80000$ so $e^{10r} = 2$. Therefore, e^r equals the seventh root of 2, which equals 1.1041 approximately. The APR is approximately 10.4%.

8.18

$$\frac{118.3}{198.7} \times \$225000 = \$133958.23.$$

8.20 After one day, you have 120 grams; after two, 60; after three, 30. So after four days there are 15 grams remaining.

Answers to Exercises A

Section 1.1

1. (i) T; (ii) F; (iii) F; (iv) T; (v) F; (vi) F; (vii) T; (viii) T; (ix) T; (x) F. **2.** (i) 2, 4, 6, 8, 10; (ii) Red, White, Blue; (iii) Sunday, Saturday. **3.** (i) Only. **5.** (i) $\mathbb{Q}$, $\mathbb{R}$; (ii) $\mathbb{Q}$, $\mathbb{R}$; (iii) $\mathbb{Z}$, $\mathbb{Q}$, $\mathbb{R}$; (iv) $\mathbb{R}$; (v) $\mathbb{N}$, $\mathbb{Z}$, $\mathbb{Q}$, $\mathbb{R}$; (vi) $\mathbb{R}$. **7.** (i) 1, 2, 3, 4, 6, 9, 12, 18, 36; (ii) 1, 2, 3, 4, 6, 8, 12, 16, 24, 48; (iii) 1, 2, 5, 10, 25, 50; (iv) 1, 3, 9, 27, 81; (v) 1, 2, 3, 4, 6, 8, 9, 12, 18, 24, 36, 72; (vi) 1, 61. **8.** (i) $2^2 \cdot 3^2 \cdot 29$; (ii) $2^2 \cdot 67$; (iii) 2^8; (iv) $3^2 \cdot 37$. **9.** (i) $2^3 \cdot 11, 2^2 \cdot 3 \cdot 11, 44$; (ii) $2^8, 2^5 \cdot 7, 32$; (iii) $2^3 \cdot 3^3 \cdot 5, 3^2 \cdot 5 \cdot 19, 45$; (iv) $2^3 \cdot 3 \cdot 7, 3 \cdot 7 \cdot 11, 21$. **10.** (i) 1; (ii) $4x^4/y^6$; (iii) xy; (iv) $2 \cdot f^3$ or 250; (v) $2b$; (vi) $41/(x+y)^3$; (vii) a^2; (viii) $\frac{1}{2}q$. **11.** (i) 117; (ii) 45; (iii) 28; (iv) -107; (v) 11.4; (vi) 2; (vii) 27; (viii) -73; (ix) 1; (x) -8.

Section 1.2

1. (i) Yes; (ii) No; (iii) No; (iv) Yes; (v) Yes; (vi) No. **2.** (i) 6; (ii) 0; (iii) 1; (iv) 5; (v) -2; (vi) 2; (vii) 6; (viii) $\frac{24}{17}$; (ix) $-\frac{1}{2}$; (x) -1; (xi) 2; (xii) $\frac{13}{2}$. **3.** (i) $\frac{2}{3}x - \frac{3}{2}$; (ii) $3 - x$; (iii) $x - 2$; (iv) $\frac{1}{3}x^2 + 1$; **4.** (i) $x > -\frac{3}{5}$; (ii) $x > 1$; (iii) $x \le 2$; (iv) $x \ge \frac{6}{5}$; (v) $x \le -1$; (vi) $x < \frac{5}{2}$; (vii) $x \ge \frac{1}{2}$; (viii) $x > 0$. **5.** (i) $y \le 2 - 2x$; (ii) $y \le 3 - 2x$; (iii) $y > 4x$; (iv) $y > \frac{1}{2}x - \frac{3}{2}$; (v) $y \ge \frac{1}{2}x - 2$; (vi) $y > 1 + x$.

Section 1.3

1. (i) $2 + 5 + 10 + 17 + 26 + 37 = 97$; (ii) $0.1 + 0.01 + 0.001 = 0.111$; (iii) $0 + 4 + 10 + 18 + 28 + 40 + 54 = 154$; (iv) $\frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} = \frac{77}{60}$; (v) $4 - 8 + 16 = 12$; (vi) $0+2+0+2 = 4$; (vii) $1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} = \frac{25}{12}$; (viii) $4+7+10+13+16+19 = 69$. **2.** (i) $\sum_{i=1}^{5} 3i - 2$; (ii) $\sum_{i=1}^{3} 4i - 2$; (iii) $\sum_{i=0}^{4} (-1)^i(3i+1)$; (iv) $\sum_{i=0}^{4} (-1)^i i^2$; (v) $\sum_{i=1}^{6} 5 + 2(-1)^i$; (vi) $\sum_{i=1}^{4} i^2 - 1$. **3.** (i) $\sum_{i=1}^{n} b_i = (1^3 - 0^3) + (2^3 - 1^3) + (3^3 - 2^3) + \cdots + ((n-1)^3 - (n-2)^3) + (n^3 - (n-1)^3) = n^3 - 0^3 = n^3$;

W.D. Wallis, *A Beginner's Guide to Finite Mathematics*,
DOI 10.1007/978-0-8176-8319-1, © Springer Science+Business Media, LLC 2012

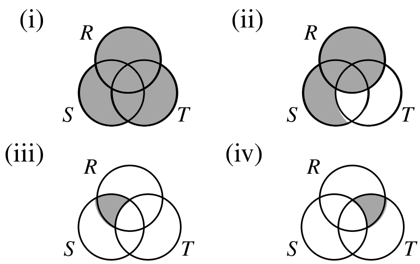
(ii) $b_i = i^3-(i-1)^3 = i^3-i^3+3i^2-3i+1 = 3i^2-3i+1$, so $\sum_{i=1}^n b_i = 3\sum_{i=1}^n i^2 - 3\sum_{i=1}^n i + \sum_{i=1}^n 1 = 3\sum_{i=1}^n i^2 - 3\sum_{i=1}^n i + n$; (iii) We know $\sum_{i=1}^n b_i = n^3$, so $n^3 = 3\sum_{i=1}^n i^2 - 3\sum_{i=1}^n i + n$. Using the formulae for $\sum_{i=1}^n i$, $n^3 = 3\cdot\sum_{i=1}^n i^2 - 3\cdot\frac{1}{2}n(n+1)+n$ so $\cdot\sum_{i=1}^n i^2 = \frac{1}{3}[n^3+3\cdot\frac{1}{2}n(n+1)-n]$. Simplifying gives the result. **4.** (i) $\sum_{i=1}^n c_i = \sum_{i=1}^n (2a_i+1) = \sum_{i=1}^n 2a_i + \sum_{i=1}^n 1 = 2A+n$; (ii) $3A-B$; (iii) $A+B+2n$; (iv) $A+B-1$ if n is odd; $A+B$ if n is even. **5.** (i) 39; (ii) 971; (iii) 124; (iv) $\frac{1}{6}n(n^2+3n+7)$; (v) $\frac{1}{2}(3^{n+1}-3)$; (vi) $n^2(2n+3)$; (vii) $n(n+1)(2n+1)/6-2\times n(n+1)/2 = n(n+1)(2n-5)/6$; (viii) $(1-\frac{1}{2}^n)/(1-\frac{1}{2})-1 = 1-1/2^n$.

Section 1.4

1. (i) 3, 5, 7, 9; (ii) Sunday, Monday, Tuesday, Wednesday, Thursday, Friday, Saturday; (iii) 12, 15, 18; (iv) 2, 3, 5, 7; (v) 2; (vi) 1, 2, 3, 4, 6, 9, 12, 18, 36. **2.** (i) Set of multiples of 3 between 1 and 10; (ii) Set of all positive multiples of 3; (iii) Solution set of $x^2+9x=10$; (iv) Set of perfect squares less than 10; (v) Solution set of $x^2-4x+3=0$. **3.** (i) a, b, c, d, e, g, i; (ii) c, e; (iii) b, d; (iv) a, b, c, d, e, g. **4.** (i) $1, 2, 3, 4, 5, 6, 7, 8, 9$; (ii) $1, 2, 4, 5, 6, 7, 9$; (iii) $1, 5$; (iv) $1, 5$. **5.** (i) $\{1, 3, 5\}$; (ii) $\{5, 6, 7\}$; (iii) $\{1, 3, 6, 7\}$; (iv) $\{1, 2, 3, 4, 5, 6, 7, 8\}$; (v) $\emptyset$; (vi) $\{1, 2, 3, 4\}$. **6.** (i) $S_3 \subseteq S_1$, $S_4 \subseteq S_1$, $S_4 \subseteq S_3$; (ii) $S_1 \cap S_2 = \{2\}$, $S_1 \cap S_3 = \{2, 3, 5, 7\}$, $S_1 \cap S_4 = \{2, 3, 5\}$, $S_1 \cap S_5 = \{2\}$, $S_2 \cap S_3 = \{2\}$, $S_2 \cap S_4 = \{2\}$, $S_2 \cap S_5 = \{2, 4, 8\}$, $S_3 \cap S_4 = \{2, 3, 5\}$, $S_3 \cap S_5 = \{2\}$, $S_4 \cap S_5 = \{2\}$. **7.** (i) No; set of all squares of multiples of 5; (ii) No; S. **8.** (i) (a,a), (a,c), (a,f), (b,a), (b,c), (b,f), (c,a), (c,c), (c,f); (ii) $(2,2)$, $(2,3)$, $(2,4)$, $(2,5)$, $(-2,2)$, $(-2,3)$, $(-2,4)$, $(-2,5)$; (iii) $(2,4)$, $(2,5)$, $(3,4)$, $(3,5)$; **9.** (i) $(1,3,5)$, $(2,3,5)$, $(1,4,5)$, $(2,4,5)$, $(1,3,6)$, $(2,3,6)$, $(1,4,6)$, $(2,4,6)$; (ii) $(x,z,1)$, $(y,z,1)$, $(x,z,2)$, $(y,z,2)$.

Section 1.5

1.



4. (i) 367; (ii) 189; (iii) 871. **5.** 25. **6.** 2. **7.** (i) 5; (ii) 10; (iii) 15; (iv) no. **8.** (i) 10; (ii) 24; (iii) 10. **9.** (i) 5; (ii) 20; (iii) 18; (iv) 17.

Section 1.6

1. (i) 22.25; 21; (ii) 16.4; 13; (iii) 19.5; 20.5. **2.** (i) 5.5; 2; 3.5; (ii) 3.125; 6; 5.5; (iii) 6.5; no mode; 8. **3.** 5; −3, −2, 0, −3, 2, 4, 2. **4.** (i) 4.796; (ii) 4.256; (iii) 4.272. **5.** 3.692; 1.727.

