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Glossary of Notation

- 1_A , characteristic function of A , 197
 A^T , transpose of a matrix A , 215
 A^* , transpose of A , 239
 $B(t)$, Brownian motion, 7
 C , Wiener space, 24
 $C_0(\mathbb{R}^n)$, continuous functions vanishing at infinity, 219
 $C_b[0, \infty)$, bounded uniformly continuous functions on $[0, \infty)$, 218
 D , “diagonal set” of T^n , 169
 D_x , differentiation operator, 157
 $E[X | Y_1, Y_2, \dots, Y_n]$, conditional expectation of X given Y_1, Y_2, \dots, Y_n , 197
 $E[X | Y_1 = y_1, Y_2 = y_2, \dots, Y_n = y_n]$, conditional expectation, 197
 $E[X|\mathcal{G}]$, $E(X|\mathcal{G})$, $E\{X|\mathcal{G}\}$, conditional expectation, 15
 $E_P[X|\mathcal{F}_t]$, conditional expectation given \mathcal{F}_t under P , 125
 E_Q , expectation with respect to Q , 139
 $H_n(x; \rho)$, Hermite polynomial of degree n with parameter ρ , 157
 $I(f)$, Itô integral, 48
 $I(f)$, Wiener integral, 11
 I_m , identity matrix of size m , 238
 $I_n(f)$, multiple Wiener–Itô integral, 172
 J_n , closure of polynomial chaos of degree $\leq n$, 161
 K_n , homogeneous chaos of order n , 162
 $L^p(\Omega)$, p th integrable random variables, 14
 $L^2[a, b]$, square integrable functions, 10
 $L^2_B(\Omega)$, L^2 -space of Brownian functions, 160
 $L_a(t)(\omega)$, local time, 135
 L_t^+ , amount of time a Brownian motion is positive during $[0, t]$, 252
 $L^2_{\text{ad}}([a, b] \times \Omega)$, a class of integrands, 43
 $L^2_{\text{pred}}([a, b]_{\langle M \rangle} \times \Omega)$, a class of integrands for $M(t)$, 84
 $L^2_{\text{sym}}(T^n)$, symmetric L^2 -functions, 176
 $L_{n-1}(t)$, Bessel process, 132, 186
 $P(A|\mathcal{G})$, conditional probability, 197
 P_X , distribution of X , 200
 $P_t(x, A)$, transition probability of a stationary Markov process, 203
 $P_{X,Y}$, joint distribution of X and Y , 200
 $P_{s,x}(t, dy)$, transition probability of a Markov process, 200
 $Q(t, x)$, diffusion coefficient, 213
 $V_p(t)$, value of a portfolio $p(t)$, 235
 $[M]_t$, quadratic variation process of $M(t)$, 82
 $[M]_t^c$, continuous part of $[M]_t$, 110
 $\llbracket n/2 \rrbracket$, integer part of $n/2$, 157
 ΔM_s , jump of a stochastic process M_s , 110
 $\Delta\varphi(t)$, jump of a function $\varphi(t)$, 91
 Δf , Laplacian of f , 129
 Δ_n , partition, 1
 $\Gamma(\alpha)$, gamma function, 131
 \mathcal{A} , infinitesimal generator, 218
 $\mathcal{B}(V)$, the Borel σ -field of V , 23

- $\mathcal{E}_h(t)$, exponential process given by h , 136
 \mathcal{F}^B , Brownian σ -field, 160
 \mathcal{F}_τ , σ -field of a stopping time τ , 204
 $\mathcal{H}_{n_1, n_2, \dots}$, multidimensional Hermite polynomial, 165
 $\mathcal{L}_{\text{ad}}(\Omega, L^1[a, b])$, adapted processes with sample paths in $L^1[a, b]$, 102
 $\mathcal{L}_{\text{ad}}(\Omega, L^2[a, b])$, a larger class of integrands, 61
 $\mathcal{L}_{\text{pred}}(\Omega, L^2[a, b]_{(M)})$, a larger class of integrands for $M(t)$, 89
 \mathcal{P} , σ -field generated by \mathbb{L} , 79
 δ_a , Dirac delta function at a , 136
 $\delta_a(B(s))$, Donsker delta function, 136
 $\ddot{B}(t)$, informal second derivative of $B(t)$, 264
 $\|\cdot\|$, L^2 -norm, 11
 $\|\sigma\|$, Hilbert–Schmidt norm of a matrix σ , 196
 $\|\cdot\|_p$, L^p -norm, 14
 $\|\cdot\|_\infty$, supremum norm, 23
 $\dot{B}(t)$, white noise, 260
 $\frac{\delta}{\delta t}\phi$, variational derivative of ϕ , 178
 \hbar , Planck constant, 249
 $\int_a^b X_t \circ dY_t$, Stratonovich integral, 120
 $\langle M \rangle_t$, compensator of $M(t)^2$, 81
 $\langle X, Y \rangle_t$, cross variation process, 113
 $\nabla\phi$, gradient of ϕ , 220
 $|v|$, Euclidean norm of $v \in \mathbb{R}$, 196
 $\rho(t, x)$, drift, 213
 $\sigma(S^{n-1})$, surface measure of the unit sphere in \mathbb{R}^n , 131
 $\sigma\{X_s; s \leq t\}$, σ -field generated by $X_s, s \leq t$, 17
 $\sigma\{Y_1, Y_2, \dots, Y_n\}$, σ -field generated by Y_1, Y_2, \dots, Y_n , 197
 \mathbb{C} , complex numbers, 258
 \mathbb{L} , adapted and left continuous stochastic processes, 79
 \mathbb{R} , real numbers, 4
 \hat{f} , symmetrization of f , 151
 $\varphi(\Phi)$, price of an option Φ , 241
 $\varphi_b(\Phi)$, buyer's maximal price, 240
 $\varphi_s(\Phi)$, seller's minimum price, 240
 \tilde{f} , Wiener integral of f , 165
 $\{A_n \text{ i.o.}\}$, A_n infinitely often, 25
 $\{T_t; t \geq 0\}$, a semigroup, 218
 e^A , exponential of a bounded operator A , 221
 $f * g$, convolution of f and g , 253
 $f \circledast_k g$, contraction of the k th variable, 184
 $g_1 \otimes \dots \otimes g_n$, tensor product, 173
 $p(t)$, portfolio, 235
 $u \cdot v$, dot product, 220
 x^T , transpose of a column vector x , 213
 $y \leq x$, partial order defined by $y_i \leq x_i$ for all $1 \leq i \leq n$, 211
 sgn , signum function, 58
 sgn^+ , positive part of the signum function, 252
 $\text{tr}A$, trace of A , 220

a.s., almost surely, 17
i.o., infinitely often, 25
SDE, stochastic differential equation, 190
SIE, stochastic integral equation, 190

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