

## R Software for Wavelets and Statistics

Here is a list and a brief description of some important R packages related to/that make use of wavelets. Such packages are important as they enable ideas to be reproduced, checked, and modified by the scientific community. This is probably just as important as the advertisement of the ideas through scientific papers and books. This is the ‘reproducible research’ view expressed by Buckheit and Donoho (1995).

Making software packages freely available is an extremely valuable service to the community and so we choose to acclaim the author(s) of each package by displaying their names! The descriptions are extracted from each package description from CRAN. The dates refer to the latest updates on CRAN, not the original publication date. Please let me know of any omissions.

**adlift** performs adaptive lifting (generalized wavelet transform) for irregularly spaced data, see Nunes et al. (2006). Written by Matt Nunes and Marina Knight, University of Bristol, UK, 2006.

**brainwaver** computes a set of scale-dependent correlation matrices between sets of preprocessed functional magnetic resonance imaging data and then performs a ‘small-world’ analysis of these matrices, see Achard et al. (2006). Written by Sophie Achard, Brain Mapping Unit, University of Cambridge, UK, 2007.

**CVThresh** carries out level-dependent cross-validation for threshold selection in wavelet shrinkage, see Kim and Oh (2006). Written by Donghoh Kim, Hongik University and Hee-Seok Oh, Seoul National University, Korea, 2006.

**DDHFm** implements the data-driven Haar–Fisz transform as described in 6.5, see references therein. Written by Efthimios Motakis, Piotr Fryzlewicz and Guy Nason, University of Bristol, Bristol, UK.

**EbayesThresh** carries out Empirical Bayes thresholding as described in section 3.10.4, see papers listed there. Written by Bernard Silverman, Oxford University, UK, 2005.

- nlt** non-decimated lifting transform (generalized wavelet transform) for irregularly spaced data, see Knight and Nason (2008). Written by Marina Knight, University of Bristol, UK, 2008.
- rwt** The Rice Wavelet Toolbox wrapper. The Rice Wavelet Toolbox is a collection of Matlab files for 1D and 2D wavelet and filter bank design, analysis, and processing. Written by P. Roebuck, Rice University, Texas, USA, 2005.
- SpherWave** carries out the wavelet transform of functions on the sphere and related function estimation techniques. See Li (1999), Oh (1999), Oh and Li (2004). Written by Hee-Seok Oh, Seoul National University and Donghoh Kim, Hongik University, Korea, 2007.
- unbalhaar** computes the unbalanced Haar transform and related function estimation, see Fryzlewicz (2007). Written by Piotr Fryzlewicz, Bristol University, UK, 2006.
- waved** wavelet deconvolution of noisy signals, see Section 4.9, Raimondo and Stewart (2007). Written by Marc Raimondo and Michael Stewart, University of Sydney, Australia, 2007.
- wavelets** computes and plots discrete wavelet transform and maximal overlap discrete wavelet transforms. Written by Eric Aldrich, Duke University, North Carolina, USA, 2007.
- waveslim** computes 1D, 2D, and 3D wavelet transforms, packet transforms, maximal overlap transforms, dual-tree complex wavelet transforms, and much more! Based on methodology described in Percival and Walden (2000) and Gencay et al. (2001). Written by Brandon Whitcer, dude, Translational Medicine and Genetics, GlaxoSmithKline, Cambridge, UK, 2007.
- wmtsa** software to accompany the book Percival and Walden (2000). Written by William Constantine, Insightful Corporation, and Donald Percival, Applied Physics Laboratory, University of Washington, Seattle, USA, 2007.

# B

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## Notation and Some Mathematical Concepts

This appendix gives some extra information, notation, and definitions on the more mathematical aspects underlying the concepts of this book. More details can be found in a number of elementary mathematical texts. In the wavelet context the book, Daubechies (1992) is precise and concise and Burrus et al. (1997) is a very readable introduction.