Section 2.1

1. 22; 11. **2.** 6; 16. **3.** 18; 36. **4.** 12; 10. **5.** 7; 5. **6.** (i) 350; (ii) 475. **7.** 50; 150. **8.** $2^8 = 256$. **9.** (i) 12; (ii) 14. **10.** (i) $26^3 \times 10^3 = 17576000$; (ii) $24^3 \times 10^3 = 13824000$. **11.** ABC, ACB, BAC, BCA, CAB, CBA. **12.** 144. **13.** (i) $3^8 = 6561$; (ii) $4^8 = 65536$. **14.** $5! = 120$. **15.** $5! = 120$. **16.** $10 \times 9 \times 8 = 720$. **17.** (i) $10! = 3628800$; (ii) $6! \times 4! = 17280$.

Section 2.2

1. (i) 336; (ii) 24; (iii) 120; (iv) 72. **2.** $26 \times 26 \times 26 = 17576$. **3.** (i) $4! = 24$; (ii) $3! \times 2 = 12$. **4.** (i) $5! = 120$; (ii) $4! \times 2 = 48$. **5.** (i) $5! \times 3! = 720$; (ii) $4! \times 4! = 576$. **6.** (i) $9!/2!2!2! = 45360$; (ii) $9!/2!2! = 90720$; (iii) $6!/2! = 360$; (iv) $10!/3!2!2! = 151200$. **7.** (i) 15!; (ii) $3! \times 4! \times 5! \times 6! = 24883200$. **8.** (i) $8 \times 10^5 = 800000$; (ii) $8 \times P(9, 5) = 120960$.

Section 2.3

1. $\{A, B, C\}$, $\{A, B, D\}$, $\{A, B, E\}$, $\{A, C, D\}$, $\{A, C, E\}$, $\{A, D, E\}$, $\{B, C, D\}$, $\{B, C, E\}$, $\{B, D, E\}$, $\{C, D, E\}$. **2.** (i) 56; (ii) 126; (iii) 20; (iv) 35; (v) 1; (vi) 28. **3.** $C(16, 4) = 1820$. **4.** $C(6, 2) \times C(12, 3) = 3300$. **5.** $C(12, 5) = 792$; $C(9, 3) = 84$. **6.** (i) $C(49, 5)$; (ii) 44. **7.** $C(5, 3) \times C(7, 3) = 350$. **8.** 20.

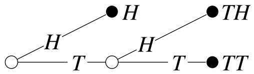
Section 2.4

2. (i) 60; (ii) $40y^2$; (iii) $-84x^2$; (iv) $-6x$. **3.** (i) $x^4 - 4x^3 + 6x^2 - 4x + 1$; (ii) $1 - 10z + 40z^2 - 80z^3 + 80z^4 - 32z^5$; (iii) $x^4 + 4x^2 + 6 + 4x^{-2} + x^{-4}$; (iv) $x^3 + y^3 + z^3 + 3x^2y + 3x^2z + 3xy^2 + 3xz^2 + 3y^2z + 3yz^2 + 6xyz$; (v) $x^2 + y^2 + z^2 + 2xy - 2xz - 2yz$. **4.** 1.030301. **5.** 1.05101. **6.** $2^7 = 128$.

Section 3.1

1. (i) $\{1H, 1T, 2H, 2T, 3H, 3T, 4H, 4T, 5H, 5T, 6H, 6T\}$; (ii) $\{2H, 2T, 4H, 4T, 6H, 6T\}$. **2.** (i) $\{BBB, BBG, BGB, BGG, GBB, GBG, GGB, GGG\}$; (ii)(a) $E = \{BBB, BBG, BGB, BGG\}$, $F = \{BBG, BGB, GBB\}$. **3.** (ii) 4; (iii) $\{F, PF\}$. **4.** (i) {22, 21, 12, 20, 11, 02, 10, 01, 00}; (ii) $E = \{21, 20, 10\}$, $F = \{20, 11, 02\}$,

$G = \{22, 20, 11, 02, 00\}$; (iii) $E \cup F = \{21, 20, 10, 11, 02\}$, $E \cap F = \{20\}$, $E \cap G = \{20\}$, $\overline{F} \cap G = \{22, 00\}$; (iv) $E \cap F$: there are exactly two heads on quarters and none on nickels, $\overline{F} \cap G$: the tosses are either all heads or all tails. **5.** (i)



(ii) $\{H, TH, TT\}$; (iii) $\{TH, TT\}$. **6.** (i) $\overline{E}$; (ii) $E \cap F$; (iii) $(E \cup F) \cap \overline{(E \cap F)}$; (iv) $E \cup F$; (v) $\overline{E} \cap \overline{F}$; (vi) $E \backslash F$. **7.** (i) Yes; (ii) No; (iii) No; (iv) Yes. **8.** Yes.

Section 3.2

1. (i) Yes; (ii) No (total > 1); (iii) Yes; (iv) No (there is a negative probability). **2.** (i) $\frac{1}{2}$; (ii) $\frac{1}{2}$; (iii) $\frac{1}{4}$. **3.** (i) $\frac{9}{19}$; (ii) $\frac{9}{19}$; (iii) $\frac{1}{19}$; (iv) $\frac{1}{38}$; (v) $\frac{6}{19}$. **4.** (i) $\frac{1}{9}$; (ii) $\frac{1}{2}$; (iii) $\frac{1}{2}$. **5.** $\frac{5}{12}$. **6.** 0.1. **7.** (i) $\frac{1}{2}$; (ii) $\frac{1}{10}$; (iii) $\frac{9}{100}$. **8.** (i) 12, 13, 14, 15, 22, 23, 24, 25, 32, 33, 34, 35, 42, 43, 44, 45; (ii) $\frac{3}{16}$; (iii) $\frac{1}{2}$. **9.** (i) $\frac{1}{5}$; (ii) $\frac{2}{5}$; (iii) $\frac{2}{5}$. **10.** (i) $\{R, R, R\}$, $\{R, R, B\}$, $\{R, B, B\}$, $\{B, B, B\}$; (ii) $\frac{1}{56}$, $\frac{15}{56}$, $\frac{30}{56}$, $\frac{10}{56}$, respectively.

Section 3.3

1. 0.3; 0.8; 0.2. **2.** 0.3, 0.3, 0.1, 0.7. **3.** 0.4; 0.9; 0.9. **4.** (i) 0.3, 0.8, 0.8; (ii) 0.2; 0.3. **5.** (i) 0.225; (ii) 0.325. **6.** 0.4. **7.** (i) 0.625; (ii) 0.35; (iii) 0.125. **8.** (i) 0.5; (ii) 0.25; (iii) 0.2. **9.** (i) $\frac{8}{81}$; (ii) $\frac{48}{81}$. **10.** $\binom{20}{16}/2^{20}$ (about 1 chance in 200). **11.** (i) $8(\frac{7}{8})^7\frac{1}{8}$ (about 39%); (ii) $8(\frac{7}{8})^7\frac{1}{8}+(\frac{7}{8})^8$ (about 74%). **12.** $\frac{160}{729}$ (about 22%). **13.** $\frac{5}{16}$. **14.** (i) $\frac{8}{27}$; (ii) $\frac{1}{27}$.

Section 3.4

1. (i) $\frac{1}{15}$; (ii) $\frac{11}{15}$. **2.** (i) $\frac{1}{55}$; (ii) $\frac{14}{55}$. **3.** (i) $\frac{1}{3}$; (ii) $1-\frac{9\cdot 9\cdot 8\cdot 7}{9000} = 0.496$. **4.** $\frac{3}{10}$. **5.** (i) $\frac{2}{5}$; (ii) $\frac{8}{15}$. **6.** $10 \times 4^5/C(52, 5)$. **7.** (i) $\frac{1}{26}$; (ii) $\frac{63}{130}$; (iii) $\frac{1}{2}$. **8.** $C(5, 2)\cdot C(4, 2)/[C(9, 4)-6] = \frac{1}{2}$. **9.** (i) $\frac{1}{6}$; (ii) $\frac{1}{3}$. **10.** $[\binom{6}{3}+\binom{4}{3}]/\binom{10}{3} = \frac{1}{5}$. **11.** (i) $\frac{1}{10}$; (ii) $\frac{9}{10}^5 = 0.59049$ (about 60%). **12.** (i) $\frac{1}{30}$; (ii) $\frac{1}{30}$; (iii) $\frac{1}{6}$. **13.** $\frac{57}{115}$ (about 50%).

Section 3.5

1. (ii) $\frac{2}{5}$, $\frac{4}{15}$, $\frac{4}{15}$, $\frac{1}{15}$, respectively; (iii) $\frac{8}{15}$. **2.** (ii) $P(WM, WF) = \frac{1}{10}$, $P(WM, GM) = \frac{1}{15}$, $P(WF, WM) = \frac{1}{10}$, $P(WF, WF) = \frac{1}{5}$, $P(WF, GM) = \frac{1}{5}$, $P(GM, WM) = \frac{1}{15}$, $P(GM, WF) = \frac{1}{5}$, $P(GM, GM) = \frac{1}{15}$; (iii) $\frac{1}{5}$; (iv) $\frac{8}{15}$; (v) $\frac{1}{3}$. **3.** (i) $P(HHH) = \frac{28}{1105}$, $P(HHM) = P(HMH) = P(MHH) = \frac{72}{1105}$, $P(HMM) = P(MHM) = P(MMH) = \frac{168}{1105}$, $P(MMM) = \frac{357}{1105}$; (ii) $\frac{244}{1105}$. **4.** (ii) $\frac{13}{18}$. **5.** (i) $\frac{17}{35}$; (ii) $\frac{11}{35}$; (iii) $\frac{1}{5}$. **6.** $\frac{1}{3}$. **7.** $\frac{1}{3}$, 1, $\frac{2}{11}$, $\frac{1}{3}$, respectively. **8.** $\frac{25}{102}$; $\frac{15}{34}$; $\frac{25}{51}$; $\frac{25}{77}$; $\frac{32}{51}$; $\frac{45}{77}$. **9.** (i) $\frac{1}{4}$; (ii) $\frac{3}{14}$. **10.** (i) $\frac{2}{5}$; (ii) $\frac{1}{5}$.

Section 3.6

1. (i) $\frac{3}{4}$; $\frac{3}{7}$; (ii) 1; $\frac{3}{7}$; (iii) $\frac{2}{3}$; $\frac{2}{3}$; (iv) $\frac{1}{3}$; $\frac{1}{3}$. **2.** (i) 0.9; 0.4; (ii) 0.85; 0.35; (iii) 0.97; 0.63. **3.** (i) (b) $\frac{1}{4}$; $\frac{1}{4}$; $\frac{12}{51}$; $\frac{12}{51}$; $\frac{3}{51}$; (c) No; (ii) (b) $\frac{1}{4}$; $\frac{1}{4}$; $\frac{1}{4}$; $\frac{1}{4}$; $\frac{1}{16}$; (c) Yes. **4.** (i) $\frac{2}{5}$; (ii) $\frac{3}{7}$; (iii) $\frac{5}{7}$; (iv) $\frac{9}{25}$. **5.** (i) 0; (ii) 0.5. **6.** (i) $\frac{1}{3}$; (ii) $\frac{2}{3}$; (iii) $\frac{1}{7}$; (iv) $\frac{6}{7}$; $\frac{1}{2}$. **7.** (i) $\frac{4}{9}$; (ii) Yes. **8.** (ii) 48%; (iii) 50%. **9.** (i) Yes; (ii) Yes. **10.** $\frac{19}{42}$.

