A

Appendix

A.1 Selected equations used in GRASS modules

This appendix section includes equations for selected GRASS modules to provide theoretical background of the methods used in these modules and give users opportunity to assess advantages and limitations of their functionality. The equations are also helpful for those who would like to improve or extend the modules.

Basic Statistics

Several GRASS modules, such as `r.univar`, `v.univar`, `r.stats`, `r.series`, `r.neighbors` compute basic statistical measures. We list the relevant equations here for reference, but check the source code to see the exact implementation. In the following equations, we are using $\bar{x}$ for mean estimated from sample of the population; replace it by μ when you are working with the entire population.

Arithmetic mean:

$$\bar{x} = \frac{1}{n}(x_1 + x_2 + \ldots + x_n) = \frac{1}{n}\sum_{i=1}^{n} x_i \qquad \text{(A.1)}$$

Arithmetic mean $\bar{x}$ has the same units as x_i.

Median:
The median is the value below which 50% of the sample lie. To find the median, the data have to be ordered from the smallest to the highest. In case of an odd number of samples, it is the middle value; in case of an even number of samples, it is as half way between the two middle samples. Median has the same units as as the samples.

Mode:
The mode is defined as the most frequently occuring measurement in a data set. Continuous data need to be discretized into intervals to compute the mode. For data with normal distribution the mean, median and mode lead to the same value (in the limit of a large data sample).

Mean absolute deviation:

$$MD = \frac{1}{n}\sum_{i=1}^{n}|x_i - \bar{x}| \tag{A.2}$$

The mean absolute deviation is an expression of dispersion about the mean. If the given values are deviations (e.g., provided by `v.surf.rst`), then we need arithmetic mean of absolute values:

$$\bar{x}_D = \frac{1}{n}(|x_1| + |x_2| + \ldots + |x_n|) = \frac{1}{n}\sum_{i=1}^{n}|x_i| \tag{A.3}$$

Arithmetic mean of absolute values has the same units as x_i.

Variance for a population and for a sample:

$$\sigma^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2 \qquad \sigma^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2 \tag{A.4}$$

The variance is another measure of dispersion, it is mean squared deviation and has the squared units of x_i.

Standard deviation for a population and for a sample:

$$\sigma = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2} \qquad \sigma = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2} \tag{A.5}$$

The standard deviation is the positive square root of the variance and has the same units as x_p.

Coefficient of variation:

$$v = \frac{\sigma}{|\bar{x}|} * 100 \tag{A.6}$$

The coefficient of variation is the ratio of the standard deviation to the mean and is dimensionless. It is expressed as a percentage when multiplied by 100 as in the above equation.

Skewness:

$$skewness = \frac{1}{n}\sum_{i=1}^{n}\left(\frac{x_i - \bar{x}}{\sigma}\right)^3 \tag{A.7}$$

Skewness is zero for any symmetric distribution. A distribution with a long tail towards larger values has a positive skewness (left skewed, typical for remote sensing images, Schowengerdt, 1997:118). Skewness is dimensionless and sensitive to outliers.

Kurtosis:

$$kurtosis = \left[\frac{1}{n}\sum_{i=1}^{n}\left(\frac{x_i - \bar{x}}{\sigma}\right)^4\right] - 3 \tag{A.8}$$

Kurtosis is zero for a normal distribution. If a distribution has a positive kurtosis, then the peak is sharper than of a Gaussian distribution. Kurtosis is dimensionless and sensitive to outliers.

Covariance:

$$covariance = \frac{1}{n-1}\sum_{i=1}^{n}(x_{im} - \bar{x}_m)(x_{in} - \bar{x}_n) \tag{A.9}$$

Interpolation

Bilinear and bicubic interpolation Bilinear interpolation uses 4 neighboring cells to compute the unknown value z by performing linear interpolation in east-west and then north-east direction, leading to the following function:

$$z = a_0 + a_1 * x + a_2 * y + a_3 * x * y \tag{A.10}$$

Note that the above function is not linear, in spite of its name. Using the notation from the figure below the function can written as follows:

$$z = z12*(1-u)*(1-v) + z22*u*(1-v) + z11*(1-u)*v + z21*u*v \tag{A.11}$$

where $u = dx/px, v = dy/py$.

```
z12 --------------- z22
 |    | dy           |
 |---+ z             |
 | dx                | py
 |                   |
 |                   |
z11 ----------------z21
          px
```

Bicubic interpolation uses 16 cells leading to the following function:

$$\begin{aligned} z = a_0 + a_1 x + a_2 y + a_3 x^2 + a_4 xy + a_5 y^2 + a_6 x^2 y + a_7 xy^2 + a_8 x^2 y^2 \\ + a_9 x^3 + a_{10} y^3 + a_{11} x^3 y + a_{12} xy^3 + a_{13} x^3 y^2 + a_{14} x^2 y^3 + a_{15} 3x^3 y^3 \end{aligned} \quad \text{(A.12)}$$

The coefficients are derived from the height at the four vertices, together with three partial derivatives at each vertex estimated using the neighboring vertices.

Inverse distance weighted interpolation (IDW) The method is based on an assumption that the value at an unsampled point can be approximated as a weighted average of values at points within a certain cut-off distance, or from a given number m of the closest points (typically 10 to 30). Weights are usually inversely proportional to a power of distance (Watson, 1992; Burrough, 1986) which, at an unsampled location $\mathbf{r} = (x, y)$, leads to an estimator:

$$F(\mathbf{r}) = \sum_{i=1}^{m} w_i z(\mathbf{r}_i) = \frac{\sum_{i=1}^{m} z(\mathbf{r}_i)/|\mathbf{r} - \mathbf{r}_i|^p}{\sum_{j=1}^{m} 1/|\mathbf{r} - \mathbf{r}_j|^p} \quad \text{(A.13)}$$

where p is a parameter (typically $p = 2$, for more details on the influence of this parameter, see Watson, 1992). GRASS modules use $p = 2$ and $m = 12$ as default values.

Regularized Spline with Tension The function is a sum of a *trend function* and a *radial basis function* with an explicit form which depends on the choice of the measure of smoothness, for more details see Mitasova and Mitas (1993) and Mitasova et al. (1995):

$$z(\mathbf{r}) = T(\mathbf{r}) + \sum_{j=1}^{N} \lambda_j R(\mathbf{r}, \mathbf{r}^{[j]}) \, . \quad \text{(A.14)}$$

The trend function $T(\mathbf{r})$ is given by

$$T(\mathbf{r}) = \sum_{l=1}^{M} a_l f_l(\mathbf{r}) \quad \text{(A.15)}$$

where $\{f_l(\mathbf{r})\}$ is a set of linearly independent functions (monomials) which have zero smooth seminorm. $R(\mathbf{r}, \mathbf{r}^{[j]})$ is a radial basis function with an explicit form which depends on the choice of weights for partial derivatives in the smooth seminorm. See Mitasova and Mitas (1993); Mitasova et al. (1995) for the RST smoothness seminorm, which includes derivatives of all orders with their weights decreasing with the increasing derivative order.

