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When combined spatial polarities activated through spatio-temporal asynchrony lead to better mathematical reasoning for addition

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Abstract

Several recent studies have supported the existence of a link between spatial processing and some aspects of mathematical reasoning, including mental arithmetic. Some of these studies suggested that people are more accurate when performing arithmetic operations for which the operands appeared in the lower-left and upper-right spaces than in the upper-left and lower-right spaces. However, this cross-over Horizontality × Verticality interaction effect on arithmetic accuracy was only apparent for multiplication, not for addition. In these studies, the authors used a spatio-temporal synchronous operand presentation in which all the operands appeared simultaneously in the same part of space along the horizontal and vertical dimensions. In the present paper, we report studies designed to investigate whether these results can be generalized to mental arithmetic tasks using a spatio-temporal asynchronous operand presentation. We present three studies in which participants had to solve addition (Study 1a), subtraction (Study 1b), and multiplication (Study 2) in which the operands appeared successively at different locations along the horizontal and vertical dimensions. We found that the cross-over Horizontality × Verticality interaction effect on arithmetic accuracy emerged for addition but not for subtraction and multiplication. These results are consistent with our predictions derived from the spatial polarity correspondence account and suggest interesting directions for the study of the link between spatial processing and mental arithmetic performances.

Keywords Mathematical reasoning · Mental arithmetic · Polarity correspondence · Integration information theory

Introduction

Cognitive sciences have recently provided a significant amount of research to investigate the link between mathematical reasoning, including mental arithmetic, and spatial processing. This new line of research aims at exploring how the horizontal and vertical dimensions of the physical space could help people to solve arithmetic operations such as addition, subtraction, and multiplication. In the present paper, we report two original studies (Studies 1a and 2) and one follow-up study (Study 1b) that strengthen previous empirical evidence supporting the hypothesis that people's mental arithmetic performance can be sensitive to two combined spatial dimensions of the situation (Verselder, Freddi, & Dru, 2017). More importantly, the studies presented here provide a valuable theoretical contribution to the field by providing preliminary evidence distinguishing between predictions that we derived from two alternative theoretical explanations of the results (i.e., the spatial polarity correspondence account and the integration information theory).

Mental arithmetic with the processing of a single spatial dimension

Researchers have acknowledged the link between spatial and numerical processing for years, and the Spatial-Numerical Association of Response Codes effect (SNARC effect) is maybe the most famous illustration of this idea (Dehaene, Bossini, & Giraux, 1993). The SNARC effect is a spatial-numerical compatibility effect leading people to be quicker to associate large numbers (e.g., 10) with their right and upper spaces and small numbers (e.g., 1) with their left and lower

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spaces than to do the opposite (Ito & Hatta, 2004). The dominant view on the SNARC effect assumes the existence of a horizontal mental number line in which numbers are progressively located from left to right, for the Western culture, according to their magnitude (Dehaene et al., 1993). By extension, the lower part of the vertical mental number line would represent small numbers and its upper part would represent large numbers.

More recently, Wiemers, Bekkering, and Lindeman (2014) extended this idea beyond numerical processing to mental arithmetic. They found that participants were faster and more accurate in providing a verbal solution to addition while moving their arm rightward or upward than while moving their arm leftward or downward. This pattern of results was reversed for subtraction. Other researchers have reported similar results for whole-body movements, like walking, along the horizontal dimension (Anelli, Lugli, Baroni, Borghi, & Nicoletti, 2014) as well as the vertical dimension (Lugli, Baroni, Anelli, Borghi, & Nicoletti, 2013). According to these researchers, the SNARC effect was the major explanation of their results (see also, McCrinck, Dehaene, & Dehaene-Lambertz, 2007; Pinhas & Fisher, 2008). All these results support the hypothesis that spatial properties of body movements – whether they rely on the whole body or not – influence concurrent mental arithmetic performance. Moreover, this influence might rely on an extension of the SNARC effect from number processing to mental arithmetic interpreted as the use of a mental number line to represent arithmetic operations.

In the same vein, Marghetis, Núñez, and Bergen (2014) obtained consistent results with a mouse-tracking paradigm. In this study, participants had to select the correct solution for various operations, including addition and subtraction, from two alternative solutions by clicking on it with the cursor of their mouse. The experimenter recorded the continuous trajectory of the participant's hand movement when they were responding. The authors observed that hand trajectory deviated from a straight line to the right for additions and to the left for subtractions. According to the authors, these results suggest that the type of arithmetic operation performed influences spatial dynamics of body movement.

Taken together, these studies support the existence of a bidirectional link between mental arithmetic and spatial processing. Mental arithmetic seems to be rooted in spatial dimensions of actions, which is consistent with embodied theories arguing that sensory-motor processes play a central role even in high-level cognitive activities (e.g., Semin & Smith, 2008). However, one major limitation of the previous studies stems from the fact that they focused only on one spatial dimension at a time. Indeed, it remains to be established whether mental arithmetic performances are even more facilitated when considering two combined spatial dimensions.

Mental arithmetic with the processing of combined spatial dimensions

To our knowledge, Verselder et al. (2017) were among the first to investigate this question. In their studies, they asked participants to solve multiplication and addition of two operands. These operands appeared simultaneously in the same space on the computer screen, what we call a spatio-temporal synchronous operand presentation (for a comparison between this procedure and those used in the present studies, see the Method section of Study 1a). The operand location varied according to the experimental condition (horizontality: left vs. right space, verticality: lower vs. upper space). The main hypothesis was to test whether participants would be more accurate when performing operations presented in spaces with compatible spatial polarities (e.g., upper-right and lower-left spaces) than with incompatible spatial polarities (e.g., upper-left and lower-right spaces). For multiplication, the authors observed a statistically significant cross-over Horizontality × Verticality interaction.[1] Indeed, the participants were more accurate in solving multiplications when the operands appeared in the lower-left and upper-right spaces than in the upper-left and lower-right spaces. For addition, in contrast, the authors observed a statistically non-significant pattern suggesting a potential main effect for horizontality and verticality without any interaction between them. Indeed, the participants were more accurate in solving addition when the operands appeared in the right space than in the left space, regardless of their location in the vertical space. Moreover, the participants were more accurate when the operands appeared in the upper space than in the lower space, regardless of their location in the horizontal space. Based on statistical significance, the authors concluded that their results supported the hypothesis that people can take advantage of two combined spatial dimensions to increase their performance, at least for multiplication. They argued that these apparently conflicting results for multiplication and addition might come from differences between operation types (for a more in-depth discussion of this point, see the overview of our Study 2). Apart from these operation differences, the authors proposed that the polarity correspondence principle (e.g., Proctor & Cho, 2006; Proctor & Xiong, 2015) might be a good candidate to explain their results, at least for multiplication.