Section 3.7

1. 0.5, 0.5. **2.** (i) $\frac{1}{6}$; $\frac{1}{2}$; $\frac{1}{3}$; $\frac{3}{4}$; $\frac{1}{4}$; 0; (ii) $\frac{1}{5}$; $\frac{1}{5}$; $\frac{3}{5}$; $\frac{3}{10}$; $\frac{3}{10}$; $\frac{2}{5}$; (iii) $\frac{2}{9}$; 0; $\frac{7}{9}$; $\frac{2}{3}$; $\frac{1}{4}$; $\frac{1}{12}$; (iv) $\frac{4}{15}$; 0; $\frac{11}{15}$; $\frac{8}{15}$; $\frac{1}{5}$; $\frac{4}{15}$. **3.** (i) 2; (ii) $\frac{1}{3}$. **4.** (i)

	Mini	Caprice	
On Time	0.72	0.14	0.86
Late	0.08	0.06	0.14
	0.8	0.2	

(ii) $\frac{7}{43}$. **5.** (i) $\frac{1}{97}$; (ii) $\frac{1}{17}$. **6.** (i) 0.011; (ii) $\frac{27}{44}$. **7.** $\frac{2}{3}$. **8.** $\frac{8}{15}$. **9.** $\frac{9}{16}$. **10.** $\frac{30}{52}$.

Section 3.8

1. $\frac{3}{8}$, $\frac{1}{4}$. **2.** $\frac{7}{31}$. **3.** $\frac{3}{143}$. **4.** (i) $\frac{1}{10}$ (10%); (ii) $\frac{1}{5}$ (20%). **5.** (i) $\frac{19}{117}$ (about 16.2%); (ii) $\frac{1}{883}$ (about 0.1%).

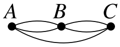
Section 3.9

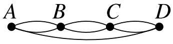
1. (i) $\frac{1}{8}$, $\frac{3}{8}$, $\frac{3}{8}$, $\frac{1}{8}$; (ii) 1.5. **2.** \$2.20. **3.** \$5. **4.** 0. **5.** \$17000. **6.** 5. **7.** 2.5. **8.** (i) $\frac{1}{9}$; (ii) $\frac{64}{81}$, $\frac{16}{81}$, $\frac{1}{81}$; (iii) $\frac{18}{81}$. **9.** 40*c*. **10.** Lose 50*c*.

Section 4.1

1. (i) $\{(1,1), (4,2), (9,3)\}$; (ii) $\{(1,1), (1,2), (1,3), (1,4), (1,5), (1,6), (1,7), (1,8), (1,9), (2,2), (2,4), (2,6), (2,8), (3,3), (3,6), (3,9), (4,4), (4,8), (5,5), (6,6), (7,7), (8,8), (9,9)\}$; (iii) $\{(2,8), (3,3)\}$. **2.** $\alpha : \{(1,1)\}$, $\beta : \{(1,2), (2,3), (3,4)\}$. **3.** (i) α; (ii) γ; (iii) α, β; (iv) α; (v) none; (vi) α. **4.** (i) S; (ii) S; (iii) ST; (iv) RST. **5.** α in Exercise 4.1A.3; none in Exercise 4.1A.4. **6.** (i) No; (ii) Yes; (iii) Yes; (iv) Yes. **7.** (i) One-to-one and onto; (ii) Not one-to-one, but onto; (iii) Not one-to-one ($f(-1) = f(1)$); onto. **8.** (i) f_4; (ii) $f^{-1}(x) = 2 - x$ if $x \geq 1$; $f^{-1}(x) = 1/x$ if $0 < x < 1$.

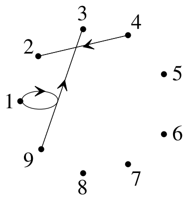
Section 4.2

1. (i) 

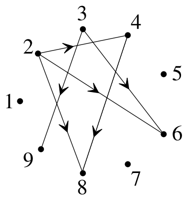
(ii) 

2. (i) $sa, st, as, at, bs, bt, tb$. $A(s) = \{a, t\}$, $B(s) = \{a, b\}$, $A(a) = \{s, t\}$, $B(a) = \{s\}$, $A(b) = \{s, t\}$, $B(b) = \{t\}$, $A(t) = \{b\}$, $B(t) = \{a, b, s\}$; (ii) sb, as, bc, ca, ce, dc, et, td. $A(s) = \{b\}$, $B(s) = \{a\}$, $A(a) = \{s\}$, $B(a) = \{c\}$, $A(b) = \{c\}$, $B(b) = \{s\}$, $A(c) = \{a, e\}$, $B(c) = \{b, d\}$, $A(d) = \{c\}$, $B(d) = \{t\}$, $A(e) = \{t\}$, $B(e) = \{c\}$, $A(t) = \{d\}$, $B(t) = \{e\}$; (iii) sa, sc, se, ab, ac, bd, ce, dc, dt, et. $A(s) = \{a, c, e\}$, $B(s) = \emptyset$, $A(a) = \{b, c\}$, $B(a) = \{s\}$, $A(b) = \{d\}$, $B(b) = \{a\}$, $A(c) = \{d, e\}$, $B(c) = \{s, a\}$, $A(d) = \{t\}$, $B(d) = \{b, c\}$, $A(e) = \{t\}$, $B(e) = \{s, c\}$, $A(t) = \emptyset$, $B(t) = \{d, e\}$.

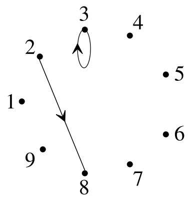
3.



(α)



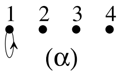
(β)



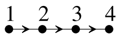
(γ)

In addition to the edges shown, β has loops on all vertices and an edge directed from 1 to each other vertex. None are graphs; all are directed, and all have loops.

4.



(α)



(β)

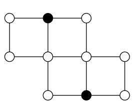
Neither is a graph; both are directed. α has a loop on 1, β is a digraph. (There is no edge from 4 to 3 in α, because $4 \notin S$.)

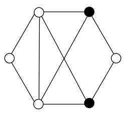
Section 4.3

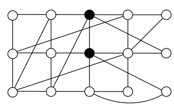
1 (i) 323303242; (ii) 1124222; (iii) 2422233; (iv) 2332332442. **2.** 4; 5. **3.** Yes. **4.** No. **5.** $2n$; n once, 3 n times. **6.** (ii) Has two bridges and one cutpoint. **7.** 21. **8.** $\frac{1}{2}n(n-1)$. **9.** $D = R = 2$. **10.** $d(s, t) = 3$. **11.** $sabct, safbt, saect, sdbct, sdbft$ (length 4); $saecbft, sdbaect$ (length 6); $d(s, t) = 4$.

Section 4.4

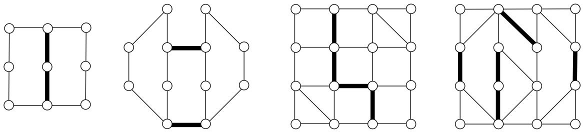
1. Graphs (ii), (v) and (viii) each have two odd vertices.





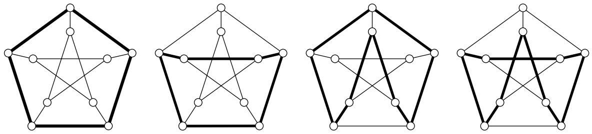


2. (i) Yes; Yes; (ii) Yes; No; (iii) Yes; Yes; (iv) No; (v) Yes; No; (vi) Yes; Yes; (vii) Yes; Yes; (viii) Yes; No. **3.** All odd n except $n = 1$. **4.** (i) 2; (ii) 2; (iii) 4; (iv) 5

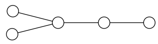


Section 4.5

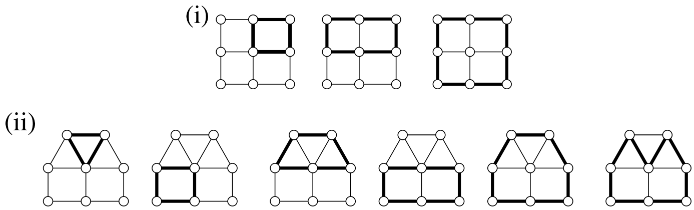
1. $D = R = n$ **2.** The diagrams show cycles of lengths 5, 6, 8, and 9, respectively.



3. One example:



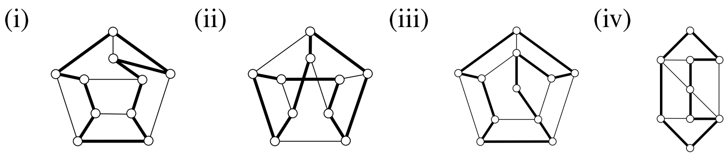
4.



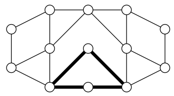
5. Say there is no cycle. Start at the vertex of degree 1. Select a (new) vertex adjacent to it. Continue in this way, never repeating a vertex. The process must stop, since the graph is finite, but it can only stop when all vertices adjacent to the new vertex have already been chosen. But if this is so, you must have a cycle. **6.** In (i), you get eight different solutions—start at any vertex, they will all be different. Case (ii) has seven solutions (the two top vertices as roots give the same diagram). In (iii), there are four different diagrams—for example, start with each vertex in the left-hand branch, including the bottom vertex. **7.** (i) Three trees (delete the edges of the triangle in turn). (ii) Three trees (delete the edges of the triangle in turn). (iii) Nine trees (delete one edge from the left triangle and one edge from the right triangle).

Section 4.6

1.



2. If there is a vertex of degree 2, any Hamiltonian cycle must contain both edges touching that vertex. So we would need to include all four heavy edges. But they form a 4-cycle, which is not allowed.



3. (i) $abcdef$, $abdcef$; (ii) $abcehgfd$, $abcfdgeh$, $abdfcegh$, $abdgfceh$, $abecfdgh$, $adbcfgeh$, $adbecfgh$, $adfcbegh$, $adgfcbeh$.

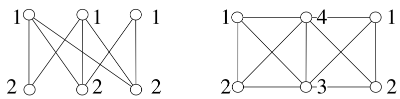
4. (i) NN: a 117, b 117, c 122, d 117, e 117, SE: 117; (ii) NN: a 112, b 113, c 112, d 113, e 116, SE: 113; (iii) NN: a 74, b 76, c 76, d 74, e 76, SE: 76; (iv) NN: a 105, b 105, c 105, d 100, e 100, SE: 105; (v) NN: a 119, b 122, c 122, d 122, e 122, SE: 122; (vi) NN: a 245, b 245, c 253, d 252, e 245, SE: 245.

Section 4.7

1. (i), (iii), (v), and (vi).