RST can be generalized to an arbitrary dimension and the corresponding d-variate formula for the radial basis function is given by

$$R_d(\mathbf{r}, \mathbf{r}_j) = R_d(|\mathbf{r} - \mathbf{r}_j|) = R_d(r) = \varrho^{-\delta}\,\gamma\,(\delta, \varrho) - \frac{1}{\delta} \tag{A.16}$$

where $r = |\mathbf{r} - \mathbf{r}_j|$, $\delta = (d-2)/2$, and $\varrho = (\varphi r/2)^2$. Further, φ is a generalized tension parameter, and $\gamma(\delta, \varrho)$ is the incomplete gamma function, not to be confused with semivariogram (Abramowitz and Stegun, 1964). For the special cases $d = 2, 3, 4$ (`s.surf.rst`, `s.vol.rst`, `s.volt.rst`, respectively), the equation A.16 can be rewritten as:

$$R_2(r) = -\left[\mathrm{E}_1(\varrho) + \ln\varrho + C_E\right] \tag{A.17}$$

$$R_3(r) = \sqrt{\frac{\pi}{\varrho}}\,\mathrm{erf}\,(\sqrt{\varrho}) - 2 \tag{A.18}$$

$$R_4(r) = \frac{1 - e^{-\varrho}}{\varrho} - 1 \tag{A.19}$$

where $C_E = 0.577215...$ is the Euler constant, $\mathrm{E}_1(\varrho)$ is the exponential integral function and $\mathrm{erf}(\sqrt{\varrho})$ is the error function (Abramowitz and Stegun, 1964), while the trend function is a constant ($M = 1$):

$$T(\mathbf{x}) = a_1\,, \quad d = 2, 3, 4 \tag{A.20}$$

The coefficients $a_1, \{\lambda_j\}$ are obtained by solving the following system of linear equations:

$$a_1 + \sum_{j=1}^{N} \lambda_j \left[R(\mathbf{r}^{[i]}, \mathbf{r}^{[j]}) + \delta_{ji} w_0/w_j\right] = z^{[i]}\,, \quad i = 1, \ldots, N \tag{A.21}$$

$$\sum_{j=1}^{N} \lambda_j = 0\,. \tag{A.22}$$

where w_0/w_j are positive smoothing weights.

Topographic analysis

Topographic parameters slope, aspect and curvatures are computed using the principles of differential geometry and derived, for example, by Krcho (1973, 1991), and Mitasova and Hofierka (1993). First we introduce the following simplifying notations:

$$f_x = \frac{\partial z}{\partial x}, \qquad f_y = \frac{\partial z}{\partial y}, \qquad f_{xx} = \frac{\partial^2 z}{\partial x^2}, \qquad f_{yy} = \frac{\partial^2 z}{\partial y^2}, \qquad f_{xy} = \frac{\partial^2 z}{\partial x \partial y} \tag{A.23}$$

and

$$p = f_x^2 + f_y^2 \ , \quad q = p + 1 \quad . \tag{A.24}$$

The steepest slope angle γ in degrees or percent, and aspect angle α in degrees are computed from gradient $\nabla f = (f_x, f_y)$ (its direction is upslope) as follows

$$\gamma = \arctan\sqrt{p} \qquad \gamma[\%] = 100.\sqrt{p} \tag{A.25}$$

$$\alpha = \arctan\frac{f_y}{f_x} \qquad (\alpha = 0 \ \text{ in west direction}) \quad . \tag{A.26}$$

Sometimes we need to compute change of the surface in a direction given by an angle α. The directional derivative of the surface $z = g(x, y)$ can be computed as

$$E = \frac{\partial g}{\partial s} = \frac{\partial g}{\partial x}\cos\alpha + \frac{\partial g}{\partial y}\sin\alpha \tag{A.27}$$

where (x, y) are the georeferenced coordinates, and α is aspect (given direction).

Curvatures In general, a surface has different curvatures in different directions. For applications in geosciences, the curvature in gradient direction (profile curvature) is important because it reflects the change in slope angle and thus controls the change of velocity of mass flowing downwards along the slope curve. The curvature in a direction perpendicular to the gradient (tangential curvature) reflects the change in aspect angle and influences the divergence/convergence of water flow. Both curvatures are measured in the normal plane. Equations for these curvatures can be derived using the general equation for curvature κ of a plane section through a point on a surface (Rektorys, 1969; Mitasova and Mitas, 1993).

The equation for the profile curvature $\kappa_s[m^{-1}]$ is

$$\kappa_s = \frac{f_{xx}f_x^2 + 2f_{xy}f_xf_y + f_{yy}f_y^2}{p\sqrt{q^3}} \quad . \tag{A.28}$$

The equation for tangential curvature $\kappa_t[m^{-1}]$ at a given point is derived as the curvature of normal plane section in a direction perpendicular to gradient (direction of tangent to the contour line)

$$\kappa_t = \frac{f_{xx}f_y^2 - 2f_{xy}f_xf_y + f_{yy}f_x^2}{p\sqrt{q}} \quad . \tag{A.29}$$

The positive and negative values of profile and tangential curvature can be combined to define the basic geometric relief forms (Krcho, 1973, 1991; Dikau, 1989). Each form has a different type of flow. Convex and concave forms in gradient direction have accelerated and deccelerated flow, respectively, and convex and concave forms in tangential direction exhibit converging and diverging flow, respectively.

Other types of curvatures, such as the principle, mean, or Gauss curvatures as well as curvatures in an arbitrary direction can be computed directly from the interpolation function.

Gradient and curvatures for volumes Volumes can be modeled by a trivariate interpolation function in the general form of $w = f(x, y, z)$. When this function is differentiable at least up to the 2nd order, the topographic parameters for volumes (3D) can be computed directly from its partial derivatives (Mitasova et al., 1995). First, we introduce simplifying notations for partial derivatives of this function:

$$\begin{aligned} f_x &= \frac{\partial f}{\partial x}, \quad f_y = \frac{\partial f}{\partial y}, \qquad f_z = \frac{\partial f}{\partial z}, \\ f_{xx} &= \frac{\partial^2 f}{\partial x^2}, f_{xy} = \frac{\partial^2 f}{\partial x \partial y}, f_{xz} = \frac{\partial^2 f}{\partial x \partial z}, \\ f_{yy} &= \frac{\partial^2 f}{\partial y^2}, f_{yz} = \frac{\partial^2 f}{\partial y \partial z}, f_{zz} = \frac{\partial^2 f}{\partial z^2}. \end{aligned} \tag{A.30}$$

Volume geometry parameters are also derived from differential geometry, using additional independent spatial coordinate (z). Theoretically, such topographic parameters can be derived up to N-dimensional space (see Hofierka, 1997b). For a three-dimensional cartesian space these parameters have the following form:
Size of gradient:

$$|\nabla f| = \sqrt{{f_x}^2 + {f_y}^2 + f_z^2} \tag{A.31}$$

Direction of gradient can be defined by two angles.
Horizontal angle A_n:

$$A_n = \arctan\left(\frac{f_y}{f_x}\right) \tag{A.32}$$

and vertical angle B_n:

$$B_n = \arctan\left(\frac{\sqrt{f_x^2 + f_y^2}}{f_z}\right) \tag{A.33}$$

The change of gradient size in its direction has the following form:

$$\frac{\partial |\nabla f|}{\partial n} = \frac{f_x^2 f_{xx} + 2 f_{xz} f_x f_z + 2 f_{xy} f_x f_y + f_y^2 f_{yy} + 2 f_{yz} f_y f_z + f_z^2 f_{zz}}{f_x^2 + f_y^2 + f_z^2} \tag{A.34}$$