### B.1 Notation and Concepts

#### B.1.1 Function spaces

In mathematical terms the book deals mostly with measurable functions  $f$  on  $\mathbb{R}$  such that

$$\int_{-\infty}^{\infty} |f(x)|^2 dx < \infty. \quad (\text{B.1})$$

The space of functions satisfying (B.1) is denoted  $L^2(\mathbb{R})$ . In engineering terms this means that the function, interpreted as a signal, has finite energy.

We let  $\mathcal{C}$  the space of continuous functions (on  $\mathbb{R}$ ),  $\mathcal{C}^k$  the space of all continuous functions with continuous derivatives of orders up to, and including,  $k$ , and  $\mathcal{C}^\infty$  the space of all continuous functions with continuous derivatives of all orders.

#### B.1.2 Support of a function

The *support* of a function,  $f$  (denoted  $\text{supp } f$ ), is the complement of the largest open set  $E$  with the property that  $x \in E \implies f(x) = 0$ . It is the ‘maximal set on which  $f$  is non-zero and inclusive’.

The function  $f$  is compactly supported if  $\text{supp } f$  is a compact set (closed and bounded).

**B.1.3 Inner product, norms, and distance**

Given two functions  $f, g \in L^2(\mathbb{R})$  we can define the *inner product* of these by

$$\langle f, g \rangle = \int_{\mathbb{R}} f(x) \overline{g(x)} dx, \quad (\text{B.2})$$

where  $\overline{g(x)}$  denotes the complex conjugate of  $g(x)$ . The inner product can be used to gauge the ‘size’ of a function by defining the  $L^2$  norm  $\|\cdot\|_2$  as follows:

$$\|f\|_2^2 = \langle f, f \rangle, \quad (\text{B.3})$$

which leads naturally to the notion of the ‘distance’ between two functions  $f, g \in L^2(\mathbb{R})$  as  $\|f - g\|_2$ .

**B.1.4 Orthogonality and orthonormality**

Two functions  $f, g \in L^2(\mathbb{R})$  are said to be *orthogonal* if and only if  $\langle f, g \rangle = 0$ . Define the *Kronecker delta* by

$$\delta_{ij} = \begin{cases} 1 & \text{if } i = j \\ 0 & \text{if } i \neq j, \end{cases} \quad (\text{B.4})$$

for integers  $i, j$ .

A set of functions  $\{\phi_1, \dots, \phi_n\}$  for some  $n = 1, 2, \dots$  forms an *orthonormal set* if and only if

$$\langle \phi_i, \phi_j \rangle = \delta_{ij}. \quad (\text{B.5})$$

**B.1.5 Vector space sums**

Let  $V, W$  be subspaces of  $L^2(\mathbb{R})$ . We define the following spaces. The vector space sum of  $V$  and  $W$  is defined as

$$V + W = \{f + g : f \in V, g \in W\}. \quad (\text{B.6})$$

The subspaces  $V$  and  $W$  are said to be orthogonal and denoted  $V \perp W$  if every function  $V$  is orthogonal to every function in  $W$  (i.e.  $V \perp W$  if and only if  $\langle f, g \rangle = 0$  for all  $f \in V, g \in W$ ). We further define

$$V \oplus W = \{f + g : f \in V, g \in W, V \perp W\}. \quad (\text{B.7})$$

**B.1.6 Fourier transform**

The Fourier transform of a function  $f \in L^2$  is given by

$$\hat{f}(\omega) = (2\pi)^{-1/2} \int_{-\infty}^{\infty} f(x) \exp(-i\omega x) dx. \quad (\text{B.8})$$

The inverse Fourier transform of  $\hat{f}(\omega)$  is given by

$$f(x) = (2\pi)^{-1/2} \int_{-\infty}^{\infty} \hat{f}(\omega) \exp(i\omega x) d\omega. \quad (\text{B.9})$$