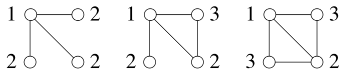
2. (i) 2; color the three top vertices in one color and the others in a second color; (ii) 4; color the leftmost and rightmost vertices the same.

3. (i) 2; (ii) 4.



4. 3 if n is even, 4 if n is odd.

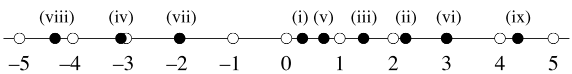
5.



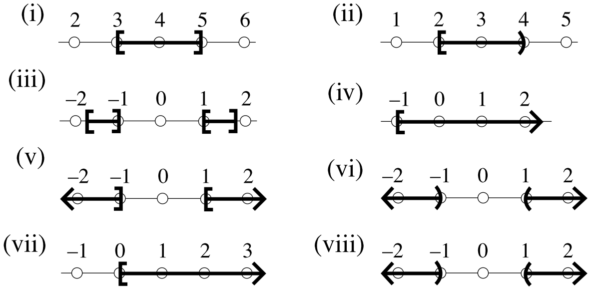
6. There are $v-1$ vertices, all connected, so at least $v-1$ colors are needed. (Consider all vertices except one endpoint of the deleted edge.) But $v-1$ colors suffice (color the endpoints of the deleted edge the same).

Section 5.1

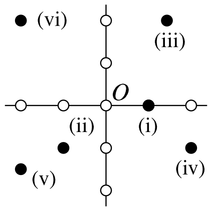
1.



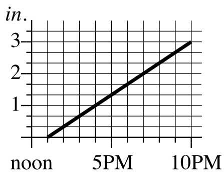
2.



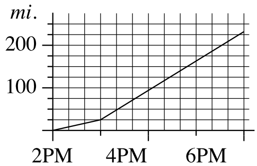
3.



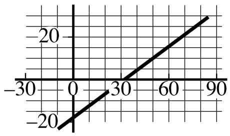
4.



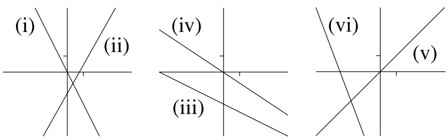
5. (i) 25 miles;(ii)



6. (i) $y = \frac{5}{9}(x - 32)$; (ii) 20°C; (iii) 77°F; (iv)



7.



8. (i) -2; $y = -2x$; (ii) 2; $y = 2x - 1$; (iii) $\frac{1}{2}$; $y = \frac{1}{2}x - 2$; (iv) $-\frac{2}{3}$; $y = -\frac{2}{3}x$; (v) 1; $y = x$; (vi) 2; $y = 2x - 3$. **9.** $y = x - 1$.

Section 5.2

1. (i) $x = 5, y = 2$; (ii) $x = -4, y = 3$; (iii) $x = \frac{1}{2}, y = \frac{3}{2}$; (iv) $x = 2, y = \frac{1}{2}$;

(v) $x = 3, y = 8$; (vi) $x = \frac{1}{2}, y = 1$; (vii) $x = 2, y = 1$; (viii) $x = 1, y = -\frac{1}{2}$; (ix) $x = -2, y = 6$; (x) $x = -1, y = 1$. **3.** (i) Inconsistent; (ii) Independent; $x = 1, y = \frac{3}{2}$; (iii) Dependent; $x = 2 - y/2$, any $y \in \mathbb{R}$; (iv) Independent; $x = 5$, $y = 4$; (v) Independent; $x = -1, y = \frac{5}{2}$; (vi) Dependent; $x = 1 - y/2$, any $y \in \mathbb{R}$; (vii) Inconsistent; (viii) Independent; $x = \frac{48}{5}, y = \frac{6}{5}$; (ix) Independent; $x = \frac{17}{21}$, $y = \frac{44}{21}$; (x) Dependent; $x = \frac{1}{5} + \frac{8}{5}y$. **4.** (i) $x = 3, y = 2, z = -1$; (ii) $x = 9$, $y = -8, z = \frac{1}{3}$; (iii) $x = 5z-2, y = 4z-3$, any $z \in \mathbb{R}$; (iv) Inconsistent; (v) $x = 2$, $y = -1, z = 1$; (vi) $x = 5z - 2, y = 4z - 3$, any $z \in \mathbb{R}$; (vii) $x = 2, y = -2$, $z = 1$; (viii) Inconsistent; (ix) $x = z - 3, y = z - 4$, any $z \in \mathbb{R}$; (x) $x = z + 1$, $y = 3z + 1$, any $z \in \mathbb{R}$; (xi) Independent; $x = 11, y = -5, z = 4$; (xii) Independent; $x = 4, y = 4, z = 0$; (xiii) Independent; $x = 0, y = 0, z = 2$; (xiv) Inconsistent.

Section 5.3

1. (i) $x = 3, y = 1$, any real z; (ii) $x = 2-z, y = 1+z$, any real z; (iii) No solutions; (iv) No solutions; (v) No solutions; (vi) $x = 2, y = 2$, any real z; (vii) $x = 4, y = 3$, $z = -3$; (viii) $x = 2 - z, y = 1 + z$, any real z; (ix) No solutions; (x) $x = 0$, $y = -1, z = 1$. **2.** (i) $x = -3, y = 2$; (ii) $x = \frac{1}{2}(5 - 3y)$, any real y; (iii) No solutions; (iv) $x = 2, y = -1$. **3.** (i) $x = 1 + 5z, y = 1 - 3z$, any real z; (ii) $x = \frac{2}{3}$, $y = \frac{1}{6}, z = \frac{1}{3}$; (iii) $x = 3, y = -1, z = -2$; (iv) $x = \frac{5}{2}, y = -\frac{1}{2}, z = 0$. **4.** (i) $x = 1 + 2z - t, y = 2 - z + 2t$, any real z, any real t; (ii) No solutions.

Section 5.4

1.

$M =$

	I	G
P	4	24
C	6	12

.

2. (i)

$M =$

	A	B	C
I	200	100	250
II	250	150	350

;

(ii) 1150 units A, 650 units B, 1550 units C.

3. (i) $\begin{bmatrix} 7 & -2 \\ -3 & 2 \end{bmatrix}$; (ii) $\begin{bmatrix} 28 & -1 \\ 8 & 19 \end{bmatrix}$;

(iii) $\begin{bmatrix} 15 & -5 & 18 \\ 10 & 11 & -13 \end{bmatrix}$; (iv) $\begin{bmatrix} -3 & 9 \\ -3 & 3 \end{bmatrix}$.

4. $x = 2, y = 1$. **5.** $x = 11, y = 9, z = 3$. **6.** (i) $(-2, 2)$; (ii) $(9, 18, 3)$; (iii) $(4, 1)$; (iv) $(-6, 0, 6)$; (v) $(7, 0, 4)$; (vi) $(5, -20, 10, 15)$.

7. (i) $\begin{bmatrix} 1 & 0 \\ -2 & -2 \end{bmatrix}$; (ii) $\begin{bmatrix} -5 \\ -14 \end{bmatrix}$; (iii) $\begin{bmatrix} 0 & -1 \\ -9 & 9 \end{bmatrix}$;

(iv) No (C, D not the same shape); (v) No (C, A not the same shape);

(vi) $\begin{bmatrix} 3 & 5 \\ 5 & -14 \end{bmatrix}$.

Section 5.5

1. (i) -1; (ii) -5; (iii) 3; (iv) 6; (v) 6; (vi) 3.

2. (i) 2×4; (ii) 2×2; (iii) No; (iv) 1×4; (v) 4×4; (vi) 2×4; (vii) No; (viii) 1×3; (ix) 2×3.

3. (i) $\begin{bmatrix} 3 & -1 \\ 4 & -1 \end{bmatrix}$; (ii) $\begin{bmatrix} 0 & -1 \\ 1 & 0 \\ 8 & 3 \end{bmatrix}$; (iii) $\begin{bmatrix} 1 & 7 & 5 \end{bmatrix}$;

(iv) $\begin{bmatrix} -1 & -1 & -2 \\ 1 & 1 & 2 \\ 5 & 3 & 4 \end{bmatrix}$; (v) $\begin{bmatrix} 9 & -7 \\ 6 & 2 \end{bmatrix}$.

4. (i) $\begin{bmatrix} 4 \\ 2 \end{bmatrix}$; (ii) $\begin{bmatrix} 4 \\ -6 \end{bmatrix}$; (iii) No; (iv) No;

(v) $\begin{bmatrix} 1 & 3 & 1 \end{bmatrix}$; (vi) No.

5. (i) \$5700; (ii) \$2440.

6. (i) $AB = \begin{bmatrix} -1 & 1 \\ -1 & 7 \end{bmatrix}$, $BA = \begin{bmatrix} 4 & 7 \\ 2 & 2 \end{bmatrix}$, No;

(ii) $AB = \begin{bmatrix} -4 & 8 \\ -5 & 2 \end{bmatrix}$, $BA = \begin{bmatrix} 0 & -4 \\ 8 & -2 \end{bmatrix}$, No;

(iii) $AB = BA = \begin{bmatrix} 0 & 10 \\ -10 & 0 \end{bmatrix}$, Yes.

7. (i) $\begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}$, $\begin{bmatrix} -2 & 2 \\ -2 & -2 \end{bmatrix}$; (ii) $\begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$, $\begin{bmatrix} -4 & -4 \\ -4 & -4 \end{bmatrix}$;

(iii) $\begin{bmatrix} 1 & -3 & 9 \\ 1 & 10 & -4 \\ -3 & -4 & -1 \end{bmatrix}$, $\begin{bmatrix} -7 & -18 & 15 \\ 6 & 34 & -11 \\ -5 & -11 & -6 \end{bmatrix}$;

(iv) $\begin{bmatrix} 7 & 0 & -2 \\ 6 & 1 & -1 \\ 4 & 0 & -1 \end{bmatrix}$, $\begin{bmatrix} 17 & 0 & -5 \\ 17 & 1 & -4 \\ 10 & 0 & -3 \end{bmatrix}$.

8. (i) $\begin{bmatrix} 0 & 3 \\ -3 & 3 \end{bmatrix}$, $\begin{bmatrix} -3 & 6 \\ -6 & 3 \end{bmatrix}$, $\begin{bmatrix} -6 & 6 \\ -6 & 0 \end{bmatrix}$.

Section 5.6

2. $\begin{bmatrix} 5 & -7 \\ -4 & 7 \end{bmatrix}$

3. (i) $\begin{bmatrix} 0 & \frac{1}{4} \\ -\frac{1}{2} & \frac{1}{4} \end{bmatrix}$; (ii) $\begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix}$; (iii) No inverse;

(iv) $\begin{bmatrix} -1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$; (v) $\begin{bmatrix} 4/7 & -1 & 2/7 \\ 3/7 & 0 & -2/7 \\ -5/7 & 1 & 1/7 \end{bmatrix}$;

(vi) $\begin{bmatrix} 3/2 & 0 & 1/2 \\ -11/6 & 1/3 & -5/6 \\ 17/6 & -1/3 & 5/6 \end{bmatrix}$; (vii) $\begin{bmatrix} 7 & -3 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$;

(viii) No inverse.