The spatial polarity correspondence account

According to Proctor and Cho (2006), when people take a binary decision, they use a coding scheme to represent the stimulus and the alternative responses according to positive and negative polarities. The polarity correspondence principle predicts that people's performance (i.e., accuracy and/or

[1] In statistics, a cross-over interaction is simply an interaction between two or more independent variables without any main effects.

reaction time) is better when the stimulus and the response polarities correspond than when they do not. This principle would be a general cognitive process involved in numerous cognitive activities ranging from word-picture verification tasks and implicit association tests to numerical judgments and metaphor comprehension (Proctor & Xiong, 2015).

In the field of numerical judgements, Santens and Gervers (2008) observed that participants were faster to categorize small numbers when using a response key located in near space than a response key located in far space, and conversely for large numbers. More important, the direction of the response movement along the horizontal dimension (i.e., leftward vs. rightward movement) did not moderate these results. According to the authors, these results were more consistent with an account based on the polarity correspondence principle than with the dominant account of the SNARC effect (i.e., the mental number line account). Following the polarity correspondence principle, people would assign a negative polarity to small numbers and a positive polarity to large numbers. Moreover, they would also assign negative polarities to left, lower, and far spaces whereas they would assign positive polarities to right, upper, and near space. Thus, people would be faster and more accurate to categorize number when the polarity of the stimulus (i.e., numbers) and the polarity of the response (i.e., its location in space) correspond than when they do not. This is precisely what Santens and Gervers (2008) found in their study.

With regard to the emergence of this new interpretation of the effect of spatial processing on numerical processing, Verselder et al. (2017) proposed that people would be more accurate to solve arithmetic operations when the operands appear in spaces with the same polarities than in space with different polarities (Fig. 1). This spatial polarity correspondence account of mental arithmetic is consistent with the fact that their own participants were more accurate at solving multiplication displayed in the lower-left and upper-right space than in the upper-left and lower-right spaces. However, we think that Verselder et al. (2017) provided only partial support for this spatial polarity correspondence account. Indeed, even if their results for multiplication are consistent with this account, they also seem consistent with the use of cognitive algebra predicted by the integration information theory (Anderson, 1981, 1982, 1996, 2008, 2013).

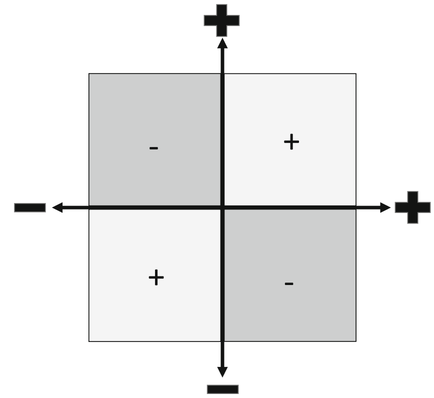

Fig. 1 Spatial polarity correspondence principle as combined polarities associated with two parts of the space. For instance, as the upper space and the right space are associated with the same positive polarities, the combined polarity of the upper-right space is positive. In contrast, as the upper space and the left space are associated with two different polarities (positive and negative, respectively), the combined polarity of the upper-left space is negative

The integration information theory

Anderson (1981, 1982, 1996, 2008, 2013) emphasized that psychology mainly deals with the complex question of how people combine information from various sources to perform a given behavior. The integration information theory is a general theory that proposes to answer this question across a wide range of psychological phenomena (e.g., moral judgment, attitude, personality, memory, visual perception). This theory assumes that information integration consists of applying algebraic rules. For instance, people would combine various pieces of information by adding or multiplying them together (an additive and multiplicative rule respectively). The additive rule implies that the combined effect of various dimensions of a stimulus on the behavior of interest will be additive in that each dimension of the stimulus would have a main effect on this behavior without interacting together. The multiplicative rule implies that the combined effect of various dimensions of the stimulus on the behavior of interest will be multiplicative in that the dimensions of the stimulus would have an interaction effect on this behavior, regardless of potential main effects. According to Anderson (1981), people would use this cognitive algebra to solve all the information integration problems they face, so that he considers this ability as "a general property of the mind" (Anderson, 1974, p. 3). In the framework of the integration information theory, Rulence-Pâques and Mullet (1998) provided fruitful considerations about the conditions under which horizontal and vertical dimensions are integrated to form a geometric judgment. In their study, participants had to estimate the area of rectangles. In one experimental condition, the participants saw rectangles with their lines joined and displayed on a single sheet of paper located in front of them, so that they could see the rectangle height and width at the same time (i.e., spatio-temporal synchrony). In another experimental condition, the participants saw two lines of a rectangle with height displayed on a sheet of paper to their left and width displayed on a sheet of paper to their right, so that they could not see the rectangle height and width at the same time (i.e., spatio-temporal asynchrony). The authors observed that participants used a multiplicative rule to combine the rectangle width (i.e., horizontal dimension) and height (i.e., vertical dimension) when the rectangle lines were joined. In contrast, the participants used an additive rule when the rectangle lines were separated. These results suggest that people can use cognitive algebra to combine spatial dimensions and use them to solve arithmetic-like tasks. More importantly for our purpose, it seems that spatio-temporal synchronous presentation of all the pieces of information promotes a multiplicative integration rule whereas spatio-temporal asynchronous presentation promotes additive integration rule.

With regard to emergence of the integration information theory, one could consider that participants in Verselder et al.'s (2017) study used a multiplicative rule and an additive rule to combine the spatial dimensions of the operands when performing multiplication and addition, respectively. This is illustrated by the results suggesting a cross-over Horizontality × Verticality interaction on multiplication accuracy and main effects of horizontality and verticality on addition accuracy without any interaction (i.e., multiplicative and additive patterns, respectively, following statistical terminology).

Spatial polarity correspondence account versus integration information theory

The spatial polarity correspondence account and the integration information theory are two distinct conceptualizations that can be useful to explain how people combine various spatial dimensions to produce a desired solution to arithmetic operations (Rulence-Pâques & Mullet, 1998; Verselder et al., 2017). Identifying whether these two accounts predict similar results or not is critical to understanding which of them better explains the observed effect of spatial dimensions on arithmetic performance. In our opinion, the similarity of the predictions stemming from these two accounts depends on the level of spatio-temporal synchrony of the operand presentation.