### B.1.7 Fourier series

Statisticians are probably most familiar with Fourier series that occur routinely in time series analysis. Given a stationary time series  $\{X_t\}$  with auto-covariance function  $\gamma(\tau)$  for integers  $\tau$ , the spectral density function,  $f(\omega)$ , can be written as

$$f(\omega) = (2\pi)^{-1} \sum_{\tau=-\infty}^{\infty} \gamma(\tau) \exp(-i\omega\tau), \tag{B.10}$$

where  $\omega \in (-\pi, \pi]$ . The inverse Fourier relationship is

$$\gamma(\tau) = \int_{-\pi}^{\pi} f(\omega) \exp(i\omega\tau) d\omega, \tag{B.11}$$

for integer  $\tau$ .

There are many other slightly different formulations of this. For example, Formulae (2.50) and (2.84) are both forms of (B.10) but with different scale factors.

### B.1.8 Besov spaces

In this book much of what we do applies to functions that belong to more interesting function spaces. The natural home for wavelets are Besov spaces; an informal discussion appears in Section 3.10.3. The Besov sequence space norm for a function  $f(x)$  is defined in terms of its wavelet coefficients  $\{d_{j,k}\}$  and coarsest-scale coefficient  $c_{0,0}$  by

$$\|d\|_{b_{p,q}^s} = |c_{0,0}| + \left\{ \sum_{j=0}^{\infty} 2^{js'q} \left( \sum_{k=0}^{2^j-1} |d_{j,k}|^p \right)^{q/p} \right\}^{1/q}, \tag{B.12}$$

if  $1 \leq q < \infty$  and

$$\|d\|_{b_{p,\infty}^s} = |c_{0,0}| + \sup_{j \geq 0} \left\{ 2^{js'} \left( \sum_{k=0}^{2^j-1} |d_{j,k}|^p \right)^{1/p} \right\}, \tag{B.13}$$

where  $s' = s + \frac{1}{2} - 1/p$ , see Abramovich et al. (1998). The sequence space norm is equivalent to a Besov function space norm on  $f$ , denoted  $\|f\|_{B_{p,q}^s}$ . Hence, one can test membership of a Besov space for a function by examining its wavelet coefficients (for a wavelet with enough smoothness).

**B.1.9 Landau notation**

We use Landau notation to express the computational efficiency of algorithms. Suppose  $f(x), g(x)$  are two functions. Then  $f(x) = \mathcal{O}\{g(x)\}$  means that there exists  $x^*$  and a constant  $K > 0$  such that for  $x \geq x^*$  we have  $|f(x)| \leq K|g(x)|$ . For example, we use the notation in the following way. Suppose that we have an algorithm whose (positive) running time is  $r(n)$ , where the size of the problem is measured by  $n$ . Then we can say that the algorithm is  $\mathcal{O}\{g(n)\}$  if there exists  $K > 0$  such that for large enough  $n$  we have  $r(n) \leq Kg(n)$ . For example, multiplication of an  $n \times 1$  vector by an  $n \times n$  matrix is  $\mathcal{O}(n^2)$ .

# C

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## Survival Function Code

This section contains the code for the remainder of the survival function estimation described in Section 4.8.

The function `uk` computes the  $U_k$  values as described in Antoniadis et al. (1999) as follows:

```
uk <- function(z, nbins=32){  
  
  anshfc <- hfc(z, nbins=nbins)  
  uk <- anshfc$bincounts/anshfc$del  
  
  ans <- list(uk=uk, bins=anshfc$bins, del=anshfc$del)  
  
  ans  
}
```

The `uk` function is essentially a  $\Delta$ -weighted version of `hfc`. The `subf` function below computes the subdensity estimator of  $f^*(t)$  as follows:

```
subf <- function(z, nbins=32, filter.number=2, lev=1){  
  
  tmp <- uk(z, nbins=nbins)  
  
  zuk <- tmp$uk  
  
  yywd <- wd(zuk, filter.number=filter.number, bc="interval")  
  
  levs <- nlevels(yywd) - (1:lev)  
  yywdT <- nullevels(yywd, levelstonull=levs)  
  
  fhat <- wr(yywdT)  
  
  fhat[fhat < 0] <- 0
```

```

l <- list(fhat = fhat, bins=tmp$bins, del=tmp$del)
l
}