When we note principal curvatures in 3D cartesian space as k_1, k_2, k_3, then the Gauss-Kronecker curvature K can by expressed as:

$$K = k_1 * k_2 * k_3 \tag{A.35}$$

The mean curvature M is:

$$M = \frac{k_1 + k_2 + k_3}{3} \tag{A.36}$$

In cartesian system these equations can be expressed as follows:

$$K = \frac{f_{xz}^2 f_{yy} + f_{yz}^2 f_{xx} + f_{xy}^2 f_{zz} - f_{xx} f_{yy} f_{zz} - 2 f_{xy} f_{yz} f_{xz}}{\left(\sqrt{1 + f_x^2 + f_y^2 + f_z^2}\right)^5} \tag{A.37}$$

$$H = \frac{\begin{vmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{vmatrix}}{3\left(1 + f_x^2 + f_y^2 + f_z^2\right)} \tag{A.38}$$

where:

$$h_{11} = \frac{-f_{xx}}{\sqrt{1 + f_x^2 + f_y^2 + f_z^2}} + 2(1 + f_x^2) \tag{A.39}$$

$$h_{12} = h_{21} = \frac{-f_{xy}}{\sqrt{1 + f_x^2 + f_y^2 + f_z^2}} + 2 f_x f_y \tag{A.40}$$

$$h_{22} = \frac{-f_{yy}}{\sqrt{1 + f_x^2 + f_y^2 + f_z^2}} + 2(1 + f_y^2) \tag{A.41}$$

$$h_{13} = h_{31} = \frac{-f_{xz}}{\sqrt{1 + f_x^2 + f_y^2 + f_z^2}} + 2 f_x f_z \tag{A.42}$$

$$h_{23} = h_{32} = \frac{-f_{yz}}{\sqrt{1 + f_x^2 + f_y^2 + f_z^2}} + 2 f_y f_z \tag{A.43}$$

$$h_{33} = \frac{-f_{zz}}{\sqrt{1 + f_x^2 + f_y^2 + f_z^2}} + 2(1 + f_z^2) \tag{A.44}$$

Estimation of partial derivatives To compute the above described equations for gradients and curvatures, we need to estimate first and second order partial derivatives.

In the RST-based modules, partial derivatives of RST functions are used. First, several definitions are introduced:

$$\eta = \frac{\varphi}{2} \tag{A.45}$$

$$R'(r_j) = 2\,\frac{1 - e^{-(\eta r_j)^2}}{r_j} \tag{A.46}$$

$$R''(r_j) = 2\,\frac{\left(2(\eta r_j)^2 + 1\right) e^{-(\eta r_j)^2} - 1}{r_j^2} \tag{A.47}$$

Partial derivatives for the bivariate RST basis function can then be expressed as follows:

$$\frac{\partial R(r_j)}{\partial x} = R'(r_j)\frac{(x - x^{[j]})}{r_j}\,, \qquad l = 1, 2 \tag{A.48}$$

$$\frac{\partial^2 R(r_j)}{\partial x_1^2} = R''(r_j)\frac{(x - x^{[j]})^2}{r_j^2} + R'(r_j)\frac{(y - y^{[j]})^2}{r_j^3} \tag{A.49}$$

whereas the derivatives, according to y, are found easily from Equation A.49 by exchange of x to y. The mixed derivative is given by

$$\frac{\partial^2 R(r_j)}{\partial x \partial x} = \left[R''(r_j) - \frac{R'(r_j)}{r_j}\right]\frac{(x - x^{[j]})(y - y^{[j]})}{r_j^2} \tag{A.50}$$

These expressions for first and second order derivatives are used for the computation of slope, aspect and curvatures in the modules `s.surf.rst`, `v.surf.rst` and `r.resamp.rst`. Optionally, the values of these partial derivatives are output by the module `s.surf.rst` when using the flag `-d`.

Partial derivatives for trivariate RST :

$$R'(r_j) = \frac{1}{r_j\sqrt{\pi}}\exp\left[-\left(\frac{\varphi r_j}{2}\right)^2\right] - \frac{1}{\varphi r_j^2}\,erf\left(\frac{\varphi r_j}{2}\right) \tag{A.51}$$

$$R''(r_j) = \frac{2}{\varphi r_j^3}\,erf\left(\frac{\varphi r_j}{2}\right) - \sqrt{\pi}\left(\frac{2}{r_j^2} + \frac{\varphi^2}{2}\right)\exp\left[-\left(\frac{\varphi r_j}{2}\right)^2\right] \tag{A.52}$$

In `r.slope.aspect`, second order polynomial approximation of a surface defined by given point and its 3 $\times$ 3 neighborhood is used leading to the following equations for the partial derivatives (as used in Horn's formula, see Horn, 1981):

$$z(x, y) = a_0 + a_1 x + a_2 y + a_3 xy + a_4 x^2 + a_5 y^2 \tag{A.53}$$

By fitting this polynomial to the 9 grid points (the given point $z_{i,j}$ and its 3 x 3 neighborhood, as shown below), using weighted least squares, we can derive the coefficients of this polynomial as well as its partial derivatives ($f_x = a_1$, $f_y = a_2$, $f_{xx} = 2a_4$, $f_{yy} = 2a_5$, $f_{xy} = a_3$):

```
i-1,j+1 ----- i,j+1 ----- i+1,j+1
   |            |            |
   |            |            |
   |            |            |
i-1,j ------- i,j --------- i+1,j
   |            |            |
   |            |            |
   |            |            |
i-1,j-1 ----- i,j-1 ----- i+1,j-1
```

$$f_x = \frac{(z_{i-1,j-1} - z_{i+1,j-1}) + (2z_{i-1,j} - 2z_{i+1,j}) + (z_{i-1,j+1} - z_{i+1,j+1})}{8\Delta x} \tag{A.54}$$

$$f_y = \frac{(z_{i-1,j-1} - z_{i-1,j+1}) + (2z_{i,j-1} - 2z_{i,j+1}) + (z_{i+1,j-1} - z_{i+1,j+1})}{8\Delta y} \tag{A.55}$$

where Δx and Δy is the resolution (grid spacing) in the east-west and north-south direction respectively.
Let us denote $D(i, \delta) = z_{i,j+1}+z_{i,j-1}-2z_{i,j}$ and $D(\delta, j) = z_{i+1,j}+z_{i-1,j}-2z_{i,j}$. Then we can write

$$f_{xx} = \frac{D(\delta, j+1) + (4z_{i-1,j} + 4z_{i+1,j} - 8z_{i,j}) + D(\delta, j-1)}{6(\Delta x)^2} \tag{A.56}$$

$$f_{yy} = \frac{D(i-1, \delta) + (4z_{i,j+1} + 4z_{i,j-1} - 8z_{i,j}) + D(i+1, \delta)}{6(\Delta y)^2} \tag{A.57}$$

$$f_{xy} = \frac{(z_{j-1,i-1} - z_{j-1,i+1}) - (z_{j+1,x-1} - z_{j+1,i+1})}{4\Delta x \Delta y} \tag{A.58}$$

where $z_{i,j}$ is the elevation value at row j column i, Δx is the east-west grid spacing and Δy is the north-south grid spacing (resolution).

Insolation

Equations for computation of solar energy related parameters used in `r.sun` (Hofierka, 1997a; Hofierka and Súri, 2002, further citations in the manual page of `r.sun`). The clear-sky solar radiation model applied in this module is based on the work undertaken for development of European Solar Radiation Atlas (Scharmer and Greif, 2000; Page et al., 2001; Rigollier et al., 2000).

