

# Appendix A

## Appendix

### Proof of Theorem 2.8

As it was mentioned after the statement of Theorem 2.8 it suffices to show that the operator  $T_m$  is of weak type (1,1), that is, there exists  $c_1 > 0$  such that for every  $f \in L^1(\mathbb{R}^n)$

$$\sup_{\alpha > 0} \alpha |\{x \in \mathbb{R}^n : |T_m f(x)| > \alpha\}| \leq c_1 \|f\|_1. \quad (\text{A.1})$$

To establish (A.1) we need the Calderón–Zygmund decomposition of  $L^1$ -functions.

**Lemma A.1 (Calderón–Zygmund lemma).** *Let  $f \in L^1(\mathbb{R}^n)$ . For any  $\alpha > 0$ ,  $f$  can be decomposed as*

$$f = g + b = g + \sum_{j=1}^{\infty} b_j \quad (\text{A.2})$$

such that

$$|g(x)| \leq 2^n \alpha \quad \text{a.e. } x \in \mathbb{R}^n, \quad (\text{A.3})$$

$$b_j \text{ is supported in a cube } Q_j, \text{ with } \int_{Q_j} b_j dx = 0, \quad (\text{A.4})$$

$$\text{the } Q_j \text{ have disjoint interior, } \sum_{j=1}^{\infty} |Q_j| \leq \alpha^{-1} \|f\|_1, \quad (\text{A.5})$$

and

$$\|g\|_1 + \sum_{j=1}^{\infty} \|b_j\|_1 \leq 6 \|f\|_1. \quad (\text{A.6})$$

*Proof.* Assume  $f \geq 0$  (otherwise  $f = f^+ - f^-$  and decompose each part). Since  $f \in L^1(\mathbb{R}^n)$  there exists  $l$  such that  $\frac{1}{|Q|} \int_Q f dy < \alpha$  for any cube of side length  $l$ .

Divide  $\mathbb{R}^n$  into a mesh of cubes of side length  $l$  parallel to the axes. Let  $Q^0$  be one of them. Divide each side of  $Q^0$  in two to get  $2^n$  cubes of side length  $l/2$ . Let  $Q^1$  be such a cube there are two possibilities:

$$(a) \quad \frac{1}{|Q^1|} \int_{Q^1} f dy < \alpha \quad \text{or} \quad (b) \quad \frac{1}{|Q^1|} \int_{Q^1} f dy \geq \alpha.$$

In case (b) one stops the subdivision, noticing that

$$\alpha \leq \frac{1}{|Q^1|} \int_{Q^1} f dy \leq \frac{2^n}{|Q^0|} \int_{Q^0} f dy \leq 2^n \alpha, \quad (A.7)$$

and collecting it in a sequence  $Q_j$ .

In case (a) the subdivision process continues. Thus, if  $x \notin \bigcup_j Q_j$  it follows from the Lebesgue differentiation theorem (Exercise 2.6 (ii)) that

$$f(x) \leq \alpha \quad \text{a.e.} \quad x \in \mathbb{R}^n \setminus \bigcup_j Q_j. \quad (A.8)$$

Finally, we define

$$g(x) = \begin{cases} \frac{1}{|Q_j|} \int_{Q_j} f dy & \text{if } x \in Q_j, \\ f(x) & \text{if } x \notin Q_j, \end{cases} \quad (A.9)$$

and

$$b_j(x) = (f(x) - g(x)) \chi_{Q_j}(x), \quad j \in \mathbb{Z}^+, \quad (A.10)$$

which yields the result.  $\square$

We shall denote by  $Q_j^*$  the cube having the same center as  $Q_j$  and twice its side length as

$$\Omega = \bigcup_j Q_j \quad \text{and} \quad \Omega^* = \bigcup_j Q_j^* \quad (A.11)$$