```

This function is very similar to the `hws` linear wavelet smooth in the main text with the addition that negative density values are prohibited and set to zero. The final, hazard estimation, function computes an estimate of the ratio  $f^*(t)/L(t)$  by dividing the outputs of `subf` by `Lest` as follows:

```

hazest <- function(z, nbins=32, levN=1, levD=1,
  filter.number=8){

L <- Lest(z=z, nbins=nbins, lev=levD)

fsub <- subf(z=z, nbins=nbins, filter.number=filter.number,
  lev=levN)

haz <- fsub$fhat / L$L

ans <- list(haz=haz, bins=fsub$bins, del=fsub$del, L=L$L,
  fsub=fsub$fhat)
ans
}

```



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# Index

- example.1(), 89
- AvBasis, 114
- BabyECG, 168, 187
- BabySS, 169
- DDHFm, 221
- DJ.EX, 34, 88
- GenW, 27, 44
- InvBasis, 74, 116
- LSWsim, 180
- MaNoVe, 74
- PsiJ, 182
- accessC, 24
- accessD, 23, 61, 94
- cns, 179
- cthresh, 124
- ddhft.np.2, 221
- denplot, 156
- denproj, 156
- denwr, 156
- draw.default, 43
- draw, 43
- ebayesthresh, 215
- ewspec, 189
- filter.select, 42
- getpacket, 62
- hft, 212
- imwd, 78, 143
- imwr, 78, 143
- ipndacw, 184
- irregwd, 147
- makegrid, 147
- mwd, 120
- mwr, 120
- numtonv, 116
- plot.irregwd, 148
- plot.wd, 24
- plot.wp, 72
- plot.wst, 65
- putD, 179
- putpacket, 74
- threshold, 94, 99
- wd3D, 78
- wd, 22, 43, 61, 78, 176
- wp, 72
- wr3D, 78
- wr, 78, 94, 176
- wst2D, 78
- wst, 62, 78
- wvmoments, 41
- airline passenger data, 221
- Allan variance, 173
- Anscombe transformation, 201
- ARMA, 134, 171
- autocorrelation, 171
- autocorrelation wavelet, 181, 183
- autocovariance, 170
  - local, 181, 193
- bandelets, 142
- bandpass filter, 134
- basis averaging, 114, 125
- Battle-Lemarié, 49
- Bayesian wavelet shrinkage, 101, 121,  
131
- beamlets, 142

- bear
  - teddy, 78, 142
- Besov space, 106, 233
- best-basis algorithm, 71, 115
- biorthogonal wavelet, 50
- block thresholding, 128, 131
- BlockJS, 131
- Blocks, 34, 88, 214, 220
- Bumps, 88, 113, 135
  
- C*, 231
- cardinal *B*-spline, 48
- chirp function, 63
- Coiflets, 49
- compression, 72
- computational efficiency, 10, 15, 26, 47, 59, 60, 97, 110, 131, 146, 157, 210
- cone of influence, 190
- confidence intervals, 104, 150
- continuous function, 231
- continuous wavelet, 78
- convolution, 47
- correlated data, 133
- cross-validation, 98
- curvelets, 142
- cycle spinning, 111, 216
  
- data
  - airline passenger, 221
  - Baby, 168, 195
  - ECG, 168, 195
  - GOES-8 X-ray, 223
  - inductance plethysmography, 29
  - motorcycle, 146
  - seismic, 169
  - teddy, 3, 142
  - unemployment, 158
  - wind speeds, 198
- data driven Haar–Fisz, 217
- Daubechies’ wavelets, 41, 45, 47, 55, 68, 92, 134, 146, 152, 176, 184
- Daubechies–Lagarias algorithm, 156
- decimation, 51
- decorrelation, 134
- density estimation, 155
- diagonal transform, 203
- dilation equation, 31, 39
- discrete wavelet transform, 34
- discrete wavelets, 176
  