According to the spatial polarity correspondence account (Proctor & Cho, 2006), we predict a multiplicative pattern consisting of a cross-over Horizontality × Verticality interaction effect on arithmetic performance without a main effect of each individual spatial dimension, regardless of the level of spatio-temporal synchrony of operand presentation (Fig. 2, upper-left and upper-right panels). Following this prediction, people would be more accurate at solving operations when the operands appear in the lower-left and upper-right spaces than when they appear in the upper-left and lower-right spaces. In contrast, according to the integration information theory, we predict two different patterns of results depending on the level of spatio-temporal synchrony of operand presentation (Rulence-Pâques & Mullet, 1998). For the spatio-temporal synchronous presentation (Fig. 2, lower-left panel), we predict a multiplicative pattern like our prediction derived from the spatial polarity correspondence account. In this way, Mullet, Cretenet, and Dru, (2014) have shown how motor congruence (two motor activations of corresponding vs. non- corresponding motivational tendencies), as a kind of polarity correspondence, impacts the way participants integrate several pieces of information with a multiplicative rule. However, for the spatio-temporal asynchronous presentation (Fig. 2, lower-right panel), we predict an additive pattern consisting of a main effect of each spatial dimension on arithmetic performance without the Horizontality × Verticality interaction effect. Following this prediction, people would be more accurate at solving operations when the operands appear in the right space than in the left space, regardless of their vertical location. Moreover, people would be more accurate at solving operations when the operands appear in the upper space than in the lower space, regardless of their horizontal location.

The results reported by Verselder et al. (2017) for multiplication are consistent with our predictions derived from the two accounts. However, as the authors only used a spatio-temporal synchronous operand presentation, it is hard to decide between the predictions derived from the spatial polarity correspondence account and the integration information theory with their results for multiplication. This would require replicating their procedure using a spatio-temporal asynchronous operand presentation.

The present studies

In the present paper, we report new studies designed to extend Verselder et al.'s (2017) work. In our three studies, participants had to solve arithmetic operations in which the operands appeared successively in different spaces along the horizontal and vertical dimensions. The purpose of using this spatio-temporal asynchronous operand presentation was to contrast our predictions derived from the spatial polarity correspondence account and the integration information theory (Fig. 2). Moreover, we aimed to explore whether the difference between the results for addition and multiplication previously observed with a spatio-temporal synchronous operand presentation would also emerge with a spatio-temporal asynchronous operand presentation.

Study 1

In this study, we aimed to compare our predictions derived from the spatial polarity correspondence account with our predictions derived from the integration information theory for addition (Study 1a) and for subtraction (Study 1b) with a spatio-temporal asynchronous operand presentation. According to the spatial polarity correspondence account, we predicted a multiplicative pattern consisting of a cross-over Horizontality × Verticality interaction effect without any main effects. Participants should be more accurate at solving addition and subtraction when the second operand appeared in the lower-left and upper-right spaces than in the upper-right and lower-left spaces (Fig. 2, upper-right panels). In contrast, according to the integration information theory, we predicted an additive pattern consisting of a main effect

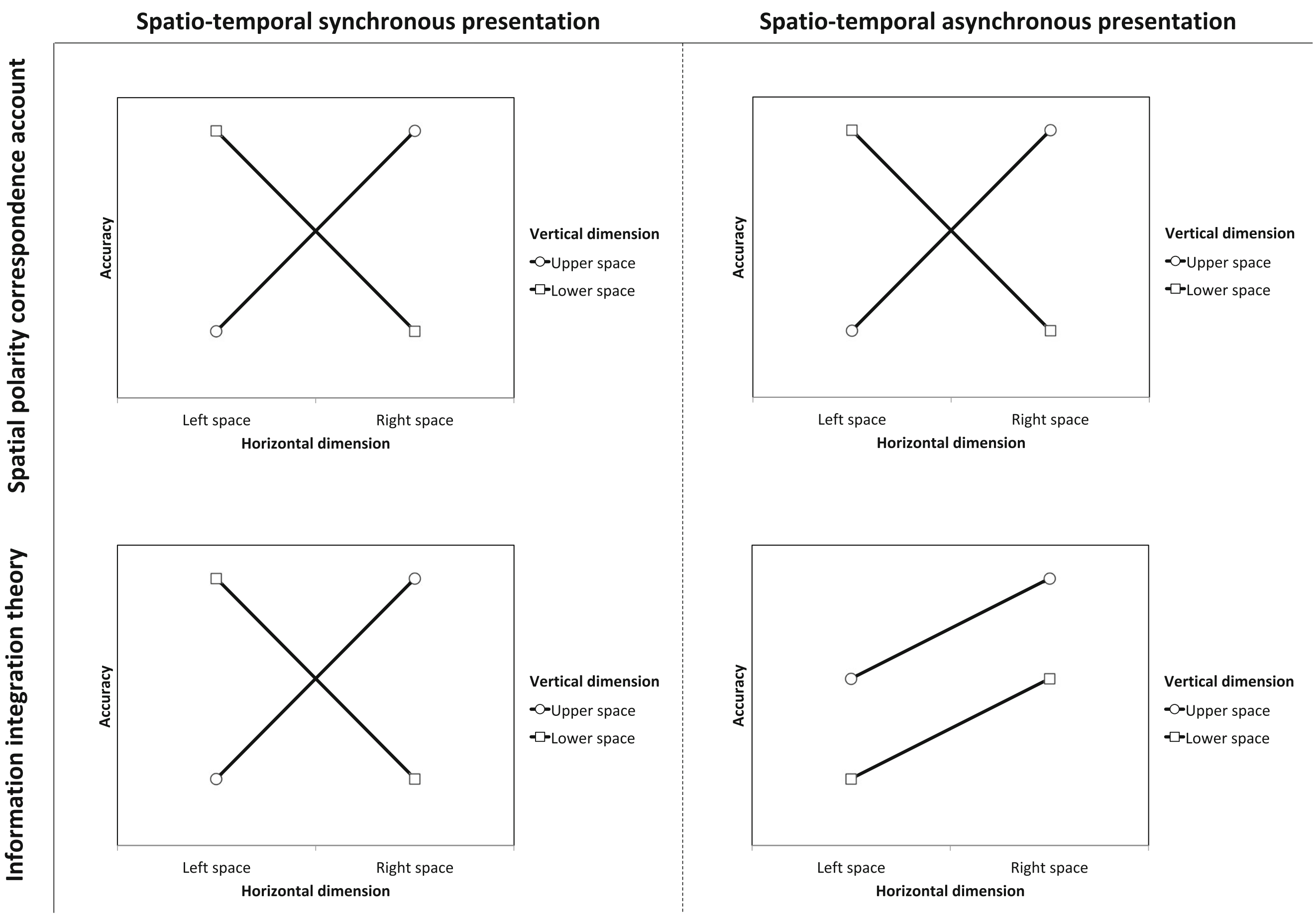

Fig. 2 Our predictions derived from the spatial polarity correspondence account and from the integration information theory for a spatio-temporal synchronous (left panels) and asynchronous (right panels) operand presentation. According to the spatial polarity account, we predicted a similar multiplicative pattern (i.e., interaction effect) for both types of presentation (upper-left and upper-right panels). According to the integration information theory, we predicted a multiplicative pattern for the spatio-temporal synchronous operand presentation (lower-left panel) and an additive pattern (i.e., main effects without interaction) for the spatio-temporal asynchronous operand presentation