4. (i) 13; $\begin{bmatrix} 3/13 & -4/13 \\ -2/13 & 7/13 \end{bmatrix}$; (ii) -10; $\begin{bmatrix} -3/10 & 1/5 \\ 1/5 & 1/5 \end{bmatrix}$;

(iii) 0; No inverse; (iv) 1; $\begin{bmatrix} 1 & -2 \\ -1 & 3 \end{bmatrix}$.

Section 5.7

3. (ii) (a) $x = 2$, $y = -1$; (b) $x = 1$, $y = 1$.

4. (ii) (a) $x = -6$, $y = 1$, $z = 2$; (b) $x = 4$, $y = 1$, $z = 2$.

5. (i) $\begin{bmatrix} 132 \\ 104 \end{bmatrix}$; (ii) $\begin{bmatrix} 68 \\ 124 \end{bmatrix}$; (iii) $\begin{bmatrix} 190 \\ 310 \end{bmatrix}$.

6. 208 tonnes of X, 20 tonnes of Y.

Section 6.1

1. x = number of Fleetwoods, y = number of Majestics. We figure in thousands of dollars.

$$\begin{aligned} \text{Maximize} \quad & P = 20x + 25y \\ \text{subject to} \quad & 500x + 600y \leq 11000, \\ & 30x + 40y \leq 700, \\ & x \geq 0, \quad y \geq 0. \end{aligned}$$

2. x = number of pounds of feed A per month, y = number of pounds of feed B per month.

$$\begin{aligned} \text{Minimize} \quad & C = 2.5x + 1.4y \\ \text{subject to} \quad & 5x + 2y \geq 70, \\ & 3x + 2y \geq 50, \\ & 4x + y \geq 40, \\ & x \geq 0, \quad y \geq 0. \end{aligned}$$

3. x_1, x_2, x_3 are the number of ornaments of types I, II, III, respectively.

$$\begin{aligned} \text{Maximize} \quad & P = 30x_1 + 45x_2 + 15x_3 \\ \text{subject to} \quad & 6x_1 + 4x_2 + x_3 \leq 200, \\ & 2x_1 + 3x_2 + x_3 \leq 100, \\ & x_1 \geq 0, \quad x_2 \geq 0, \quad x_3 \geq 0. \end{aligned}$$

4. x_1, x_2, x_3 are the acres of corn, linseed and oats, respectively.

$$\begin{aligned} \text{Maximize} \quad & P = 330x_1 + 340x_2 + 270x_3 \\ \text{subject to} \quad & x_1 + x_2 + x_3 \leq 60, \\ & 5x_1 + 4x_2 + 3x_3 \leq 160, \\ & 90x_1 + 110x_2 + 80x_3 \leq 6000, \\ & x_1 \geq 0, \quad x_2 \geq 0, \quad x_3 \geq 0. \end{aligned}$$

5. x_1, x_2 and x_3 are amounts (in millions of dollars) to be invested in real estate, stocks and treasury bills, respectively. P is the expected income in *millions of cents.*

$$\begin{aligned} \text{Maximize} \quad & P = 8x_1 + 5x_2 + 3x_3 \\ \text{subject to} \quad & x_1 + x_2 + x_3 \leq 100, \\ & x_1 - x_2 \leq 0, \\ & 2x_1 + 2x_2 - x_3 \leq 0, \\ & x_1 \geq 0, \quad x_2 \geq 0, \quad x_3 \geq 0. \end{aligned}$$

6. x = standard clocks, y = number of mantel clocks.

$$\begin{aligned} \text{Maximize} \quad & P = 60x + 11y \\ \text{subject to} \quad & 40x + 8y \leq 5,000, \\ & 5x + 2y \leq 600, \\ & x \geq 0, \quad y \geq 0. \end{aligned}$$

7. x = number of student desks, y = number of faculty desks.

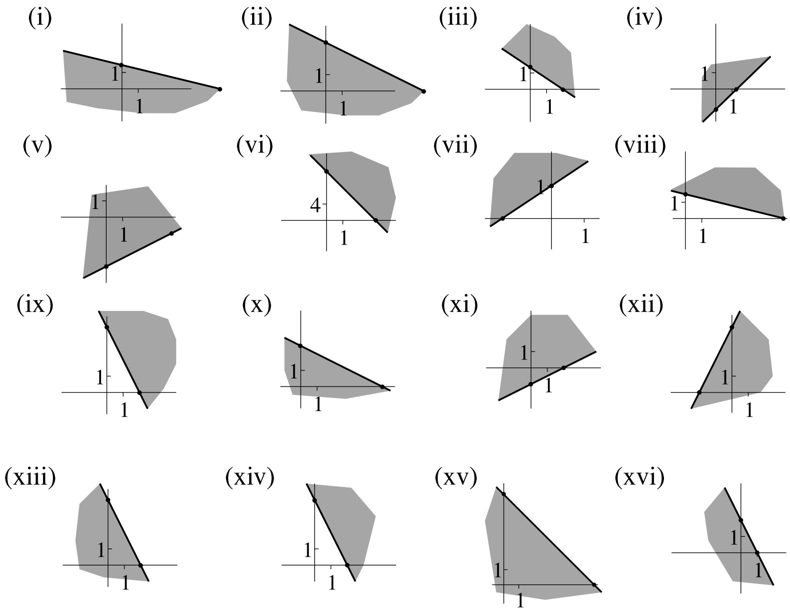
$$\begin{aligned} \text{Maximize} \quad & P = 10x + 12y \\ \text{subject to} \quad & 10x + 12y \le 8000, \\ & 2x + 2y \le 2000, \\ & 1.5x + 2y \le 1800 \\ & x \ge 0, \quad y \ge 0. \end{aligned}$$

8. x_1, x_2, x_3 are the number of acres of types potatoes, corn, and peppers, respectively.

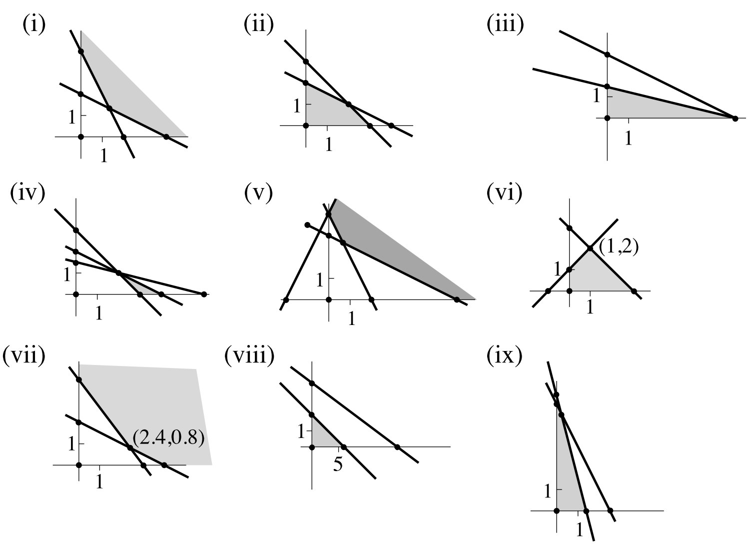
$$\begin{aligned} \text{Maximize} \quad & P = 80x_1 + 40x_2 + 70x_3 \\ \text{subject to} \quad & x_1 + x_2 + x_3 \le 100, \\ & x_1 \le 30, \\ & 300x_1 + 160x_2 + 280x_3 \le 20000, \\ & x_1 \ge 0, \quad x_2 \ge 0, \quad x_3 \ge 0. \end{aligned}$$

Section 6.2

1.



2.



3. (i) $C = \frac{8}{3}$ at $x = \frac{4}{3}$, $y = \frac{4}{3}$; (ii) $P = 3$ at $x = 3$, $y = 0$; (iii) $P = 6$ at $x = 6$, $y = 0$; (iv) $C = 11$ at $x = 2$, $y = 1$; (v) $C = \frac{10}{3}$ at $x = \frac{2}{3}$, $y = \frac{8}{3}$.

4. (i) $2x + y \le 10000$; (ii) $x + 2y \le 8000$; (iii) The problem is to maximize $P = 3x + 5y$ subject to the given constraints (including $x, y \ge 0$). The maximum is 22000, achieved at $x = 4000$, $y = 2000$.

Section 6.3

1. (i)

$$\begin{aligned} &\text{Maximize} && P = 4x_1 + 2x_2 + 0x_3 + 0x_4 \\ &\text{subject to} && x_1 + 2x_2 + x_3 + 0x_4 = 5, \\ &&& 2x_1 + 6x_2 + 0x_3 - x_4 = 12, \\ &&& x_1 \ge 0, \quad x_2 \ge 0, \quad x_3 \ge 0, \quad x_4 \ge 0. \end{aligned}$$

(ii)

$$\begin{aligned} &\text{Minimize} && C = x_1 + x_2 + 0x_3 + 0x_4 \\ &\text{subject to} && 2x_1 + x_2 - x_3 + 0x_4 = 4, \\ &&& x_1 + 2x_2 + 0x_3 - x_4 = 4, \\ &&& x_1 \ge 0, \quad x_2 \ge 0, \quad x_3 \ge 0, \quad x_4 \ge 0. \end{aligned}$$

(iii)

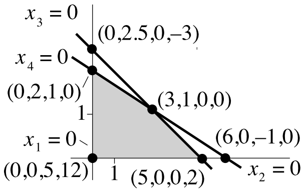
$$\begin{aligned} &\text{Maximize} && C = x_1 + x_2 + 0x_3 + 0x_4 \\ &\text{subject to} && 2x_1 + x_2 + x_3 + 0x_4 = 4, \\ &&& 4x_1 + 4x_2 + 0x_3 + x_4 = 12, \\ &&& x_1 \ge 0, \quad x_2 \ge 0, \quad x_3 \ge 0, \quad x_4 \ge 0. \end{aligned}$$

(iv)

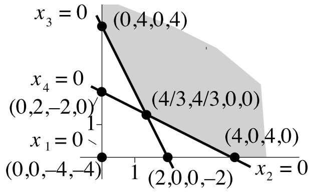
$$\begin{aligned} \text{Maximize} \quad & P = 2x_1 + 3x_2 + 0x_3 + 0x_4 \\ \text{subject to} \quad & 8x_1 + 2x_2 + x_3 + 0x_4 = 11, \\ & 2x_1 + x_2 + 0x_3 + x_4 = 5, \\ & x_1 \geq 0, \quad x_2 \geq 0, \quad x_3 \geq 0, \quad x_4 \geq 0. \end{aligned}$$

2.