- Donoho and Johnstone functions, 88
- Doppler, 34, 88
- downsampling, 51
- DWT, 84, 152
- dyadic decimation, 51
  
- EbayesThresh, 215
- error measure, 85
- evolutionary wavelet spectrum (EWS), 179
  
- false discovery rate, 100
- FDR, 136
- filter
  - bandpass, 134
- filtering, 51
- Fisz transform, 203
- forecasting, 192
- Fourier
  - series, 233
  - transform, 39, 232
- functions
  - continuous, 231
  - distance, 232
  - inner product, 232
  - norm, 232
  - orthogonal, 232
  - orthonormal, 232
  - support, 231
  
- GOES-8 X-ray data, 223
  
- Hölder space, 106
- Haar MA process, 178
- Haar wavelet, 2, 41, 46, 55, 80, 110, 131, 138, 151, 159, 204, 209
  - father, 29, 207
  - mother, 33, 207
- Haar–Fisz transform, 209
  - data driven, 217
- hazard function estimation, 158
- Heavisine, 88
- heteroscedastic error, 149, 201
  
- initial wavelet coefficients, 52
- inner product, 232
- inverse problems, 47, 163
- inverse wavelet transform, 20, 54
- irregularly spaced data, 80, 143

- Kolmogorov formula, 193
- Kovac-Silverman method, 145
- Kronecker delta, 177, 232
  
- $L^2(\mathbb{R})$ , 231
- Landau notation, 234
- level, 17
- level-dependent, 133
- lifting, 80, 144
- linear wavelet smoothing, 109, 110, 158
- local autocovariance, 181
  - estimates, 194
- localization, 3, 209
- locally stationary wavelet process, 177
- LSW predictor, 192
  
- MAD, 133
- marginal maximum likelihood, 108
- matching pursuit, 197
- maximal overlap, 60
- mean square prediction error, 192
- measurable function, 231
- Meyer wavelet, 47
- MISE, 85, 98
- monotonic regression, 219
- Montserrat, 169, 190
- multiple wavelets, 66
- multiresolution analysis, 37
- multiwavelet, 66
  - shrinkage, 118, 131
  - TI denoising, 120
  
- noise
  - correlated, 133
  - double exponential, 138
  - heavy tailed, 138
  - heteroscedastic, 149, 201
  - iid, 83
  - multiplicative, 138
  - non-Gaussian, 138
  - Poisson, 138
- non-decimated wavelet packets, 75
- non-decimated wavelet transform, 58, 110
- non-negative wavelets, 157
- norm, 232
- Nyquist frequency, 190
  
- $\mathcal{O}$ , 234
- oracle, 87
- orthogonal functions, 232
- orthogonalization trick, 49
- orthonormal functions, 232
  
- Parseval's relation, 26, 84
- piecewise polynomial, 5, 88
- Poisson
  - variance stabilization, 201
- power law processes, 174
- prediction coefficients, 192
- primary resolution, 96, 124, 136, 155
  
- quadrature mirror filters, 55
  
- regression model
  - basic, 83
  - correlated, 133
  - multidimensional, 140
  - non-Gaussian, 138
  - wavelet, 84
- rescaled time, 179, 183, 197
- resolution, 17
- risk
  - ideal, 87
- RiskShrink, 96, 130
- RSAM, 169, 190
  
- S+Wavelets, 13
- saddlepoint, 153
- scale, 17
- Shannon entropy, 72
- sleep state, 169
- Slepian semi-wavelets, 157
- sparsity, 5, 20, 36, 41, 73, 84
  - prior, 101
- spectrum, 170, 179
  - evolutionary wavelet, 179
  - Fourier, 172, 179
  - wavelet, 172
- stationary, 133, 134, 169, 171
  - second order, 170, 197
  - strictly, 169
- Stein unbiased risk, 97
- Strömberg, 49
- support, 231
- SureBlock, 131
- SureShrink, 96