of horizontality and verticality without the cross-over Horizontality × Verticality interaction effect. Participants should be more accurate at solving addition when the second operand appeared in the right and upper spaces than in the left and lower spaces (Fig. 2, lower-right panels). Subtraction and addition are formally similar but follow a reversed logic. Thus, we expected a reversed additive pattern for subtraction in that participants should be more accurate at solving subtraction in the left and lower spaces than in the right and upper spaces.

Study 1a

Method

Participants To determine an adequate minimum sample size to detect the Horizontality × Verticality interaction, we conducted an a priori power analysis using G*Power 3 (Faul, Erdfelder, Lang, & Buchner, 2007). Our power analysis indicated that a minimum of 16 participants was required to detect an effect size as large as $\eta_p^2 = .22$ in a 2 × 2 within-subject design with 95% power ($\alpha = .05$, $\varepsilon = 1$, $\rho_{\text{among repeated measures}} = .04$)[2]. As our study did not represent any threat for the participants, we decided to recruit as many participants as we could to get the best possible effect size estimate.

Fifty-two students from the Université Paris Nanterre volunteered to participate in this study (27 men, 25 women; $m_{age} = 21.85$ years, $s_{age} = 3.18$ years, $min_{age} = 19$ years, $max_{age} = 32$ years). All participants were right-handed as indicated by their answers to Porac and Cohen's (1981) handedness questionnaire. This study was conductedin accordance with the ethical principles of the American Psychological Association

[2] To avoid being too optimistic about our sample size, we followed Perugini, Gallucci, and Costantini's (2014) recommendations to perform a safeguard power analysis. They proposed using the lower bound of the confidence interval (CI) of the effect size obtained in previous studies rather than point estimates. We used the data collected by Verselder et al. (2017, Study 2a) to estimate our minimum effect size of interest and the correlation among repeated measures. We used the lower bound of the 95% CI of each parameter to be as conservative as possible as the power analysis did not suggest an immoderately large sample size.

(2017). All participants provided informed consent before taking part to this study.

Apparatus We used 15 additions for the practice block and 48 additions for the test block. These additions came from Verselder et al. (2017, Study 1b) and are presented in Table 1. Based on a pilot study, these authors considered the additions of the practice block as easy and the additions of the test block as moderately difficult. These additions consisted of adding two operands that could either be two numbers below 10 or one number below and the other above 10. None of the additions of the practice block required carrying the unit over. In contrast, all additions of the test block required carrying the unit over (e.g., 5 + 8, 15 + 8). Furthermore, we did not include any additions containing the same two digits (e.g., 8 + 8, 8 + 18). The stimuli were presented in a 40-point black Calibri font on a white background. The experiment was conducted with a computer with a 17-in. VGA monitor.

Procedure We presented the study to the participants as a pretest to validate stimuli for a future study with elderly people. This cover story was provided to limit the evaluative threat of a mathematical test for the participants. The participants worked individually in a quiet room at the university. They were seated at 50 cm from the screen, with their eyes directed to its center.

The experiment consisted of two successive blocks of trials, including a practice block immediately followed by a test block. The procedure was identical for the practice and the test blocks except that the practice block consisted of 15 easy operations whereas the test block consisted of 48 moderately difficult operations. Moreover, whereas the operator was presented during the practice block to be sure that the participants clearly identified the arithmetic operation they had to solve, it was not presented during the test block. The additions were randomly presented in four experimental conditions: lower-left space, upper-left space, lower-right space, and upper-right space. Each participant saw all experimental conditions.

Contrary to Verselder et al. (2017) who used a spatio-temporal synchronous operand presentation (Fig. 3, left panel), we used a spatio-temporal asynchronous operand presentation in that the two operands appeared successively at different locations (Fig. 3, right panel). Each trial began with a fixation point presented at the center of the screen, lasting for 3,000 ms, to draw participants' attention to the center of the screen. A first black dot followed this fixation point and lasted for 500 ms. This first black dot appeared either in the left or in the right part of the screen and was centered according to the vertical axis. It served as a spatial cue to direct the participants' attention to the future location of the first operand, which then appeared for 500 ms. A second black dot followed it and lasted for 500 ms. This second black dot appeared either in the upper or in the lower part of the space and remained at the same horizontal location as the first operand. It served as a spatial cue to direct the participants' attention to the future location of the second operand, which then appeared then over 500 ms. A white screen then appeared and lasted for 1,500 ms. The participants had to respond when this white screen appeared by providing the first response that came to mind. Each trial ended with a black screen lasting for 5,000 ms, offering a short break to the participants before the next trial. At the end of the experiment, the experimenter thanked and debriefed the participants who could then ask any questions they had about the experiment.

During the experiment, a second experimenter blind to the hypotheses recorded the arithmetic accuracy of the participants by coding 0 or 1 for an incorrect or a correct response, respectively. An incorrect response immediately corrected by the participants was still considered as incorrect. We chose this measure following many studies that used the frequency of correct responses to analyse accuracy (e.g., Bae, Cho, & Proctor, 2009; Proctor & Cho, 2003). Some previous studies indicated that requiring verbal response motivated participants to achieve a greater accuracy at the expense of their response speed (Kirk & Ashcraft, 2001; Russo, Johnson, & Stephens, 1989). Thus, when participants must generate the correct solution themselves, an accuracy measure would be better suited than a response speed measure to capture arithmetic performance. The reverse would be true when participants have only to detect the correct solution. As we were more interested in participants' accuracy at generating solutions than in response speed, we did not record reaction times.

The independent variables of the study were the horizontality and the laterality of the location where the operands appeared. We manipulated these independent variables within-subject. Thus, our study followed a 2 (Horizontality: left vs. right spaces) × 2 (Verticality: lower vs. upper space) within-subject experimental design.