Solar geometry

Declination d [rad]:

$$\delta = \arcsin(0.3978 \sin(j - 1.4 + 0.0355 \sin(j - 0.0489))) \quad (A.59)$$

where:
$j = 2\pi day/365.25$ [rad]

Position of the sun in respect to a horizontal plane:

$$sinh_0 = C_{31} \cos T + C_{33} \quad (A.60)$$

$$cosA_0 = \frac{C_{11} \cos T + C_{13}}{\sqrt{(C_{22} \sin T)^2 + (C_{11} \cos T + C_{13})^2}} \quad (A.61)$$

where:
$C_{11} = \sin \varphi \cos \delta$
$C_{13} = - \cos \varphi \sin \delta$
$C_{22} = \cos \delta$
$C_{31} = \cos \varphi \cos \delta$
$C_{33} = \sin \varphi \sin \delta$

Position of the sun in respect to an inclined plane:

$$\sin \delta_{exp} = C'_{31} \cos(T - \lambda') + C'_{33} \quad (A.62)$$

where:
$C'_{31} = \cos \varphi' \cos \delta$
$C'_{33} = \sin \varphi' \sin \delta$
$\sin \varphi' = - \cos \varphi \sin \gamma_N \cos A_N + \sin \varphi \cos \gamma_N$
$\tan \lambda' = -\frac{\sin \gamma_N \sin A_N}{\sin \varphi \sin \gamma_N \cos A_N + \cos \varphi \cos \gamma_N}$

Sunrise/sunset over a horizontal plane:

$$\cos(Th_{r,s}) = -\frac{C_{33}}{C_{31}} \quad (A.63)$$

Sunrise/sunset over an inclined plane:

$$\cos(Tp_{r,s} - \lambda') = -\frac{C'_{33}}{C'_{31}} \quad (A.64)$$

Extraterrestrial irradiance on a plane perpendicular to the solar beam G_0 [W/m²]

$$G_0 = I_0 \epsilon \tag{A.65}$$

where:

$\epsilon = 1 + 0.03344 \cos(j - 0.048869)$

values j and 0.048869 are in radians.

Extraterrestrial irradiance on a horizontal plane G_{0h} [W/m²]

$$G_{0h} = G_0 \sin h_0 \tag{A.66}$$

Beam irradiance on a horizontal plane B_h [W/m²]

$$B_h = G_0 e^{(-0.8662 \ T_{LK} \ m \ \delta_R(m))} \sin h_0 \tag{A.67}$$

where:

$p/p_0 = e^{(-z/8434.5)}$

$\Delta h_0^{ref} = 0.061359(0.1594 + 1.123h_0 + 0.065656h_0^2)/(1 + 28.9344h_0 + 277.3971h_0^2)$

$h_0^{ref} = h_0 + \Delta h_0^{ref}$

$m = (p/p_0)/(\sin h_0^{ref} + 0.50572(h_0^{ref} + 6.07995)^{-1.6364})$

where values h_0^{ref} and 6.07995 are in degree

$\delta_R(m) = 1/(6.6296 + 1.7513m - 0.1202m^2 + 0.0065m^3 - 0.00013m^4)$

if $m \leq 20$

$\delta_R(m) = 1/(10.4 + 0.718m)$ if $m > 20$

Beam irradiance on an inclined plane B_i [W/m²]

$$B_i = G_0 e^{(-0.8662 \ T_{LK} \ m \ \delta_R(m))} \sin \delta_{exp} \tag{A.68}$$

Diffuse irradiance on a horizontal plane B_h [W/m²]

$$D_h = G_0 F_d(h_0) Tn(T_{LK}) \tag{A.69}$$

where:

$Tn(T_{LK}) = -0.015843 + 0.030543 \ T_{LK} + 0.0003797 \ T_{LK}^2$

$F_d(h_0) = A_1 + A_2 \sin h_0 + A_3 \sin^2 h_0$

$A_1' = 0.26463 - 0.061581T_{LK} + 0.0031408T_{LK}^2$

$A_1 = 0.0022/Tn(T_{LK})$

$$if\, A'_1\ Tn(T_{LK}) < 0.0022$$

$$A_1 = A'_1$$

$$if\, A'_1\ Tn(T_{LK}) \geq 0.0022$$

$$A_2 = 2.04020 + 0.018945\ T_{LK} + 0.011161\ T_{LK}^2$$
$$A_3 = -1.3025 + 0.039231\ T_{LK} + 0.0085079\ T_{LK}^2$$

Diffuse irradiance on an inclined plane D_i [W/m²]

$$D_i = D_h F_x \tag{A.70}$$

where:

if plane is in shade (e.g. $\delta_{exp} < 0$ and $h_0 \geq 0$):

$$F_x = F(\gamma_N)$$
$$F(\gamma_N) = r_i(\gamma_N) + \left(\sin\gamma_N - \gamma_N \cos\gamma_N - \pi \sin^2\left(\tfrac{\gamma_N}{2}\right)\right) 0.252271$$

if plane is sunlit under clear sky:

$if\, h_0 \geq 0.1 rad$:

$$F_x = F(\gamma_N)(1 - K_b) + K_b \sin\delta_{exp} / \sin h_0$$

$if\, h_0 < 0.1 rad$:

$$F_x = F(\gamma_N)(1 - K_b) + K_b \sin\gamma_N \cos A_{LN} / (0.1 - 0.008 h_0)$$
$$A_{LN}^* = A_0 - A_N$$
$$A_{LN} = A_{LN}^*$$

$$if -\pi \leq A_{LN}^* \leq \pi$$

$$A_{LN} = A_{LN}^* - 2\pi$$

$$if\, A_{LN}^* > \pi$$

$$A_{LN} = A_{LN}^* + 2\pi$$

$$if\, A_{LN}^* < -\pi$$

$$F(\gamma_N) = r_i(\gamma_N)$$
$$+ \left(\sin\gamma_N - \gamma_N \cos\gamma_N - \pi \sin^2\left(\tfrac{\gamma_N}{2}\right)\right) (0.00263 - 0.712 K_b - 0.6883 K_b^2)$$
$$K_b = B_h / G_{0h}$$
$$r_i(\gamma_N) = (1 + \cos\gamma_N)/2$$

Diffuse ground reflected irradiance on an inclined plane R_i [W/m²]

$$R_i = \rho_g G_h r_g(\gamma_N) \tag{A.71}$$

where:

$$r_g(\gamma_N) = (1 - \cos\gamma_N)/2$$
$$G_h = B_h + D_h$$

with G_h in [W/m^2]

Symbols

- Position of the grid cell (solar plane):
  φ geographical latitude [rad],
  z elevation above sea level [m],
  γ_N slope angle [rad],
  A_N aspect (orientation, azimuth) – angle between the projection of the normal on the horizontal plane and east [rad],
  φ' relative geographical latitude of an inclined plane [rad],
  λ' relative geographical longitude [rad].
- Parameters of the surface (plane):
  ρ_g mean ground albedo.
- Date-related parameters:
  day day number 1-365 (366),
  j Julian day number expressed as a day angle [rad],
  T time of computation [decimal hours/rad],
  $Th_{r,s}$ time of sunrise and sunset over the local horizon,
  $Tp_{r,s}$ time of sunrise and sunset over the inclined grid cell (plane),
  δ solar declination [rad],
  ϵ correction of the variation of sun-earth distance from its mean value.
- Solar position:
  h_0 solar altitude – an angle between sun and horizon [rad],
  A_0 solar azimuth – an angle between sun and meridian measured from east [rad],
  A_{LN} angle between the vertical plane containing the normal to the surface and vertical plane passing through the center of the solar disc [rad],
  δ_{exp} solar incidence angle - an angle between sun and the (inclined) plane [rad].
- Solar radiation:
  I_0 solar_const $= 1367$ W/m^2,
  G_0 extraterrestrial irradiance on a plane perpendicular to the solar beam [W/m^2],
  G_h $G_h = B_h + D_h$ – global solar irradiance on a horizontal plane [W/m^2],
  G_i $G_i = B_i + D_i + R_i$ – global solar irradiance on an inclined plane [W/m^2],
  B_h beam irradiance on a horizontal plane [W/m^2],
  B_i beam irradiance on an inclined plane [W/m^2],
  D_h diffuse irradiance on a horizontal plane [W/m^2],
  D_i diffuse irradiance on an inclined plane [W/m^2],
  R_i diffuse ground reflected irradiance on an inclined plane [W/m^2].
- Parameters of the atmosphere:
  p/p_0 correction of station elevation [-],
  T_{LK} Linke turbidity factor [-],
  T_L corrected Linke turbidity factor ($T_L = 0.8662\, T_{LK}$), see Kasten (1996),
  m relative optical air mass [-],
  $\delta_R(m)$ Rayleigh optical thickness [-].