- survival function estimation, 158
- symmlet, 41
- teddy bear, 76, 78, 142
- threshold, 85
  - Bayesian shrinkage, 101, 121, 131
  - block, 128, 131
  - BlockJS, 131
  - complex-valued wavelets, 120, 131
  - cross-validation, 98
  - false discovery rate, 100, 131
  - for densities, 156
  - hazard function, 158
  - level-dependent, 133
  - multiwavelet, 118, 131
  - NeighBlock, 130
  - NeighCoeff, 130
  - RiskShrink, 96
  - SURE, 96, 135
  - SureBlock, 131
  - survival function, 158
  - universal, 87, 88, 135
- thresholding, 85
  - hard, 85
  - soft, 85
- TI denoising, 216
  - multiwavelets, 120
- time series
  - forecasting, 192
  - Haar MA, 178
  - locally stationary, 177
  - purely random, 171
  - SLEX, 198
  - transfer function models, 197
  - tv ARCH, 198
  - wavelet packets, 197
- time-scale, 37
- translation-invariant wavelet shrinkage, 111, 216
- triangular process array, 179
- unconditional basis, 8, 20
- universal threshold, 88
- vaguelette-wavelet decomposition, 163
- vanishing moments, 40
  - wvmoments, 41
- variance stabilization, 189, 201
  - Anscombe transform, 201
  - data driven Haar–Fisz, 217
  - Fisz transform, 203
  - Haar–Fisz transform, 209
  - kurtosis, 205
  - Poisson, 201
  - skewness, 205
- VisuShrink, 94, 130
- WaveD, 164
- WaveLab, 13
- wavelet
  - 2D, 140
  - autocorrelation, 181, 183
  - Battle–Lemarié, 49
  - biorthogonal, 50
  - cardinal spline, 49
  - Coiflets, 49
  - complex-valued, 45
  - continuous, 78
  - Daubechies, 41, 92, 134, 146, 152, 176, 184
  - decorrelation, 134
  - density estimation, 155
  - dilation equation, 31, 39
  - discrete, 176
  - discrete Haar, 176
  - Haar, 2, 41, 46, 55, 80, 110, 131, 138, 151, 159, 174, 204, 209
  - Littlewood Paley, 47
  - matrix, 25
  - maximal overlap, 60
  - Meyer, 47
  - multiple, 66
  - non-decimated discrete, 177
  - non-negative, 157
  - packet, 68
  - periodogram, corrected, 184
  - periodogram, raw, 183
  - powers of, 152
  - Shannon, 47, 184
  - shrinkage, 84
  - Slepian, 157
  - spectrum, 172
  - spline, 48
  - Strömberg, 49
  - two-scale relation, 31
  - vanishing moments, 40
  - variance, 174

- wavelet packets, 68, 172, 174, 197
  - best-basis, 71
  - non-decimated, 75
  - shrinkage, 130
  - Yates, 15
- wavelet processes, 174, 196
  - locally stationary, 177
- wavelet shrinkage, 83, 84, 186
  - basis averaging, 114, 125
  - block thresholding, 128
  - complex-valued wavelets, 120
  - cycle spinning, 111
  - inverse problem, 163
  - level-dependent, 133, 135
  - multiwavelet, 118
  - non-Gaussian, 138
  - translation-invariant, 111
- wavelet transform, 50, 84
  - $\epsilon$ -decimated, 57
  - boundaries, 55
  - continuous, 78
  - discrete, 34
  - inverse, 20, 176
  - multiple, 67
  - non-decimated, 58, 110
  - of noise, 8
  - three-dimensional, 78
  - two-dimensional, 76
- wavelet variance, 174
- wavelet-vaguelette decomposition, 163
- wedgelets, 141
  
- Yule-Walker equations, 193