Results and discussion

A lot of criticism has been raised against the hegemonic Null Hypothesis Significance Testing (NHST) and the overconfidence placed in p-values since their primary introduction in empirical science (e.g., Fidler, 2006). Some researchers have proposed reforming statistical practices by using effect size estimation (e.g., Cumming, 2014) and Bayesian statistical inference (Wagenmakers, Marsman, et al., 2017) to overcome the main limitations of NHST. In this paper, we present Bayesian analyses of variance (ANOVA) conducted with JASP, using the default multivariate Cauchy prior recommended by Wagenmakers, Love, et al. (2017, p.16-17; for more technical details, see Rouder, Morey, Speckman, & Province, 2012). As the main effect of each independent

Table 1 Generation of additions and subtractions for the Study 1

Operations + or -		Second Operand								Total of Possible Operations
		2	3	4	5	6	7	8	9	
First Operand	2								x	1
	3							x	x	2
	4						x	x	x	3
	5					(x)	x	x	x	4
	6						(x)	x	x	3
	7							(x)	x	2
	8								(x)	1
	12								x	1
	13							x	x	2
	14						x	x	x	3
	15					x	x	x	x	4
	16				(x)		x	x	x	4
	17			x	x	(x)		x	x	5
	18		x	x	x	x	(x)		x	6
	19	x	x	x	x	x	x	(x)		7
	22								x	1
	23							x	x	2
	24						x	x	x	3
	25					x	x	x	x	4
	26				(x)		x	x	x	4
	27			x	x	(x)		x	x	5
	28		x	x	x	x	(x)		x	6
	29	x	x	x	x	x	x	(x)		7
Total of Possible Operations		2	4	6	8	9	13	17	21	80

Study 1a. The additions consisted of adding two units with a carry, or one unit with a decade also with a carry on the units. There were no additions of operands composed with the same digits as 8 + 18. We did not use reversed additions like 2 + 9 and 9 + 2. The crosses in parentheses were used only for addition of two positive operands (and not for addition of two operands with different polarity)

Study 1b. The same operations were used for subtraction, except operations in parentheses

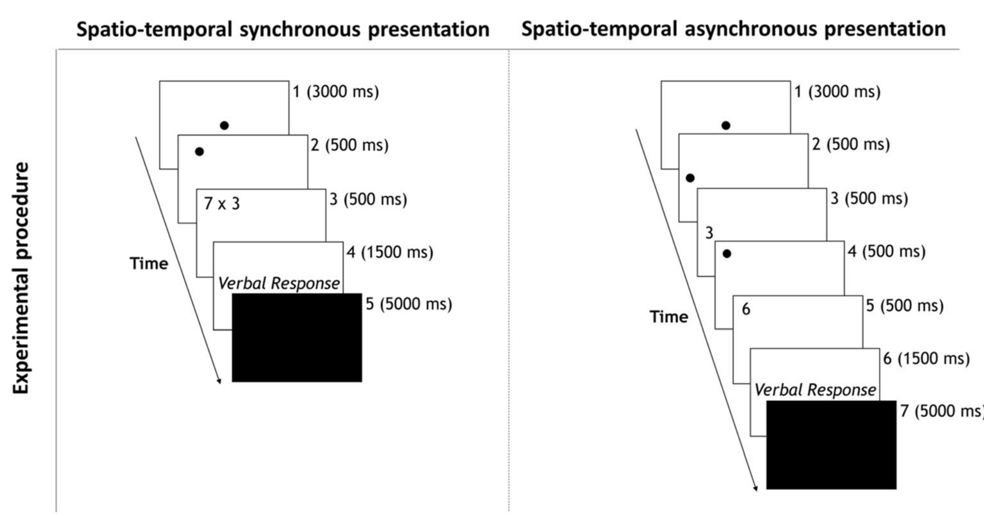

Fig. 3 Spatio-temporal synchronous operand presentation (left panel) used by Verselder et al. (2017) and spatio-temporal asynchronous operand presentation (right panel) used in the present studies. With synchronous presentation, the two operands appeared at the same time at the same location. With asynchronous presentation, the two operands appeared successively at different locations

variable and their interaction were included in more than just one model comparison in the Bayesian ANOVA, we ran a Bayesian Model Averaging (BMA) across matched models to reach a clearer conclusion about the relative probability of each effect of interest (Wagenmakers, Love, et al., 2017, p. 15).[3] We focused our interpretation on the inclusion Bayes factors (BF_{incl}). We interpreted it as the strength of evidence supporting the alternative hypothesis predicting the existence of an effect (H_1) relative to the strength of evidence supporting the null hypothesis predicting the non-existence of an effect (H_0). We used the classification proposed by Wagenmakers, Love, et al. (2017, p. 14) to qualify this strength of evidence as null (BF_{incl} = 1), anecdotal (BF_{incl} = [1/3, 1] or [1, 3]), moderate (BF_{incl} = [1/10, 1/3] or [3, 10]), strong (BF_{incl} = [1/30, 1/10] or [10, 30]), very strong (BF_{incl} = [1/100, 1/30] or [30, 100]), and extreme (BF_{incl} < 1/100 or > 100).

For readers who are more familiar with NHST, we also present classical frequentist analyses conducted with a significance threshold at .05 and supplemented with standardized effect sizes ($\hat{\eta}^2_p$). We used Wuensch's (2012) SPSS script to estimate the 90% confidence interval (CI) for η^2_p based on the procedure proposed by Smithson (2001). We also used Lakens' (2013) spreadsheet to compute $\hat{\eta}^2_G$ as proposed by Olejnik and Algina (2003), which is an alternative to $\hat{\eta}^2_p$ that is not sensitive to research design. This allows comparison of effect sizes from studies with various designs more easily.

Participants were more accurate when performing additions with a spatio-temporal asynchronous operand presentation in the lower-left space (m = .86, s = .13) than in the upper-left space (m = .82, s = .15). In contrast, participants were more accurate when performing additions with a spatio-temporal asynchronous operand presentation in the upper-right space (m = .87, s = .13) than in the lower-right space (m = .81, s = .15). This pattern suggests a cross-over Horizontality × Verticality interaction effect on addition accuracy without any main effects, as illustrated in Fig. 4 (upper-right panel).

We ran a Bayesian repeated measures ANOVA with the mean frequency of correct response (i.e., accuracy) as dependent variable and verticality and horizontality as within-subject independent variables. According to the BMA across matched models, our data suggest that the existence of the cross-over Horizontality × Verticality interaction effect is 250.45 times more likely than its non-existence. This BF_{incl} can be interpreted as extreme relative evidence for the existence of the Horizontality × Verticality interaction effect. Our data also suggest that the existence of the main effects of horizontality and verticality are, respectively, 0.44 and 0.79 times more likely than their non-existence. These BF_{incl} can be interpreted as anecdotal relative evidence for the non-existence of both main effects.