(i)



(ii)



3. (i)

Non-basic variables	Basic variables	Basic solution	Feasible?	P
x_1, x_2	x_3, x_4	$(0, 0, 5, 12)$	Yes	0
x_1, x_3	x_2, x_4	$(0, \frac{5}{2}, 0, -3)$	No	5
x_1, x_4	x_2, x_3	$(0, 2, 1, 0)$	Yes	4
x_2, x_3	x_1, x_4	$(5, 0, 0, 2)$	Yes	20
x_2, x_4	x_1, x_3	$(6, 0, -1, 0)$	No	24
x_3, x_4	x_1, x_2	$(3, 1, 0, 0)$	Yes	14

Maximum $P = 20$ at $x_1 = 5, x_2 = 0$.

(ii)

Non-basic variables	Basic variables	Basic solution	Feasible?	C
x_1, x_2	x_3, x_4	$(0, 0, -4, -4)$	No	0
x_1, x_3	x_2, x_4	$(0, 4, 0, 4)$	Yes	4
x_1, x_4	x_2, x_3	$(0, 2, -2, 0)$	No	2
x_2, x_3	x_1, x_4	$(2, 0, 0, -2)$	No	2
x_2, x_4	x_1, x_3	$(4, 0, 4, 0)$	Yes	4
x_3, x_4	x_1, x_2	$(\frac{4}{3}, \frac{4}{3}, 0, 0)$	Yes	$\frac{8}{3}$

Minimum $C = 4$ at $x_1 = 4, x_2 = 0$ or $x_1 = 0, x_2 = 4$.

(iii)

Non-basic variables	Basic variables	Basic solution	Feasible?	C
x_1, x_2	x_3, x_4	$(0, 0, 4, 8)$	Yes	0
x_1, x_3	x_2, x_4	$(0, 4, 0, 0)$	Yes	16
x_1, x_4	x_2, x_3	$(0, 4, 0, 0)$	Yes	16
x_2, x_3	x_1, x_4	$(2, 0, 0, 2)$	Yes	6
x_2, x_4	x_1, x_3	$(\frac{8}{3}, 0, -\frac{4}{3}, 0)$	No	8
x_3, x_4	x_1, x_2	$(0, 4, 0, 0)$	Yes	16

Maximum $P = 16$ at $x_1 = 0, x_2 = 4$. (Note: three boundary lines have a common point. There are actually only four boundary points.)

(iv)

Non-basic variables	Basic variables	Basic solution	Feasible?	C
x_1, x_2	x_3, x_4	$(0, 0, 2, 4)$	Yes	0
x_1, x_3	x_2, x_4	$(0, 1, 0, 3)$	Yes	3
x_1, x_4	x_2, x_3	$(0, 4, -6, 0)$	No	12
x_2, x_3	x_1, x_4	$(-2, 0, 0, 8)$	No	-4
x_2, x_4	x_1, x_3	$(2, 0, 4, 0)$	Yes	4
x_3, x_4	x_1, x_2	$(\frac{6}{5}, \frac{8}{5}, 0, 0)$	Yes	$\frac{36}{5}$

Maximum $C = \frac{36}{5}$ at $x_1 = \frac{6}{5}, x_2 = \frac{8}{5}$.

Section 6.4

1. (i)

$$\begin{aligned} x_1 + 2x_2 + x_3 + 0x_4 &= 3, \\ 2x_1 + 3x_2 + 0x_3 + x_4 &= 4, \end{aligned} \quad (0, 0, 3, 4);$$

(ii)

$$\begin{aligned} 2x_1 + 2x_2 + x_3 + 0x_4 &= 3, \\ x_1 + 3x_2 + 0x_3 - x_4 &= 5, \end{aligned} \quad \text{no};$$

(iii)

$$\begin{aligned} 2x_1 + x_2 + x_3 + 0x_4 &= 5, \\ x_1 + 3x_2 + 0x_3 + x_4 &= 8, \end{aligned} \quad (0, 0, 5, 8);$$

(iv)

$$\begin{aligned} 3x_1 + 2x_2 + x_3 + 0x_4 &= 3, \\ x_1 - 2x_2 + 0x_3 - x_4 &= 5, \end{aligned} \quad \text{no};$$

(v)

$$\begin{aligned} 3x_1 + 2x_2 + x_3 + 0x_4 &= 6, \\ x_1 + 3x_2 + 0x_3 - x_4 &= 4, \end{aligned} \quad \text{no};$$

(vi)

$$\begin{aligned} x_1 - x_2 + 2x_3 + x_4 + 0x_5 &= 1, \\ 2x_1 + x_2 + x_3 - 0x_4 - x_5 &= 3, \end{aligned} \quad \text{no;}$$

(vii)

$$\begin{aligned} x_1 + x_2 + 2x_3 + x_4 + 0x_5 + 0x_6 &= 1, \\ 3x_1 - 3x_2 + 2x_3 + 0x_4 + x_5 + 0x_6 &= 1, \\ -x_1 + 2x_2 + 3x_3 + 0x_4 + 0x_5 + x_6 &= 3, \end{aligned} \quad (0, 0, 0, 1, 1, 3);$$

(viii)

$$\begin{aligned} 3x_1 + 2x_2 + x_3 + 0x_4 + 0x_5 &= 3, \\ x_1 + 5x_2 + 0x_3 + x_4 + 0x_5 &= 2, \\ 2x_1 - 2x_2 + 0x_3 + 0x_4 + x_5 &= 1, \end{aligned} \quad (0, 0, 3, 2, 1).$$

2. (i)

BV	x_1	x_2	x_3	x_4	b
x_3	1	2	1	0	3
x_4	2	3	0	1	4

;

(iii)

BV	x_1	x_2	x_3	x_4	b
x_3	2	1	1	0	5
x_4	1	3	0	1	8

;

(vii)

BV	x_1	x_2	x_3	x_4	x_5	x_6	b
x_4	1	1	2	1	0	0	1
x_5	3	-3	2	0	1	0	1
x_6	-1	2	3	0	0	1	3

;

(viii)

BV	x_1	x_2	x_3	x_4	x_5	b
x_4	3	2	1	0	0	3
x_5	1	5	0	1	0	2
x_6	2	-2	0	0	1	1

.

3. (i) x_1; (ii) x_4; (iii) No positive entry in column x_2; (iv) x_5 *or* x_4; (v) x_1; (vi) x_5.

4. (i) (2, 1, 0, 0)

BV	x_1	x_2	x_3	x_4	b
x_3	$\frac{1}{4}$	0	1	$\frac{1}{4}$	$\frac{1}{2}$
x_2	$-\frac{1}{4}$	1	0	$\frac{15}{4}$	$\frac{1}{2}$

$(0, \frac{1}{2}, \frac{1}{2}, 0)$;

(ii) (2, 0, 0, 2)

BV	x_1	x_2	x_3	x_4	b
x_3	0	2	1	$\frac{1}{2}$	1
x_1	1	−1	0	$-\frac{1}{2}$	1

(1, 0, 1, 0);

(iv) (2, 0, 0, 6, 2)

BV	x_1	x_2	x_3	x_4	x_5	x_6	b
x_1	1	3	0	0	1	5	4
x_3	0	2	1	0	1	1	2
x_4	0	−8	0	1	−3	−1	0

or

BV	x_1	x_2	x_3	x_4	x_5	x_6	b
x_5	1	$-\frac{1}{3}$	0	$\frac{1}{3}$	0	$\frac{14}{3}$	4
x_3	0	$\frac{8}{3}$	0	$-\frac{2}{3}$	1	$\frac{1}{3}$	0
x_1	0	$-\frac{2}{3}$	1	$\frac{1}{3}$	0	$\frac{2}{3}$	2

(4, 0, 2, 0, 0);

(v) (2, 0, 0, 0, 6)

BV	x_1	x_2	x_3	x_4	x_5	b
x_4	$\frac{1}{2}$	1	0	1	0	1
x_5	−1	1	−2	0	1	4

(0, 0, 0, 1, 4).

(vi) (2, 0, 0, 0, 2, 3)

BV	x_1	x_2	x_3	x_4	x_5	x_6	b
x_1	1	$-\frac{1}{2}$	$-\frac{3}{2}$	0	$-\frac{1}{2}$	0	1
x_4	0	$\frac{1}{2}$	$\frac{1}{2}$	1	$\frac{1}{2}$	0	1
x_6	0	$-\frac{1}{2}$	$\frac{5}{2}$	0	$-\frac{1}{2}$	1	2

.

(1, 0, 0, 1, 0, 2)

5. (i)

BV	x_1	x_2	x_3	x_4	b
x_2	0	1	1	1	1
x_1	1	0	−2	1	2

;

(ii)

BV	x_1	x_2	x_3	x_4	b
x_4	$\frac{4}{3}$	$-\frac{2}{3}$	0	1	2
x_3	1	1	1	0	3

;

(iii)

BV	x_1	x_2	x_3	x_4	x_5	x_6	b
x_2	-2	1	0	0	-1	2	1
x_4	$\frac{1}{2}$	0	0	1	$\frac{5}{2}$	$-\frac{1}{2}$	$\frac{3}{2}$
x_3	$\frac{1}{2}$	0	1	0	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$

;

(iv)

BV	x_1	x_2	x_3	x_4	x_5	x_6	b
x_1	1	-1	0	0	-1	3	1
x_4	0	0	0	1	3	4	3
x_3	0	1	1	0	2	-1	1

;

(v)

BV	x_1	x_2	x_3	x_4	x_5	x_6	b
x_2	-2	1	0	0	-1	2	1
x_4	$\frac{1}{2}$	0	0	1	$\frac{5}{2}$	$-\frac{1}{2}$	$\frac{3}{2}$
x_3	$\frac{1}{2}$	0	1	0	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$

;

(iv)

BV	x_1	x_2	x_3	x_4	x_5	x_6	b
x_1	1	-1	0	0	-1	3	1
x_4	0	0	0	1	3	4	3
x_3	0	1	1	0	2	-1	1

.

6. (i) 2, 1, (2, 0, 1, 0); (ii) 4; (iii) 2; (iv) 2; (v) (1, 2, 0, 0).

Section 6.5

1. (i)

BV	x_1	x_2	x_3	x_4	P	b
x_3	1	4	1	0	0	2
x_4	1	2	0	1	0	4
	-3	-5	0	0	1	0

;

(ii)

BV	x_1	x_2	x_3	x_4	P	b
x_3	2	2	1	0	0	4
x_4	3	5	0	1	0	1
	−1	−3	0	0	1	0

;

(iii)

BV	x_1	x_2	x_3	x_4	P	b
x_3	3	3	1	0	0	2
x_4	1	4	0	1	0	3
	−3	−2	0	0	1	0

;

(iv)

BV	x_1	x_2	x_3	x_4	x_5	P	b
x_4	1	1	1	1	0	0	3
x_5	3	2	3	0	1	0	4
	4	−3	2	0	0	1	0

;

(v)

BV	x_1	x_2	x_3	x_4	x_5	P	b
x_3	2	1	1	0	0	0	6
x_4	3	3	0	1	0	0	7
x_5	1	2	0	0	1	0	5
	−2	−5	0	0	0	1	0

;

(vi)

BV	x_1	x_2	x_3	x_4	x_5	P	b
x_3	5	3	1	0	0	0	8
x_4	3	1	0	1	0	0	4
x_5	1	4	0	0	1	0	5
	6	−1	0	0	0	1	0

.