with

$$|\Omega^*| \leq \sum_j |Q_j^*| = 2^n \sum_j |Q_j|. \quad (A.12)$$

*Proof of inequality (A.1).* First we notice that using Calderón–Zygmund Lemma

$$\begin{aligned}
 & |\{x \in \mathbb{R}^n : |T_m f(x)| > \alpha\}| \\
 & \leq |\{x \in \mathbb{R}^n : |T_m g(x)| > \alpha/2\}| + |\{x \in \mathbb{R}^n : |T_m b(x)| > \alpha/2\}| \\
 & \leq |\{x \in \mathbb{R}^n : |T_m g(x)| > \alpha/2\}| + |\{x \notin \Omega^* : |T_m b(x)| > \alpha\}| + |\Omega^*| \\
 & = E_1 + E_2 + E_3.
 \end{aligned} \tag{A.13}$$

From (A.12) and (A.5) in Calderón–Zygmung Lemma we have that

$$E_3 = |\Omega^*| \leq 2^n \sum_j |Q_j| \leq 2^n \alpha^{-1} \|f\|_1. \tag{A.14}$$

Tchebychev’s inequality and (A.3) in the Calderón–Zygmund lemma yield

$$\begin{aligned}
 E_1 & = |\{x \in \mathbb{R}^n : |T_m g(x)| > \alpha/2\}| \leq c \left( \frac{\|T_m g\|_2}{\alpha/2} \right)^2 \leq c \frac{\|g\|_2^2}{\alpha^2} \\
 & \leq \frac{c}{\alpha^2} \|g\|_1 \|g\|_\infty \leq \frac{c}{\alpha} \|g\|_1 \leq \frac{c}{\alpha} \|f\|_1.
 \end{aligned} \tag{A.15}$$

Hence it remains to prove that

$$E_2 = |\{x \notin \Omega^* : |T_m b(x)| > \alpha/2\}| \leq c \alpha^{-1} \|f\|_1. \tag{A.16}$$

It will suffice to show that

$$\int_{x \notin Q_j^*} |T_m b_j(x)| dx \leq c \|b_j\|_1, \quad j \in Z^+. \tag{A.17}$$

To establish (A.17) we follow the argument in [Sg].

Let  $\varphi \in C_0^\infty(\{\xi : |\xi| < 2\})$  such that  $\varphi(\xi) = 1$  for  $|\xi| \leq 1$ . Let  $\beta(\xi) = \varphi(\xi) - \varphi(2\xi)$ . Thus

$$\sum_{l=-\infty}^{\infty} \beta(2^{-l}\xi) = 1 \quad \text{for } \xi \neq 0. \tag{A.18}$$

If  $m_l(\xi) = \beta(\xi) m(2^l \xi)$ , then by hypothesis (2.18) it follows that

$$\int |(1 - \Delta)^{s/2} m_l(\xi)|^2 d\xi < c. \tag{A.19}$$

Thus by Plancherel’s identity using the notation  $K_l(x) = \widehat{m}_l(x)$ , one gets that

$$\int (1 + |x|^2)^s |K_l(x)|^2 dx < c, \tag{A.20}$$

which combined with the Cauchy–Schwarz inequality yields the estimate

$$\int_{\{x: \max_m |x_m| > R\}} |K_l(x)| dx < c R^{n/2-s}, \tag{A.21}$$

which is a good estimate for  $R \gg 1$ .

Reapplying the estimates (A.19)–(A.20) for  $\xi_l m_l(\xi)$  instead of  $m_l(\xi)$  one finds that

$$\int |\nabla K_l(x)| dx < c. \tag{A.22}$$

Consequently, it follows that

$$\int |K_l(x+y) - K_l(x)| dx < c|y|. \tag{A.23}$$

We observe that as a temperate distribution,

$$K(x) = \sum_{l=-\infty}^{\infty} 2^{nl} K_l(2^l x) = \sum_{l=-\infty}^{\infty} \widehat{m}_l(2^{-l} x). \tag{A.24}$$