- Parameters of the radiation transmission:
  $F_d(h_0)$ diffuse solar elevation function,
  $Tn(T_{LK})$ diffuse transmission function,
  $F(\gamma_N)$ function accounting for the diffuse sky irradiance distribution,
  K_b proportion between beam irradiance and extraterrestrial solar irradiance on a horizontal plane,
  $r_i(\gamma_N)$ fraction of the sky dome viewed by an inclined plane [-],
  $r_g(\gamma_N)$ fraction of the ground viewed by an inclined plane [-].

Walking person

Anisotropic movement of a person between different geographic locations:

$$T = a.\Delta S + b.\Delta H_u + c.\Delta H_d + d.\Delta H_s \tag{A.72}$$

where T is time of movement in seconds, ΔS is the distance covered in meters, $\Delta H_u, H_d, H_s$ is the altitude difference in meters when going uphill, downhill, and steep downhill respectively. The a, b, c, d parameters represent speed under different conditions and are linked to a underfoot condition (a=1/walking_speed), b underfoot condition and cost associated to movement uphill, c underfoot condition and cost associated to movement moderate downhill, d underfoot condition and cost associated to movement steep downhill. Moving downhill is beneficial up to a specific slope value threshold, after that it becomes unfavourable. The default slope value threshold (slope factor) is -0.2125, corresponding to tan (12) downslope. The default values for a, b, c, d are those proposed by Langmuir (0.72, 6.0, 1.9998, -1.9998), based on man walking effort in standard conditions. Total cost is estimated as a linear equation combining movement and friction costs using the λ parameter:

$$TC = MC + \lambda * FC \tag{A.73}$$

where TC is total cost, MC is movement time cost and FC is friction costs, (see Aitken, 1977; Fontanari, 2001).

A.2 Landscape process modeling

Wetness index

$$W = \ln \frac{A}{\tan \beta} \tag{A.74}$$

where A is the upslope area per unit contour width [m] and β is the slope angle in [deg].

Universal Soil Loss Equation (USLE, RUSLE)

$$A = RKLSCP \tag{A.75}$$

where A is average annual soil loss in ton/(acre.year)=0.2242kg/(m^2.year), R is rainfall factor in (hundreds of ft-tonf.in)/(acre.hr.year)=17.02 (MJ.mm)/(ha.hr.year), K is soil erodibility factor in (ton acre.hr)/(hundreds of acre ft-tonf.in)=0.1317(ton.ha.hr)/(ha.MJ.mm), LS is a dimensionless topographic (length-slope) factor, C is a dimensionless land cover factor, and P is a dimensionless prevention measures factor. The modified factor, representing topographic potential for erosion at a point on the hillslope, is a function of the upslope area per unit width and the slope angle:

$$LS = (m+1)\left(\frac{U}{22.1}\right)^m \left(\frac{\sin\beta}{0.09}\right)^n \tag{A.76}$$

where U is the upslope area per unit width (measure of water flow) in meters (m^2/m), β is the slope angle in degree, 22.1 is the length of the standard USLE plot in meters and $0.09 = 9\% = 5.15°$ is the slope of the standard USLE plot. The values of exponents range for $m = 0.2 - 0.6$ and $n = 1.0 - 1.3$, where the lower values are used for prevailing sheet flow and higher values for prevailing rill flow. When nothing is known about the type of flow, $m = 0.4$ and $n = 1.3$ are usually used (see RUSLE for ArcView[1]).

Unit Stream Power Based Erosion/Deposition model (USPED)
The *Unit Stream Power Based Erosion/Deposition model* (*USPED*, Mitasova and Mitas, 2001) estimates a simplified case of erosion/deposition using the idea originally proposed by Moore and Burch (1986). It combines the $RUSLE$ parameters and upslope contributing area per unit width A to estimate the sediment flow T:

$$T \approx RKCPA^m(sin\beta)^n. \tag{A.77}$$

The upslope area and slope are not normalized, because T is an estimate of sediment flow $[kg/(ms)]$ (rather than soil detachment $[kg/(m^2s)]$). The net erosion/deposition $D[kg/(m^2s)]$ is then computed as a divergence of sediment flow:

$$D = \nabla \cdot (T\mathbf{s_0}) = \frac{d(T\cos\alpha)}{dx} + \frac{d(T\sin\alpha)}{dy}, \tag{A.78}$$

where α in degrees is the aspect of the terrain surface (direction of flow). The exponents m, n control the relative influence of water and slope terms and reflect the impact of different types of flow. The typical range of values is $m = 1.0 - 1.6, n = 1.0 - 1.3$, with the higher values reflecting the pattern for

[1] RUSLE for ArcView,
`http://abe.www.ecn.purdue.edu/~engelb/agen526/gisrusle/gisrusle.html`

prevailing rill erosion with more turbulent flow when erosion sharply increases with the amount of water. Lower exponent values close to $m = n = 1$ better reflect the pattern of compounded, long term impact of both rill and sheet erosion and averaging over a long term sequence of large and small events.

A.3 Definition of SQLite-ODBC connection

At time of this writing, there is only OpenOffice.org Base which can be used as powerful graphical user interface to SQL databases. While GRASS can be directly connected to SQLite, OpenOffice.org Base needs an ODBC based driver, the "sqliteodbc" library[2] and of course ODBC. In this example, we define a sample SQLite-ODBC connection.

After installation of the "sqliteodbc" library, on UNIX-based systems, add the driver to `/etc/odbcinst.ini` (either add next lines or create as new file if the file does not yet exist):

```
[SQLite]
Description=SQLite ODBC Driver
Driver=/usr/local/lib/libsqlite3odbc.so
Setup=/usr/local/lib/libsqlite3odbc.so
```

Now the SQLite driver is available for ODBC. The next step is to add the definition(s) of the database you want to connect to. Add an ODBC Data Source Name (DSN) to your definition file at `$HOME/.odbc.ini` (replace "user" and "mymapset" with the correct entries):

```
[nc_sqlite]
Description=North Carolina SQLite DB
Driver=SQLite
Database=/home/user/grassdata/nc_spm/mymapset/sqlite.db
# optional lock timeout in milliseconds
Timeout=2000
```

See Section 6.2.1 for GRASS related usage notes of the SQLite driver. For installation under MS-Windows see the "sqliteodbc" Web page.

[2] Web site of "sqliteodbc" library, `http://www.ch-werner.de/sqliteodbc/`

References

Abramowitz, M. and Stegun, I. 1964. *Handbook of Mathematical Functions*. New York: Dover.

Aitken, R. 1977. *Wilderness areas in Scotland*. Ph.D. thesis, University of Aberdeen, U.K.

Albrecht, J. 1992. GTZ-handbook GRASS. Technical report, Univ. of Vechta. Electr. Doc. `http://grass.itc.it/gdp/`.

Alexandrov, A., Kolmogorov, A., and Lavrent'ev, M. 1989. *Mathematics: Its content, methods and meanings*, volume 2. Cambridge (MA): MIT Press, 6th edition.

Arge, L., Chase, J., Halpin, P., Toma, L., Vitter, J., Urban, D., and Wickremesinghe, R. 2003. Efficient flow computation on massive grid terrain datasets. *GeoInformatica* **7** (4): 283–313.