According to the frequentist repeated measures ANOVA, the cross-over Horizontality × Verticality interaction effect was statistically significant, $F(1,51) = 16.66$, $p = .000$, $\hat{\eta}^2_p = .25$, 90% CI for η^2_p [.09, .39], $\hat{\eta}^2_G = .08$. However, the main effect of horizontality, $F(1,51) = 0.00$, $p = .99$, $\hat{\eta}^2_p = .00$, 90% CI for η^2_p [.00, .00], $\hat{\eta}^2_G = .00$, and the main effect of verticality, $F(1,51) = 1.00$, $p = .32$, $\hat{\eta}^2_p = .02$, 90% CI for η^2_p [.00, .12], $\hat{\eta}^2_G = .00$, were not statistically significant.

The results of this study suggest that people are more accurate when performing additions with a spatio-temporal asynchronous operand presentation when the second operand appeared in the lower-left and upper-right spaces than in the upper-left and lower-right spaces. In other words, people seem to be more accurate when performing additions with compatible spatial polarities (i.e., two negative or two positive spatial polarities) than with incompatible ones (i.e., one positive and one negative spatial polarity). This cross-over Horizontality × Verticality interaction effect on addition accuracy corroborates our predictions derived from the spatial polarity correspondence account rather than those derived from the information integration theory.

Our results observed with a spatio-temporal asynchronous operand presentation differ from those observed by Verselder et al. (2017) for addition with a spatio-temporal synchronous operand presentation. Thus, spatial polarity correspondence seems to be relevant for people who perform addition with an asynchronous rather than a synchronous operand presentation.

Study 1b

Method

Participants To determine an adequate minimum sample size to detect the Horizontality × Verticality interaction, we used the results of the a priori power analysis presented in Study 1a. As our study did not represent any threat for the participants, we decided to recruit as many participants as we could to get the best possible effect size estimate.

Fifty-five students from the Paris Nanterre University volunteered to participate in this study (30 men, 27 women; m_{age} = 21.88 years, s_{age} = 3.23 years, min_{age} = 18 years, max_{age} = 33 years). All participants were right-handed as indicated by their answers to Porac and Cohen's (1981) handedness questionnaire. This study was conducted in accordance with the ethical principles of the American Psychological Association (2017). All participants provided informed consent before taking part in this study.

[3] See Mathôt (2017) for a discussion of the advantages of BMA across matched models over BMA across all models.

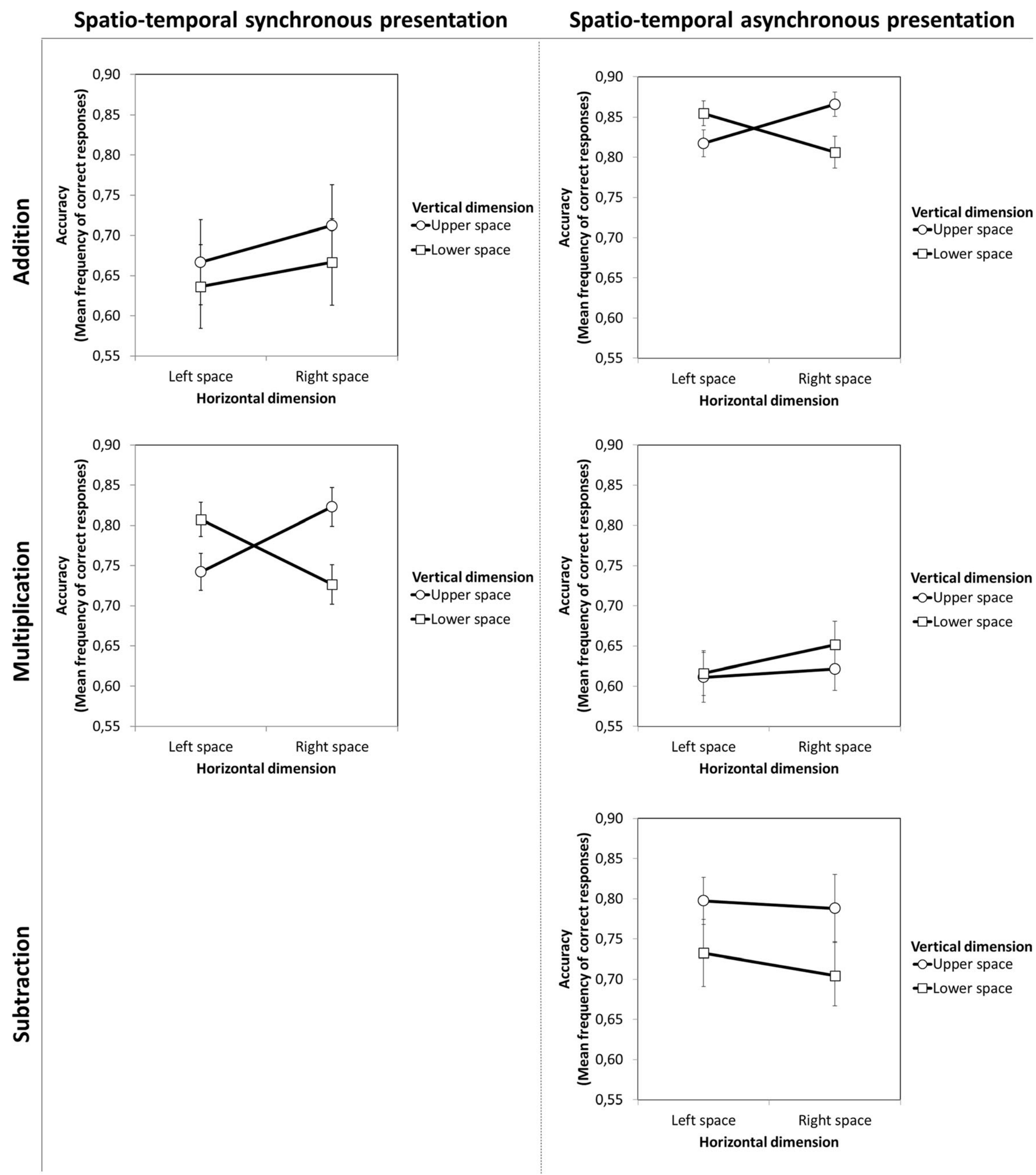

Fig. 4 Results observed by Verselder et al. (2017) for addition (upper-left panel) and multiplication (middle-left panel) and results reported in the present paper for addition (Study 1a, upper-right panel), multiplications (Study 2, middle-right panel), and subtractions (Study 1b, lower-right panel). Error bars represent correlation- and difference-adjusted standard error (SE) of the means calculated with WSPLOT for SPSS (O'Brien & Cousineau, 2014; for a standard terminology for SE and CI, see Cousineau, 2017)

Apparatus and procedure The apparatus, the procedure, and experimental design were the same as in Study 1a with the only exception that the participants had to solve subtractions instead of additions. We used 15 subtractions for the practice block and 48 subtractions for the test block. Based on a pilot study, we considered the subtractions of the practice block as easy and the subtractions of the test block as moderately difficult. These subtractions were presented as additions like those used in Study 1a, except that one of the operands was positive whereas the other was negative (e.g., 9 + (-3) or -9 + 3).