Assume that  $Q_j$  is a cube of side  $R$  centered at the origin. From (A.21) one has that

$$\begin{aligned} \int_{x \notin Q_j^*} |2^{nl} K_l(2^l \cdot) * b_j| dx &\leq \int_{Q_j} \int_{x \notin Q_j^*} |2^{nl} K_l(2^l(x-y))| |b_j(y)| dx dy \\ &\leq \|b_j\|_1 \int_{\{x: \max_m |x_m| \geq 2^l R\}} |K_l(x)| dx \\ &\leq c(2^l R)^{n/2-s} \|b_j\|_1. \end{aligned} \tag{A.25}$$

Now using that  $\int_{Q_j} b_j dy = 0$  it follows that

$$\begin{aligned} &\int_{x \notin Q_j^*} 2^{nl} \int_{y \in Q_j} K_l(2^l(x-y)) b_j(y) dy dx \\ &= \int_{x \notin Q_j^*} 2^{nl} \int_{y \in Q_j} (K_l(2^l(x-y)) - K_l(2^l x)) b_j(y) dy dx. \end{aligned} \tag{A.26}$$

Therefore, (A.23) yields

$$\begin{aligned} &\int_{x \notin Q_j^*} |2^{nl} K_l(2^{nL} \cdot) * b_j| dx \\ &\leq \int_{y \in Q_j} \int_{x \notin Q_j^*} 2^{nl} |K_l(2^l(x-y)) - K_l(2^l x)| |b_j(y)| dx dy \\ &\leq c(2^l R) \|b_j\|_1. \end{aligned} \tag{A.27}$$

Adding in  $l$  in (A.25) for  $2^l R > 1$  and in (A.27) for  $2^l R \leq 1$  one gets that

$$\int_{x \notin Q_j^*} |T_m b_j(x)| dx \leq c \|b_j\|_1, \quad (\text{A.28})$$

which completes the proof.

□

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# Index

- N*-solitons, 186
- A-regular, 72
- A-super regular, 72
- almost conserved quantities, 176
- asymptotic flatness, 218, 228
  
- Benjamin–Ono equation, vii, 81, 191, 196
  - generalized, 199
- Bicharacteristic flow, 53, 80
  - nontrapped, 82
- bilinear estimates, 141, 163, 177
- blow up (KdV), 173, 183
- blow up (NLS), 119, 120, 122, 123, 131–133
  
- Calderón–Zygmund lemma, 233
- Camassa–Holm equation, 208
- Christoffel symbol, 212
- classical symbols, 51
- Cole–Hopf transformation, 210
- commutator estimates, 50, 87
- compactons, 208
- concentration, 120, 129, 130
  
- Davey–Stewartson systems, vii, 191
- decay properties, 108
- defocusing, 90
- differential operator, 5
- distribution function, 30
- Duhamel’s principle, 86
  
- embedding, 47
  
- focusing, 90
- Fourier transform, 1
- fractional chain rule, 146
- fractional Leibniz rule, 146
  
- Gagliardo–Nirenberg inequality, 50, 56, 101, 121, 174, 180
- Galilean invariance, 91, 115
- Gauss summation method, 5
- global smoothing, 64
  
- Hamiltonian flow, 218
- Hamiltonian system, 196
- Hamiltonian vector field, 53, 54
- Hardy’s inequality, 20, 57
- Hardy–Littlewood maximal function, 33
- Hardy–Littlewood theorem, 34
- Hardy–Littlewood–Sobolev theorem, 35, 65, 144
- Hausdorff–Young’s inequality, 29
- Heisenberg’s inequality, 57, 128
- higher order KdV equations, 191
- Hilbert transform, 12, 22, 40, 145
- homogeneous smoothing effect, 79
  
- I-method, 176
- ill-posedness, 154
- inhomogeneous smoothing effect, 79
- instability, 182, 185
- inverse scattering method, 203
- Ishimori equations, vii, 191, 193
  
- k-gKdV equation, 139, 140
- KP equations, 191
- Korteweg–de Vries equation, vi, 139, 155
  - blow up, 183
  - critical, 149, 151, 173
  - generalized, vii, 149
  - modified, 139, 146, 170
- KP equations, vii, 194
  