Bailey, T. and Gatrell, A. 1995. *Interactive Spatial Data Analysis*. Essex: Pearson.

Baker, W. 2001. The r.le programs for multiscale analysis of landscape structure using the GRASS geographical information system. Technical report, Department of Geography and Recreation, University of Wyoming.

Baker, W. and Cai, Y. 1992. The r.le programs for multiscale analysis of landscape structure using the GRASS geographical information system. *Landscape Ecology* **7** (4): 291–302.

Barsi, J., Schott, J., Palluconi, F., Helder, D., Hook, S., Markham, B., Chander, G., and O'Donnell, E. 2003. Landsat TM and ETM+ thermal band calibration. *Canadian Journal of Remote Sensing* **29** (2): 141–153.

Bartelme, N. 1995. *Geoinformatik: Modelle, Strukturen, Funktionen*. Heidelberg: Springer.

Becker, R., Chambers, J., and Wilks, A. 1988. *The New S Language*. London: Chapman & Hall.

Bivand, R. 2000. Using the R statistical data analysis language on GRASS 5.0 GIS database files. *Computers & Geosciences* **26** (9-10): 1043–1052.

Bivand, R. 2007. Using the R–GRASS interface. *OSGeo Journal* **1**: 36–38. Electr. Doc. `http://www.osgeo.org/journal/volume1`.

Bivand, R. and Gebhardt, A. 2000. Implementing functions for spatial statistical analysis using the R language. *Journal of Geographical Systems* **2** (3): 307–317.

Bivand, R. and Neteler, M. 2000. Open Source Geocomputation: Using the R Data Analysis Language Integrated with GRASS GIS and PostgreSQL Data Base Systems. In *Proc., 5th conference on GeoComputation, University of Greenwich, U.K.*, pp. 23–25. Electr. Doc. `http://reclus.nhh.no/gc00/gc009.htm`.

Blazek, R., Neteler, M., and Micarelli, R. 2002. The new GRASS 5.1 vector architecture. In *Proc., Open Source Free Software GIS – GRASS users conference, Trento, Italy, Sept. 2002* (eds. Ciolli, M. and Zatelli, P.), pp. 11–13. Electr. Doc. `http://www.ing.unitn.it/~grass/conferences/GRASS2002/proceedings/proceedings/pdfs/Blazek_Radim.pdf`.

Brandon, R., Kludt, T., and Neteler, M. 1999. Archaeology and GIS – the Linux way: Using GRASS and Linux to analyze archaeological data. *Linux Journal* **7**: 50–54. Feature article.

Brovelli, M., Cannata, M., and Longoni, U. 2004. LIDAR data filtering and DTM interpolation within GRASS. *Transactions in GIS* **8** (2): 155–174.

Brown, W., Astley, M., Baker, T., and Mitasova, H. 1995. GRASS as an integrated GIS and visualization environment for spatio-temporal modeling. In *Proc., Auto-carto XII, ACSM/ASPRS, Charlotte, NC*, pp. 89–99.

Bugayevskiy, L. and Snyder, J. 2000. *Map Projections: A reference manual*. London, Philadelphia: Taylor & Francis.

Burrough, P. 1986. *Principles of GIS for Land Resources Assessment*. Oxford: Clarendon Press.

Burrough, P. and McDonnell, R. 1998. *Principles of Geographical Information Systems*. New York: Oxford University Press.

Cannata, M. 2006. *A GIS embedded approach for Free & Open Source Hydrological Modelling*. Ph.D. thesis, Department of Geodesy and Geomatics, Polytechnic of Milan, Italy.

Chambers, J. and Hastie, T. 1992. *Statistical Models in S*. London: Chapman & Hall.

Chavez, P., Guptill, S., and Bowell, J. 1984. Image processing techniques for thematic mapper data. *Proc., Am. Soc. Photogr.* **2**: 728–743.

Chavez Jr., P. 1996. Image-based atmospheric corrections – revisited and improved. *Photogr. Eng. & Rem. Sens.* **62** (9): 1025–1036.

Clarke, K. 2002. *Getting Started with Geographic Information Systems*. New Jersey: Prentice Hall, 4th edition.

Cressie, N. 1993. *Statistics for Spatial Data*. New York: Wiley.

Curtin, K. M. 2007. Network Analysis in Geographic Information Science: Review, Assessment, and Projections. *Cartography and Geographic Information Science* **34** (2): 103–111.

Desmet, P. and Govers, G. 1996. A GIS procedure for automatically calculating the USLE LS factor on topographically complex landscape units. *J. Soil and Water Conservation* **51** (5): 427–433.

Dikau, R. 1989. The application of a digital relief model to landform analysis in geomorphology. In *Three dimensional applications in Geographic Information Systems* (ed. Raper, J.), pp. 51–77. Taylor & Francis, London.

Ehlschlaeger, C. and Goodchild, M. 1994. Dealing with uncertainty in categorical coverage maps: Defining, visualizing, and managing errors. *Proc., Workshop on Geographic Information Systems at the Conf. on Information and Knowledge Management, Gaithersburg, MD* pp. 86–91.

Erle, S., Gibson, R., and Walsh, J. 2005. *Mapping Hacks*. Cambridge: O'Reilly & Associates.

Evenden, G. 1995. Cartographic projection procedures for the Unix environment – a user's manual. Technical report, U.S. Geological Survey Open File Report. Electr. Doc. `http://proj.maptools.org`.

Fontanari, S. 2001. Svilluppo di metodologie GIS per la determinazione dell'accessibilità territoriale come supporto alle decisioni nella gestione ambientale, M.Sc. thesis, University of Trento, Italy.

Fortune, S. 1987. A sweepline algorithm for Voronoi diagrams. *Algorithmica* **2**: 153–174.

Furlanello, C., Neteler, M., Merler, S., Menegon, S., Fontanari, S., Donini, A., Rizzoli, A., and Chemini, C. 2003. GIS and the RandomForest predictor: Integration in R for tick-borne disease risk assessment. In *Proc., Distributed Statistical Computing, Vienna, Austria, March 2003* (eds. Hornik, K. and Leisch, F.).

Gebbert, S. 2007. Groundwater modeling. Dipl. Ing. thesis, Technical University Berlin, Germany.

GRASS Development Team, ed. 2006. *GRASS 6.2 Programmer's Manual. Geographic Resources Analysis Support System*. FBKITC-irst, Trento, Italy. Electr. Doc. `http://grass.itc.it/devel/`.

GSFC/NASA 2001. Landsat-7 Science Data User's Handbook. Technical report, NASA. Electr. Doc. `http://ltpwww.gsfc.nasa.gov/IAS/handbook/handbook_htmls/chapter11/chapter11.html`.

Haan, C., Barfield, B., and Hayes, J. 1994. *Design Hydrology and Sedimentology for Small Catchments*. New York: Academic Press.

Hake, G. and Grünreich, D. 1994. *Kartographie*. Berlin: de Gruyter, 7th edition.

Hofierka, J. 1997a. Direct solar radiation modelling within an Open GIS environment. *Proc., Geographical Information '97: Third Joint European Conference & Exhibition on Geographical Information., Vienna, Austria, April 1997* pp. 575–584.

Hofierka, J. 1997b. *Modeling Natural Phenomena in a GIS Environment*. Ph.D. thesis, Comenius University, Bratislava.

Hofierka, J., Mitasova, H., and Neteler, M. 2007. Terrain parameterization in GRASS. In *Geomorphometry: concepts, software, applications* (eds. Hengl,

T. and Reuter, H.), pp. 301–318. Office for Official Publications of the European Communities, Luxembourg, EUR 22670 EN. Electr. Doc. http://www.geomorphometry.org.