Results and discussion

Participants were more accurate when performing subtractions with a spatio-temporal asynchronous operand presentation when the second operand appeared in the upper-left space ($m = .80$, $s = .13$) and upper-right space ($m = .78$, $s = .26$) than in the lower-left space (m = .73, s = .26) and lower-right space (m = .71, s = .25). These patterns suggest a main effect of verticality on subtraction accuracy, as illustrated in Fig. 4 (bottom-right panel).

We ran a Bayesian repeated measures ANOVA with the mean frequency of correct response (i.e., accuracy) as dependent variable and horizontality and verticality as within-subject independent variables. According to the BMA across matched models, our data suggest that the existence of the Horizontality × Verticality interaction effect is 0.20 times more likely than its non-existence. This BF_{incl} can be interpreted as moderate relative evidence for the non-existence of the Horizontality × Verticality interaction effect. Our data also suggest that the existence of the main effects of horizontality and verticality are, respectively, 0.11 and 16.78 times more likely than their non-existence. These BF_{incl} can be interpreted as moderate relative evidence for the non-existence of horizontality and as strong relative evidence for the existence of a main effect of verticality.

According to the frequentist repeated measures ANOVA, neither the Horizontality × Verticality interaction effect, $F(1,54) = 0.13$, $p = .72$, $\hat{\eta}^2_p = .00$, 90% CI for η^2_p [.00, .06], $\hat{\eta}^2_G = .00$, nor the main effect of horizontality, $F(1,54) = 0.51$, $p = .48$, $\hat{\eta}^2_p = .01$, 90% CI for η^2_p [.00, .09], $\hat{\eta}^2_G = .00$, were statistically significant. However, the main effect of verticality was statistically significant, $F(1,54) = 6.65$, $p = .01$, $\hat{\eta}^2_p = .11$, 90% CI for η^2_p [.01, .24], $\hat{\eta}^2_G = .03$.

The results of this study suggest that people are only sensitive to the vertical location of the operands when performing subtractions with a spatio-temporal asynchronous operand presentation. Not only do people not seem to combine vertical and horizontal dimensions, but they also do not seem sensitive to the horizontal location of the operands at all. Indeed, people were more accurate at performing subtraction with a spatio-temporal asynchronous operand presentation when the second operand appeared in the upper space than in the lower space, regardless of the horizontal dimension. These results are neither consistent with our predictions derived from the spatial polarity correspondence account nor with those derived from the integration information theory.

Discussion

The results from our Study 1a are consistent with our predictions derived from the spatial polarity correspondence account. However, our Study 1b suggests a more nuanced view as this pattern seems to be only limited to addition of positive operands and does not seem to generalize to subtraction, which is addition of positive and negative operands. We think that this apparent discrepancy could come from a sampling error or could reflect that the way spatial dimensions are combined depends on the specific properties of each operation type. Although these two alternative explanations required further replications to be investigated, our own data could still convey preliminary insights. Thus, we conducted a between-study analysis to estimate whether the operation type (i.e., addition vs. subtraction) moderates the Horizontality × Verticality interaction effect that we predicted according to the spatial polarity correspondence account.

We ran a Bayesian mixed-design ANOVA with the mean frequency of correct response (i.e., accuracy) as dependent variable, horizontality and verticality as within-subject independent variables, and operation type as between-group independent variable. According to the BMA across matched models, our data suggest that it is 0.48 more likely that the operation type moderates the Horizontality × Verticality interaction than it does not. This BF_{incl} can be interpreted as anecdotal relative evidence for the non-existence of Operation Type × Horizontality × Verticality interaction effect. According to the frequentist mixed-design ANOVA, this three-way interaction effect was not statistically significant, $F(1, 105) = 1.81$, $p = .18$, $\hat{\eta}^2_p = .02$, 90% CI for η^2_p [.00, .08], $\hat{\eta}^2_G = .00$. However, our data suggest that it is 1.13 more likely that the operation type moderates the effect of verticality. This BF_{incl} can be interpreted as anecdotal relative evidence for the existence of Operation Type × Verticality interaction effect. According to the frequentist mixed-design ANOVA, this two-way interaction effect was marginal, $F(1, 105) = 3.94$, $p = .05$, $\hat{\eta}^2_p = .04$, 90% CI for η^2_p [.00, .11], $\hat{\eta}^2_G = .01$.

Contrary to what our results apparently suggested, this between-study analysis does not clearly support the conclusion that the cross-over Horizontality × Verticality interaction effect on operation accuracy reliably differs for addition and subtraction. However, it supports the conclusion that the main effect of verticality on operation accuracy is larger for subtraction than for addition, irrespective of horizontality. This implies that operation type might be a critical aspect to consider when considering how people use spatial dimensions when performing arithmetic tasks. More studies are needed to better understand the moderating role of operation type on the way people combine vertical and horizontal dimensions of the operands. Nevertheless, the results of our Study 1a are more consistent with our predictions derived from the spatial polarity correspondence account than with those derived from the integration information theory.

Study 2

At first sight, our results for addition (Study 1a) seem inconsistent with Verselder et al.'s results (2017) for addition, whereas they seem consistent with their results for multiplication. In their Discussion section, they speculated that the observed difference between their own results for multiplication and addition might come from the fact that these two types of operations rely on distinct representations.

Dixon, Deets, and Banfert (2001) proposed a distinction between arithmetic and intuitive representations. Arithmetic representations would deal with formal principles that people apply to get the correct solution to the operation. In contrast, intuitive representations would deal with non-formal relations between the operation components (i.e., operands and operators) like the level of spatio-temporal synchrony of the operand presentation. Following Rulence-Pâques and Mullet (1998), multiplication and addition might differ on the relative role played by the level of spatio-temporal synchrony of operand presentation for each type of operation. Indeed, people are more prone to add elements (e.g., rectangle height and width) when they are presented asynchronously and to multiply elements when they are presented synchronously. Thus, people might intuitively represent addition as more sequential in nature than multiplication.