- Liouville’s type theorem, 181

- local smoothing, 67, 79
- Marcinkiewicz interpolation theorem, 29
- Maximal function estimates, 143
- Mihlin–Hörmander’s theorem, 38
- Minkowski integral inequality, 19
- Miura transformation, 139, 167
- Morawetz’s estimate, 138
- multiplier, 38, 41
  
- Nash–Moser techniques, 217
- nonisotropic, 211
- Nonlinear Schrödinger equation, vi
  - nontrapped, 54
  - nontrapping condition, 217, 219, 220
  
- oscillatory integrals, 13
  
- Paley–Wiener theorem, 21
- parabolic regularization, 199, 219
- Plancherel theorem, 6
- Pohozaev’s identity, 90, 116
- Poisson bracket, 52
- pseudoconformal invariance, 91, 129
  
- Riemann–Lebesgue lemma, 1
- Riesz potentials, 35
- Riesz transform, 41
- Riesz–Thorin theorem, 27, 37
  
- Schrödinger equation, vi
  - potential, 76, 108
- Schrödinger flow, 211
  
- Schwartz space, 9, 46
- sharp Garding’s inequality, 223, 229
- Sobolev boundedness, 52
- Sobolev embedding, 156
- Sobolev spaces, 45
- solitary waves, 91, 140, 154
- soliton theory, vi
- stability, 182, 185
- standing waves, 90, 135
  - instability, 135
  - stability, 135
- Stein interpolation theorem, 37, 145
- Stein–Tomas restriction theorem, 144
- Stone theorem, 62
- Strichartz estimates, 75
- Strichartz estimates, 73, 76, 106, 110
- sublinear operator, 28
- symbolic calculus, 52
  
- tempered distributions, 8, 9
  
- unitary group of operators, 62
  
- van der Corput lemma, 15
- Vitali’s covering lemma, 33
  
- weak derivatives, 47
- weak type operator, 31
  
- Young’s inequality, 6, 19, 28
  
- Zakharov system, vii, 191

- Aguilar, M.; Gitler, S.; Prieto, C.:* Algebraic Topology from a Homotopical Viewpoint
- Ahlsvede, R.; Blinovskiy, V.:* Lectures on Advances in Combinatorics
- Aksoy, A.; Khamsi, M. A.:* Methods in Fixed Point Theory
- Alevras, D.; Padberg M. W.:* Linear Optimization and Extensions
- Andersson, M.:* Topics in Complex Analysis
- Aoki, M.:* State Space Modeling of Time Series
- Arnold, V. I.:* Lectures on Partial Differential Equations
- Arnold, V. I.; Cooke, R.:* Ordinary Differential Equations
- Audin, M.:* Geometry
- Aupetit, B.:* A Primer on Spectral Theory
- Bachem, A.; Kern, W.:* Linear Programming Duality
- Bachmann, G.; Narici, L.; Beckenstein, E.:* Fourier and Wavelet Analysis
- Badescu, L.:* Algebraic Surfaces
- Balakrishnan, R.; Ranganathan, K.:* A Textbook of Graph Theory
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- Boltyanskii, V. G.; Efremovich, V. A.:* Intuitive Combinatorial Topology
- Bonnans, J. F.; Gilbert, J. C.; Lemarchal, C.; Sagastizbal, C. A.:* Numerical Optimization
- Booss, B.; Bleecker, D. D.:* Topology and Analysis
- Borkar, V. S.:* Probability Theory
- Brides/Vita:* Techniques of Constructive Analysis
- Bruiner, J. H.:* The 1-2-3 of Modular Forms
- Brunt B. van:* The Calculus of Variations
- Bühlmann, H.; Gisler, A.:* A Course in Credibility Theory and its Applications
- Carleson, L.; Gamelin, T. W.:* Complex Dynamics
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- Chae, S. B.:* Lebesgue Integration
- Chandrasekharan, K.:* Classical Fourier Transform
- Charlap, L. S.:* Bieberbach Groups and Flat Manifolds
- Chern, S.:* Complex Manifolds without Potential Theory
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