Hofierka, J., Parajka, J., Mitasova, H., and Mitas, L. 2002. Multivariate interpolation of precipitation using regularized spline with tension. *Transactions in GIS* **6** (2): 135–150.

Hofierka, J. and Súri, M. 2002. The solar radiation model for Open Source GIS: Implementation and applications. In *Proc., Open Source Free Software GIS – GRASS users conference, Trento, Italy, Sept. 2002* (eds. Ciolli, M. and Zatelli, P.), pp. 11–13.

Horn, B. 1981. Hill shading and the reflectance map. *Proc. of the IEEE* **69** (1): 14–47.

Hutchinson, M. and Bischof, R. 1983. A new method for estimating the spatial distribution of mean seasonal and annual rainfall applied to the hunter valley, new south wales. *Australian Meteorological Magazine* **31**: 179–184.

Ihaka, R. and Gentleman, R. 1996. R: A language for data analysis and graphics. *J. of Comp. and Graph. Stat.* **5** (3): 299–314.

Jolma, A., Ames, D., Horning, N., Mitasova, H., Neteler, M., Racicot, A., and Sutton, T. 2007. Environmental Modeling and Management using Free and Open Source Geospatial Tools. In *Encyclopedia of GIS* (eds. Shekhar, S. and Xiong, H.). Springer.

Jolma, A., Ames, D., Horning, N., Neteler, M., Racicot, A., and Sutton, T. 2006. Free and Open Source geospatial tools for environmental modeling and management. In *Proc., iEMSs 2006, 3rd Biennial meeting of the Intl. Env. Mod. and Softw. Soc. July 9-13, 2006, Burlington, Vermont, USA, W13: Open geospatial tools and methods in environmental modeling and management* (ed. Voinov, A.). Electr. Doc. http://www.iemss.org/iemss2006/papers/w13/pp.pdf.

Journel, A. 1996. Modelling uncertainty and spatial dependence: Stochastic imaging. *Int. J. of Geogr. Inf. Sys.* **10** (5): 517–522.

Kasten, F. 1996. The linke turbidity factor based on improved values of the integral Rayleigh optical thickness. *Solar Energy* **56** (3): 239–244.

Kinner, D., Mitasova, H., and Stallard, R. 2005. GIS Database and Stream Network Analysis for the Upper Río Chagres Basin, Panama. In *The Río Chagres: A Multidisciplinary Perspective of a Tropical River Basin* (ed. Harmon, R.), pp. 83–95. New York: Kluwer Academic/Plenum Publishers.

Krcho, J. 1973. Morphometric analysis of relief on the basis of geometric aspect of field theory. *Acta Geographica Universitae Comenianae* Geographica Physica 1, Bratislava, SPN.

Krcho, J. 1991. Georelief as a subsystem of landscape and the influence of morphometric parameters of georelief on spatial differentiation of landscape-ecological processes. *Ecology /CSFR/* **10**: 115–157.

Longley, P., Goodchild, M., Maguire, D. J., and Rhind, D. 2005. *Geographical Information Systems: Principles, Techniques, Management and Applications*. London: Wiley.

Maling, D. 1992. *Coordinate Systems and Map Projections*. Elmsford, New York: Pergamon Press, 2nd edition.

Mandelbrot, B. 1983. *The Fractal Geometry of Nature*. New York: Freeman.

Mather, P. 1999. *Computer Processing of Remotely-sensed Images*. Chichester: Wiley.

McCauley, J. and Engel, B. 1995. Comparison of Scene Segmentations: SMAP, ECHO and Maximum Likelihood. *IEEE Trans. on Geosc. & Rem. Sens.* **33** (6): 1313–1316.

Mitas, L., Brown, W., and Mitasova, H. 1997. Role of dynamic cartography in simulations of landscape processes based on multi-variate fields. *Comp. & Geosc.* **23**: 437–446. Electr. Doc. `http://skagit.meas.ncsu.edu/~helena/gmslab/lcgfin/cg-mitas.html`.

Mitas, L. and Mitasova, H. 1998. Distributed soil erosion simulation for effective erosion prevention. *Water Resources Research* **34** (3): 505–516.

Mitas, L. and Mitasova, H. 1999. Spatial interpolation. In *Geographical Information Systems: Principles, Techniques, Management and Applications.* (eds. Longley, P., Goodchild, M., Maguire, D., and Rhind, D.), pp. 481–492. New York: Wiley.

Mitasova, H. and Hofierka, J. 1993. Interpolation by Regularized Spline with Tension: II. Application to Terrain Modeling and Surface Geometry Analysis. *Math. Geol.* **25**: 657–667. Electr. Doc. `http://skagit.meas.ncsu.edu/~helena/gmslab/papers/listsj.html`.

Mitasova, H., Hofierka, J., Zlocha, M., and Iverson, L. 1996. Modeling topographic potential for erosion and deposition using GIS. *Int. J. of Geogr. Inf. Sci.* **10** (5): 629–641.

Mitasova, H. and Mitas, L. 1993. Interpolation by Regularized Spline with Tension: I. Theory and Implementation. *Math. Geol.* **25**: 641–655. Electr. Doc. `http://skagit.meas.ncsu.edu/~helena/gmslab/papers/listsj.html`.

Mitasova, H. and Mitas, L. 2001. Multiscale soil erosion simulations for land use management. In *Landscape erosion and landscape evolution modeling.* (eds. Harmon, R. and Doe, W.), pp. 321–347. New York: Kluwer Academic/Plenum Publishers.

Mitasova, H., Mitas, L., Brown, W., Gerdes, D., Kosinovsky, I., and Baker, T. 1995. Modeling spatially and temporally distributed phenomena: New methods and tools for GRASS GIS. *Int. J. of GIS, Special issue on Integrating GIS and Environmental modeling* **9** (4): 433–446.

Mitasova, H., Mitas, L., and Harmon, R. 2005a. Simultaneous Spline Interpolation and Topographic Analysis for LiDAR Elevation Data: Methods for Open Source GIS. *IEEE GRSL* **2** (4): 375–379. Electr. Doc. `http://skagit.meas.ncsu.edu/~helena/gmslab/papers/listsj.html`.

Mitasova, H., Overton, M., and Harmon, R. 2005b. Geospatial analysis of a coastal sand dune field evolution: Jockey's Ridge, North Carolina. *Geomorphology* **72**: 204–221. Electr. Doc. http://skagit.meas.ncsu.edu/~helena/gmslab/papers/listsj.html.

Mitasova, H., Thaxton, C., Hofierka, J., McLaughlin, R., Moore, A., and Mitas, L. 2005c. Path sampling method for modeling overland water flow, sediment transport, and short term terrain evolution in Open Source GIS. In *Proc., XVth International Conf. on Computational Methods in Water Resources (CMWR XV)* (eds. Miller, C., Farthing, M., Gray, V., and Pinder, G.), pp. 1479–1490. Elsevier.

Mitchell, T. 2005. *Web Mapping Illustrated*. Cambridge: O'Reilly & Associates.

Moore, I. and Burch, G. 1986. Modeling erosion and deposition: Topographic effects. *Transactions ASAE* **29**: 1624–1640.

Moore, I. and Wilson, J. 1992. Length-slope factors for the revised universal soil loss equation: Simplified method of estimation. *Journal of Soil and Water Conservation* **47**: 423–428.

Moran, M., Jackson, R., Slater, P., and Teillet, P. 1992. Evaluation of simplified procedures for retrieval of land surface reflectance factors from satellite sensor output. *Rem. Sens. Env.* **41**: 169–184.