2. (i)

BV	x_1	x_2	x_3	x_4	P	b
x_2	0	1	−2	−3	0	1
x_1	1	0	1	2	0	1
	0	0	2	3	1	4

;

(ii)

BV	x_1	x_2	x_3	x_4	P	b
x_3	0	$\frac{1}{2}$	1	$\frac{1}{2}$	0	1
x_1	1	$\frac{1}{2}$	0	$-\frac{1}{2}$	0	0
	0	$\frac{1}{2}$	0	$\frac{3}{2}$	1	6

;

(iii)

BV	x_1	x_2	x_3	x_4	x_5	P	b
x_2	0	1	0	0	1	0	2
x_3	1	0	1	0	1	0	3
x_4	−1	0	0	1	2	0	1
	1	0	0	0	2	1	5

;

(iv)

BV	x_1	x_2	x_3	x_4	x_5	x_6	P	b
x_5	0	0	2	1	1	1	0	3
x_1	1	0	0	1	0	−1	0	7
x_2	0	1	−1	−1	0	−2	0	1
	0	0	7	4	0	3	1	17

.

3. (i)

BV	x_1	x_2	x_3	x_4	P	b
x_3	1	0	1	$-\frac{1}{2}$	0	8
x_2	$\frac{3}{2}$	1	0	$\frac{1}{4}$	0	2
	20	0	0	6	1	48

;

(ii) Unbounded;

(iii)

BV	x_1	x_2	x_3	x_4	P	b
x_3	$\frac{1}{2}$	0	1	$-\frac{1}{2}$	0	4
x_2	$\frac{3}{2}$	1	0	$\frac{1}{2}$	0	2
	10	0	0	6	1	24

;

(iv) Finished: $P = 18$ at $x_1 = 4, x_2 = 3$;

(v)

BV	x_1	x_2	x_3	x_4	P	b
x_2	0	1	$\frac{1}{2}$	$-\frac{1}{2}$	0	2
x_1	0	0	$-\frac{1}{4}$	$\frac{3}{4}$	0	3
	0	0	$\frac{1}{4}$	$\frac{5}{4}$	1	13

;

(vi) Finished: $P = 7$ at $x_1 = 1, x_2 = 2$;
(vii) Finished: $P = \frac{9}{2}$ at $x_1 = \frac{3}{2}, x_2 = 0$; (viii) unbounded; (ix) unbounded;

(x)

BV	x_1	x_2	x_3	x_4	P	b
x_2	0	1	2	−1	0	2
x_1	1	0	−1	1	0	1
	0	0	−11	8	1	23

;

(xi) Finished: $P = 96$ at $x_1 = 10, x_2 = 19, x_3 = 0$;

(xii)

BV	x_1	x_2	x_3	x_4	P	b
x_1	1	0	−3	4	0	2
x_2	0	1	1	−1	0	2
	0	0	3	2	1	20

;

(xiii) Finished: $P = 3$ at $x_1 = 0, x_2 = 1$;

(xiv) Unbounded;
(xv) Unbounded;

(xvi)

BV	x_1	x_2	x_3	x_4	x_5	x_6	P	b
x_3	$-\frac{1}{2}$	0	1	0	2	2	0	2
x_1	$\frac{1}{2}$	0	0	1	−1	1	0	1
x_2	0	1	0	0	−1	−1	0	4
	$\frac{3}{2}$	0	0	0	−5	4	1	18

.

4. (i) $x_1 = 6, x_2 = 0, P = 36$; (ii) $x_1 = 1, x_2 = 2, P = 16$; (iii) $x_1 = 0, x_2 = 4, P = 20$; (iv) $x_1 = 0, x_2 = 0, x_3 = 2, P = 6$; (v) $x_1 = 9, x_2 = 27, x_3 = 9, P = 261$.

Section 6.6

1. (i) $4x_1 + 3x_2 - x_3 + A_1 = 5$; (ii) $x_1 + x_2 - x_3 + A_1 = 2$; (iii) $7x_1 - 2x_2 + 3x_3 + A_1 = 7$; (iv) $4x_1 + 5x_2 + x_3 - x_4 + A_1 = 4$; (v) $4x_1 - 4x_2 + 2x_3 + x_5 = 5$; (vi) $7x_1 + 3x_2 - 3x_3 + 7x_4 + A_1 = 5$; (vii) $3x_1 - 2x_2 + 4x_3 - 3x_4 - x_5 + A_1 = 1$; (viii) $11x_1 + 9x_2 + 5x_3 + 2x_4 + x_5 = 3$.

2. (i)

BV	x_1	x_2	x_3	A_1	A_2	P	b
	4	2	−1	1	0	0	5
	3	2	0	0	1	0	10
	2	3	0	M	M	1	0

;

BV	x_1	x_2	x_3	A_1	A_2	P	b
A_1	4	2	−1	1	0	0	5
A_2	3	2	0	0	1	0	10
	2	3	0	0	0	1	0
	$-7M$	$-4M$	M	0	0	0	$-15M$

;

(ii)

BV	x_1	x_2	x_3	x_4	A_1	A_2	P	b
	4	2	1	0	0	0	0	8
	2	10	0	−1	1	0	0	11
	3	−1	0	0	0	1	0	7
	2	5	0	0	M	M	1	0

BV	x_1	x_2	x_3	x_4	A_1	A_2	P	b
x_3	4	2	1	0	0	0	0	8
A_1	2	10	0	−1	1	0	0	11
A_2	3	−1	0	0	0	1	0	7
	2	5	0	0	0	0	1	0
	$-5M$	$-9M$	0	M	0	0	0	$-18M$

;

(iii)

BV	x_1	x_2	x_3	x_4	x_5	A_1	A_2	P	b
	1	2	3	1	0	0	0	0	6
	2	1	−1	0	−1	1	0	0	4
	0	1	1	0	0	0	1	0	2
	1	2	1	0	0	M	M	1	0

;

BV	x_1	x_2	x_3	x_4	x_5	A_1	A_2	P	b
x_3	1	2	3	1	0	0	0	0	6
A_1	2	1	−1	0	−1	1	0	0	4
A_2	0	1	1	0	0	0	1	0	2
	1	2	1	0	0	0	0	1	0
	$-2M$	$-2M$	0	0	M	0	0	0	$-6M$

;

(iv)

BV	x_1	x_2	x_3	x_4	x_5	x_6	A_1	A_2	P	b
	1	2	3	2	1	0	0	0	0	6
	2	3	1	1	0	−1	1	0	0	9
	2	2	−1	−1	0	0	0	1	0	4
	2	3	1	1	0	0	M	M	1	0

;

BV	x_1	x_2	x_3	x_4	x_5	x_6	A_1	A_2	P	b
x_5	1	2	3	2	1	0	0	0	0	6
A_1	2	3	1	1	0	−1	1	0	0	9
A_2	2	2	−1	−1	0	0	0	1	0	4
	2	3	1	1	0	0	0	0	1	0
	$-4M$	$-5M$	0	0	0	M	0	0	0	$-13M$

.

3. (i) Pivot A_1, x_1

BV	x_1	x_2	x_3	x_4	A_2	P	b
x_3	0	$-\frac{1}{2}$	1	$-\frac{1}{2}$	$\frac{1}{2}$	0	4
x_1	1	$-\frac{1}{2}$	0	$\frac{3}{2}$	$\frac{1}{2}$	0	2
	0	1	0	−6	−1	1	0
	0	$\frac{1}{2}M$	0	$\frac{7}{2}M$	$\frac{1}{2}M$	0	$-2M$

;

(ii) Infeasible;

(iii) Pivot A_1, x_3

BV	x_1	x_2	x_3	x_4	A_1	P	b
x_1	1	−1	0	−1	1	0	6
x_3	0	−3	1	−1	1	0	2
	0	7	0	3	−3	1	−18
	0	0	0	0	M	0	0

;

(iv) Infeasible;

(v) Pivot A_1, x_3

BV	x_1	x_2	x_3	x_4	A_1	P	b
x_1	1	0	0	−1	1	0	6
x_3	0	−3	1	−1	1	0	2
P	0	10	4	4	−4	1	28
	0	0	0	0	M	0	0

;

(vi) Pivot A_2, x_3

$$\left[\begin{array}{c|cccccccc|c} \text{BV} & x_1 & x_2 & x_3 & x_4 & x_5 & A_1 & A_2 & P & b \\ \hline A_1 & 1 & 1 & 0 & -1 & 0 & 1 & 0 & 0 & 1 \\ x_5 & \frac{3}{2} & \frac{1}{2} & 0 & 0 & 1 & 0 & -\frac{1}{2} & 0 & 2 \\ x_3 & \frac{1}{2} & \frac{1}{2} & 1 & 0 & 0 & 0 & \frac{1}{2} & 0 & 1 \\ \hline & \frac{3}{2} & \frac{3}{2} & 0 & 0 & 0 & 0 & -\frac{1}{2} & 1 & -1 \\ & 0 & -M & 0 & M & 0 & 0 & 2M & 0 & -4M \end{array}\right];$$

(vii) Pivot x_3, A_1

$$\left[\begin{array}{c|cccccccc|c} \text{BV} & x_1 & x_2 & x_3 & x_4 & x_5 & A_1 & A_2 & P & b \\ \hline x_4 & 4 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 5 \\ x_5 & -1 & 5 & 0 & 0 & 1 & -2 & 0 & 0 & 4 \\ x_3 & 2 & -2 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ A_2 & -7 & 9 & 0 & 0 & 0 & -3 & 1 & 0 & 2 \\ \hline & 1 & -6 & 0 & 0 & 0 & 2 & 0 & 1 & 2 \\ & 7M & -9M & 0 & 0 & 0 & 4M & 0 & 0 & -2M \end{array}\right].$$

4. (i) $P = 8$ at $x_1 = 4$, $x_2 = 0$; (ii) unbounded; (iii) $P = 9$ at $x_1 = 3$, $x_2 = 0$, $x_3 = 0$; (iv) $C = 52$ at $x_1 = 1, x_2 = 9, x_3 = 0$; (v) infeasible; (vi) $C = \frac{101}{3}$ at $x_1 = 0, x_2 = \frac{10}{3}, x_3 = \frac{11}{3}$; (vii) $C = 14$ at $x_1 = 3, x_2 = 2, x_3 = 0, x_4 = 2$.

Section 7.1

1.

$$\begin{array}{|cc|} \hline 2 & -3 \\ -3 & 4 \\ \hline \end{array}.$$

2.

$$\begin{array}{|ccc|} \hline -200 & 100 & 100 \\ 100 & -200 & 100 \\ 100 & 100 & -200 \\ \hline \end{array}.$$

3. (i) Strategy R2C1, value 0; (ii) Strategy R2C2, value 1; (iii) Strategy R1C2, value 1; (iv) Strategy R1C3, value 0; (v) Strategy R1C2, value 0; (vi) Strategy R2C2, value 0; (vii) Strategy R3C3, value 1; (viii) Strategy R3C2, value 1.

4.

(i) $\begin{array}{|cc|} \hline 2 & -1 \\ -2 & 3 \\ \hline \end{array}$; (ii) $\begin{array}{|cc|} \hline 1 & -1 \\ -1 & 1 \\ \hline \end{array}$; (iii) $\begin{array}{|cc|} \hline 1 & -3 \\ -1 & 2 \\ \hline \end{array}$ (iv) $\begin{array}{|cc|} \hline 2 & 3 \\ 3 & -1 \\ \hline \end{array}$;

(v) $\begin{array}{|cc|} \hline 2 & -2 \\ -1 & 1 \\ \hline \end{array}$; (vi) $\begin{array}{|cc|} \hline 1 & 0 \\ 0 & 1 \\ \hline \end{array}$; (vii) $\begin{array}{|cc|} \hline -2 & -1 \\ -1 & -2 \\ \hline \end{array}$; (viii) $\begin{array}{|cc|} \hline -1 & 1 \\ 1 & 0 \\ \hline \end{array}$.

5. (i) Saddle point R2C2, value 0; (ii) Saddle point R3C1, value 1; (iii) Saddle point R2C2, value -1; (iv) Saddle point R2C3, value 1.

Section 7.2

1. (i) R($\frac{5}{8}$, $\frac{3}{8}$)C($\frac{1}{2}$, $\frac{1}{2}$) value $\frac{3}{8}$; (ii) Saddle point: R1C1 value 3; (iii) R($\frac{1}{6}$, $\frac{5}{6}$)C($\frac{2}{3}$, $\frac{1}{3}$) value $\frac{4}{3}$; (iv) R($\frac{1}{2}$, $\frac{1}{2}$)C($\frac{1}{2}$, $\frac{1}{2}$) value -1; (v) R($\frac{1}{3}$, $\frac{2}{3}$)C($\frac{2}{3}$, $\frac{1}{3}$) value $-\frac{2}{3}$; (vi) Saddle point: R1C2 value -1; (vii) R($\frac{1}{2}$, $\frac{1}{2}$)C($\frac{1}{3}$, $\frac{2}{3}$) value 0; (viii) R($\frac{1}{5}$, $\frac{4}{5}$)C($\frac{3}{5}$, $\frac{2}{5}$) value $\frac{7}{5}$; (ix) R($\frac{1}{2}$, $\frac{1}{2}$)C($\frac{1}{2}$, $\frac{1}{2}$) value 3; (x) R($\frac{3}{7}$, $\frac{5}{7}$)C($\frac{4}{7}$, $\frac{2}{7}$) value $\frac{22}{7}$; (xi) R($\frac{4}{9}$, $\frac{5}{9}$)C($\frac{1}{3}$, $\frac{2}{3}$) value $\frac{2}{3}$; (xii) Saddle point: R1C2 value -1; (xiii) Saddle point: R2C1 value 1; (xiv) R($\frac{3}{4}$, $\frac{1}{4}$)C($\frac{1}{2}$, $\frac{1}{2}$) value $\frac{3}{2}$; (xv) R($\frac{7}{8}$, $\frac{1}{8}$)C($\frac{7}{8}$, $\frac{1}{8}$) value $\frac{17}{8}$; (xvi) R($\frac{7}{12}$, $\frac{5}{12}$)C($\frac{1}{4}$, $\frac{3}{4}$) value $\frac{1}{4}$.

3. (i) $a \leq 3$; (ii) all a; (iii) all a; (iv) $a \geq 0$.

Section 7.3

1. (i) R($\frac{2}{3}$, $\frac{1}{3}$)C($\frac{1}{3}$, 0, $\frac{2}{3}$) value $-\frac{1}{3}$; (ii) R($\frac{3}{8}$, $\frac{5}{8}$)C(0, $\frac{5}{8}$, $\frac{3}{8}$, 0) value $-\frac{1}{8}$; (iii) R($\frac{5}{9}$, $\frac{4}{9}$)C(0, $\frac{4}{9}$, $\frac{5}{9}$) value $-\frac{2}{9}$; (iv) R($\frac{3}{4}$, $\frac{1}{4}$)C($\frac{1}{4}$, 0, $\frac{3}{4}$, 0) value $\frac{7}{4}$.

2. (i) R($\frac{7}{15}$, $\frac{8}{15}$, 0)C($\frac{3}{5}$, $\frac{2}{5}$) value $-\frac{1}{5}$; (ii) R(0, $\frac{2}{5}$, $\frac{3}{5}$)C($\frac{1}{5}$, $\frac{4}{5}$) value $-\frac{3}{5}$; (iii) R($\frac{1}{2}$, $\frac{1}{2}$)C(0, $\frac{1}{4}$, 0, $\frac{3}{4}$) value 3; (iv) R($\frac{2}{5}$, $\frac{3}{5}$)C(0, $\frac{1}{2}$, $\frac{1}{2}$, 0) value 0.

3. (i) R($\frac{6}{7}$, $\frac{1}{7}$)C($\frac{3}{7}$, 0, $\frac{4}{7}$) value $\frac{3}{7}$; (ii) R($\frac{1}{3}$, $\frac{2}{3}$)C($\frac{7}{15}$, $\frac{8}{15}$, 0) value $\frac{2}{3}$; (iii) R($\frac{7}{8}$, 0, $\frac{1}{8}$)C($\frac{5}{8}$, 0, $\frac{3}{8}$) value $-\frac{3}{4}$; (iv) R($\frac{1}{4}$, 0, $\frac{3}{4}$)C($\frac{1}{2}$, $\frac{1}{2}$, 0) value 0.

4. For each player, say the strategies are "penny, nickel, dime" in that order.

1	5	-1
1	5	-5
-10	-10	10

R($\frac{10}{11}$, 0, $\frac{1}{11}$)C($\frac{1}{2}$, 0, $\frac{1}{2}$) value 0.

Section 7.4

3. (i) R($\frac{1}{2}$, $\frac{1}{2}$, 0)C($\frac{1}{14}$, $\frac{4}{7}$, $\frac{5}{14}$) value 0; (ii) R($\frac{1}{8}$, $\frac{7}{16}$, $\frac{7}{16}$)C($\frac{1}{6}$, $\frac{1}{2}$, $\frac{1}{3}$) value $-\frac{1}{2}$; (iii) R($\frac{1}{2}$, 0, $\frac{1}{2}$)C(0, $\frac{1}{2}$, $\frac{1}{2}$) value 0; (iv) R($\frac{1}{4}$, 0, $\frac{3}{4}$)C(0, $\frac{1}{4}$, $\frac{3}{4}$) value $\frac{1}{2}$.

4. (i) R($\frac{3}{11}$, $\frac{5}{11}$, $\frac{3}{11}$)C($\frac{5}{22}$, $\frac{4}{11}$, $\frac{9}{22}$) value $-\frac{1}{11}$; (ii) R($\frac{2}{3}$, $\frac{1}{6}$, $\frac{1}{6}$C(0, $\frac{1}{6}$, $\frac{1}{6}$, $\frac{2}{3}$) value $-\frac{1}{3}$; (iii) R($\frac{3}{11}$, $\frac{8}{11}$, 0)C($\frac{7}{11}$, 0, $\frac{4}{11}$) value $\frac{1}{11}$; (iv) R($\frac{10}{21}$, $\frac{8}{21}$, $\frac{1}{7}$C($\frac{1}{2}$, 0, $\frac{5}{14}$, $\frac{1}{7}$) value $\frac{1}{7}$.

5. (i) R(0, $\frac{1}{3}$, $\frac{1}{3}$, $\frac{1}{3}$)C($\frac{5}{9}$, $\frac{2}{9}$, 0, $\frac{2}{9}$) value $-\frac{1}{3}$; (ii) R($\frac{3}{20}$, $\frac{9}{20}$, $\frac{2}{5}$, 0)C(0, $\frac{1}{2}$, $\frac{1}{2}$) value $\frac{1}{2}$; (iii) R($\frac{2}{3}$, 0, $\frac{1}{3}$, 0)C0, 0, $\frac{7}{12}$, $\frac{5}{12}$) value $\frac{2}{3}$; (iv) R($\frac{5}{9}$, 0, $\frac{7}{18}$, $\frac{1}{18}$)C($\frac{2}{9}$, 0, $\frac{3}{9}$, $\frac{4}{9}$) value $\frac{7}{9}$.

Section 8.1

1. (i) \$5350; (ii) \$6050; (iii) \$6750. **2.** (i) \$384; (ii) \$960; (iii) \$1536. **3.** (i) \$12040; (ii) \$12120; (iii) \$12200. **4.** (i) \$ 2512.50; (ii) \$2525; (iii) \$2575. **5.** 8%. **6.** 10%. **7.** (i) \$2200; (ii) \$1500; (iii) \$1200. **8.** 24%. **9.** 8.16%. **10.** \$113.33. **11.** \$236. **12.** \$40000; \$666.67. **13.** \$21917.81; \$608.83.

Section 8.2

1. (i) \$1125.51; (ii) \$1123.60; (iii) \$1120. **2.** (i) \$27.05; (ii) \$26.90; (iii) \$26.25. **3.** \$3588.26. **4.** \$991.92. **5.** \$225.36. **6.** \$2338.10. **7.** \$2419.99. **8.** \$7440.94. **9.** 12 (actually about $11\frac{2}{3}$ years). **10.** After 6 years. **11.** (i) 5.095%; (ii) 6.136%; (iii) 3.042%. **12.** (i) 12%; (ii) 12.55%; (iii) 12.68%; (iv) 12.75%; (v) 12.75%.

Section 8.3

1. \$27223.76. **2.** \$24664.64. **3.** \$91.68. **4.** (i) \$2501.52; (ii) \$8230.54; (iii) \$15084.83; (iv) \$27971.23. **5.** (i) \$2028.53; (ii) \$1609.25. **6.** \$758.48. **7.** \$200; \$193.43; the credit union is better. **8.** (i) 28951.22 (about 29%); (ii) 45996.46 (about 46%); (iii) 56641.35 (about 57%).

Section 8.4

1. (i) \$1025.32; (ii) \$1221.40; (iii) \$2718.28. **2.** (i) \$12544; (ii) \$17623.42. **3.** \$176622.22. **4.** The CD (it returns \$1159.27 for each \$1000 invested; housing returns \$1129.74). **5.** (i) $5376741 \times (1.012)^{10} = 6057930$; (ii) $5376741 \times (1.012)^{50} = 9762135$. **6.** 2550×1.2; 2550×1.2^4. **7.** April 5th. **8.** 20 grams.

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