Neteler, M. 1999. Spectral Mixture Analysis von Satellitendaten zur Bestimmung von Bodenbedeckungsgraden im Hinblick auf die Erosionsmodellierung. M.Sc. thesis, Univ. of Hannover, Germany.

Neteler, M. 2000. *GRASS-Handbuch*. Geosynthesis 11. Univ. of Hannover. Der praktische Leitfaden zum Geographischen Informationssystem GRASS.

Neteler, M. 2001. Towards a stable Open Source GIS: Status and Future Directions in GRASS Development. In *The Geomatics Workbook* (ed. Brovelli, M.). Polytec. di Milano, Italy, 2nd edition. Electr. Doc. http://geomatica.ing.unico.it/workbooks2/.

Neteler, M. 2005. Time series processing of MODIS satellite data for landscape epidemiological applications. *International Journal of Geoinformatics* **1** (1): 133–138.

Oliver, C. and Quegan, S. 1998. *Understanding Synthetic Aperture Radar Image*. London: Artech House.

O'Rourke, J. 1998. *Computational Geometry in C*. Cambridge: Cambridge University Press, 2nd edition.

Page, J., Albuisson, M., and Wald, L. 2001. The European Solar Radiation Atlas: A Valuable Digital Tool. *Solar energy* **71**: 81–83.

Pebesma, E. 2001. Gstat user's manual. Technical report, Dept. of Physical Geography, Faculty of Geosciences, Utrecht University. Gstat 2.3.3., Electr. Doc. http://www.gstat.org.

Pebesma, E. J. and Wesseling, C. G. 1998. Gstat: a Program for Geostatistical Modelling, Prediction and Simulation. *Comp. & Geosc.* **24** (1): 17–31.

Poeter, E., Hill, M., Banta, E., Mehl, S., and Christensen, S. 2005. UCODE_2005 and six other computer codes for Universal Sensitivity Analysis, calibration, and uncertainty evaluation. Technical report, USGS.

Pohl, C. and van Genderen, J. 1998. Multisensor image fusion in remote sensing: Concepts, methods and application. *Int. J. of Rem. Sens.* **19**: 823–854.

Powell, M. 1992. Tabulation of thin plate splines on a very fine two-dimensional grid. In *Numerical Methods of Approximation Theory* (eds. Braess, D. and Schumaker, L.), pp. 221–244. Univ. of Cambridge, U.K.

Ramsey, P. 2006. The State of Open Source GIS. Technical report, Refractions Research Inc. Electr. Doc. `http://www.refractions.net/white_papers/oss_briefing/`.

Rase, W. 1998. Visualisierung von Planungsinformationen: Modellierung und Darstellung immaterieller Oberflächen. Technical report, Bundesamt für Bauwesen und Raumordnung, Forschungen H. 89, Bonn.

Raymond, E. 1987. The cathedral and the bazaar. Technical report. Electr. Doc. `http://www.ccil.org/~esr/writings/cathedral-paper.html`.

Raymond, E. 1999. *The Cathedral and the Bazaar. Musings on Linux and Open Source by an accidental revolutionary*. Cambridge: O'Reilly & Associates.

Redslob, M. 1998. *Radarfernerkundung in Niedersächsischen Hochmooren*. Ph.D. thesis, Inst. f. Landschaftspfl. u. Natursch., Univ. of Hannover.

Rektorys, K. 1969. *Survey of Applicable Mathematics*. Cambridge, MA: MIT Press and London: Iliffe Books Ltd.

Richards, J. and Xiuping, J. 1999. *Remote Sensing Digital Image Analysis: An Introduction*. Heidelberg: Springer, 3rd edition.

Rigollier, C., Bauer, O., and Wald, L. 2000. On the clear sky model of the ESRA – European Solar Radiation Atlas – with respect to the Heliosat method. *Solar energy* **68**: 33–48.

Rigon, R., Ghesla, E., Tiso, C., and Cozzini, A. 2006. *The Horton machine: a system for DEM analysis*. Università degli studi di Trento. Dipartimento di Ingegneria Civile e Ambientale, Trento.

Ripley, B. 1996. *Pattern Recognition and Neural Networks*. Cambridge: Cambridge University Press.

Rizzoli, A., Neteler, M., Rosà, R., Versini, W., Cristofolini, A., Bregoli, M., Buckley, A., and Gould, E. 2007. Early detection of TBEv spatial distribution and activity in the Province of Trento assessed using serological and remotely-sensed climatic data. *Geospatial Health* **1** (2): 169–176.

Robinson, A., Morrison, J., Muehricke, P., Kimerling, A., and Guptill, S. 1995. *Elements of Cartography*. New York: Wiley, 6th edition.

Scharmer, K. and Greif, J. 2000. *The European Solar Radiation Atlas*. Paris: Les Presses de l'Ecole des Mines. Vol. 2: Database and exploitation software.

Schowengerdt, R. 1997. *Remote Sensing: Models And Methods for Image Processing*. San Diego: Academic Press, 2nd edition.

Shapiro, M. and Westervelt, J. 1992. r.mapcalc. An algebra for GIS and image processing. Technical report, US-Army CERL, Champaign, Illinois. Electr. Doc. `http://grass.itc.it/gdp/`.

Snyder, J. 1987. Map projections, a working manual. Technical report, U.S. Geological Survey Professional Paper 1395. Department of the Interior. Washington, D.C.

Súri, M. and Hofierka, J. 2004. A new GIS-based Solar Radiation Model and its application to photovoltaic assessments. *Transactions in GIS* **8** (2): 175–190.

Talmi, A. and Gilat, G. 1977. Method for smooth approximation of data. *J. of Computational Physics* **23**: 93–123.

Ullah, I., Barton, M., and Mitasova, H. 2007. Mediterranean Landscape Dynamics Project. Technical report, Arizona State University.

U.S. Army CERL 1993. GRASS 4.1 Reference Manual. Technical report, U.S. Army Corps of Engineers, Construction Engineering Research Laboratories, Champaign, Illinois.

Venables, W. and Ripley, B. 2000. *S Programming*. New York: Springer.

Venables, W. and Ripley, B. 2002. *Modern Applied Statistics with S*. New York: Springer, 4th edition. Supplements: `http://www.stats.ox.ac.uk/pub/MASS4/`.

Vermote, E., Tanré, D., Deuzé, J., Herman, M., and Morcrette, J. 1997. Second Simulation of the Satellite Signal in the Solar Spectrum, 6S: An Overview. *IEEE Trans. Geosc. Rem. Sens.* **35** (3): 675–686.

Wahba, G. 1990. Spline models for observational data. Technical report, CNMS-NSF Regional Conference series in applied mathematics, 59, SIAM, Philadelphia, Pennsylvania.

Watson, D. 1992. *Contouring: A Guide to the Analysis and Display of Spatial Data*. Oxford: Pergamon.

Webster, R. and Oliver, M. 2001. *Geostatistics for Environmental Scientists*. Chichester: Wiley.

Wheeler, D. 2003. Why open source: Look at the numbers. Technical report. Electr. Doc. `http://www.dwheeler.com/oss_fs_why.html`.

Wood, J. 1996. *The Geomorphological Characterisation of Digital Elevation Models*. Ph.D. thesis, Dep. of Geogr., Univ. of Leicester, U.K. Electr. Doc. `http://www.geog.le.ac.uk/jwo/research/dem_char/thesis/`.

Zhou, J., Civico, D., and Silander, J. 1998. A wavelet transform method to merge LANDSAT-TM and SPOT panchromatic data. *Int. J. of Rem. Sens.* **19** (4): 743–757.

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