According to Verselder et al. (2017, p. 922), the simultaneous nature of their spatio-temporal synchronous operand presentation should be more consistent with the intuitive representation of multiplication than with the more sequential nature of the intuitive representation of addition. Thus, this type of presentation could have facilitated multiplication rather than addition, resulting in the predicted Horizontality × Verticality interaction effect on multiplication accuracy but not on addition accuracy. If so, the sequential nature of the spatio-temporal asynchronous operand presentation should be more consistent with the intuitive representation of addition than with the intuitive representation of multiplication. Thus, this type of presentation should facilitate addition rather than multiplication, resulting in the predicted Horizontality × Verticality interaction effect on addition accuracy but not on multiplication accuracy. The purpose of the present study was to investigate whether the multiplicative pattern that we predicted according to the spatial polarity correspondence account vanishes when people solve multiplication with a spatio-temporal asynchronous operand presentation.

Method

Participants

To determine an adequate minimum sample size to detect the Horizontality × Verticality interaction, we conducted an a priori power analysis. This analysis was like those conducted for Study 1, except for the research design that includes one more independent variable with three modalities. Our power analysis indicated that a minimum of nine participants was required to detect an effect size as large as $\eta^2_p = .22$ in a 2 × 2 × 3 within-subject design with 95% power ($\alpha = .05$, $\varepsilon = 1$, $\rho_{among\ repeated\ measures} = .04$). As our study did not represent any threat for the participants, we decided to recruit as many participants as we could to get the best possible effect size estimate.

Thirty-three right-handed French students from a college in Paris participated in this study (27 men, 6 women; m_{age} = 14.42 years, s_{age} = 1.23 years, min_{age} = 13 years, max_{age} = 17 years). All participants were right-handed as indicated by their answers to Porac and Cohen's (1981) handedness questionnaire. This study was conducted in accordance with the ethical principles of the American Psychological Association (2017). All participants provided informed consent before taking part to this study.

Apparatus and procedure

The apparatus and the procedure were the same as in Study 1 with the only exception that the participants had to solve multiplication instead of addition or subtraction. We used 15 multiplications for the practice block and 48 multiplications for the test block from Verselder et al. (2017, Study 1a). Based on a pilot study, these authors considered the multiplications of the practice block as easy and the multiplications of the test block as moderately difficult. These multiplications consisted of multiplying two operands below 10. The multiplications of the practice block required multiplying two operands ranging from 1 to 5. The multiplications of the test block required multiplying one operand ranging from 2 to 5 with another operand ranging from 6 to 9. Considering all the possible combinations of two operands resulted in 16 different multiplications. We presented these multiplications for the three possible combinations of operand signs (e.g., 2 × 6, - 2 × 6, - 2 × -6).

The independent variables of the study were the horizontality and the verticality of the location where the operands appeared and the combination of the operand signs. We manipulated these independent variables within-subject. Thus, our study followed a 2 (horizontality: left vs. right spaces) × 2 (verticality: lower vs. upper space) × 3 (operand signs: ++, –, +-) within-subject experimental design.

Results

Participants were nearly as accurate when performing multiplication with a spatio-temporal asynchronous operand presentation when the second operand appeared in the lower-left space (m = .62, s = .26) as in the upper-left space (m = .61, s = .24). Similarly, participants were nearly as accurate when performing multiplication with a spatio-temporal asynchronous operand presentation when the second operand appeared in the lower-right space (m = .65, s = .26) than in the upper-right space (m = .62, s = .25). These patterns do not clearly suggest the presence of main effects of horizontality and verticality or their interaction on accuracy, as illustrated in Fig. 4 (middle-right panel). However, participants were slightly more accurate for multiplication of two positive operands (m = .67, s = .07) than for multiplication of positive and negative operands (m = .62, s = .09), for which they were even more accurate than for multiplication of two negative operands (m = .57, s = .09). This pattern is compatible with the presence of a main effect of operand signs on accuracy.

We ran a Bayesian repeated measures ANOVA with the mean frequency of correct response (i.e., accuracy) as dependent variable, verticality, laterality, and operand signs as within-subject independent variables. According to the BMA across matched models, our data suggest that the existence of a Horizontality × Verticality interaction effect is 0.19 times more likely than its non-existence. This BF_{incl} can be interpreted as moderate relative evidence for the non-existence of the Horizontality × Verticality interaction effect. Our data also suggest that the existence of the main effects of horizontality and verticality is,, respectively 0.20 and 0.16 times more likely than their non-existence. These BF_{incl} can be interpreted as moderate relative evidence for the non-existence of both main effects. In contrast, our data suggest that the existence of a main effect of operand signs is 23.10 times more likely than its non-existence. These BF_{incl} can be interpreted as strong relative evidence for the existence of the main effect of operand signs. More importantly, our data suggest that it is 0.21 times more likely that the operand signs moderate the Horizontality × Verticality interaction effect than it does not. This BF_{incl} can be interpreted as moderate relative evidence for the non-existence of the null Operand Signs × Horizontality × Verticality interaction effect.

According to the frequentist repeated measures ANOVA, the Horizontality × Verticality interaction effect was not statistically significant, $F(1, 32) = 0.39$, $p = .54$, $\hat{\eta}^2_p = .01$, 90% CI for η^2_p [.00, .13], $\hat{\eta}^2_G = .00$. Neither the main effect of horizontality, $F(1, 32) = 1.60$, $p = .22$, $\hat{\eta}^2_p = .05$, 90% CI for η^2_p [.00, .20], $\hat{\eta}^2_G = .00$, nor the main effect of verticality, $F(1, 32) = 0.63$, $p = .44$, $\hat{\eta}^2_p = .02$, 90% CI for η^2_p [.00, .15], $\hat{\eta}^2_G = .00$, were statistically significant. The main effect of operand signs was statistically significant, $F(2, 64) = 5.37$, $p = 0.01$, $\hat{\eta}^2_p = .14$, 90% CI for η^2_p [.02, .26], $\hat{\eta}^2_G = .04$. The Operand Signs × Horizontality × Verticality interaction effect was not statistically significant, $F(2, 64) = 1.09$, $p = 0.34$, $\hat{\eta}^2_p = .03$, 90% CI for η^2_p [.00, .11], $\hat{\eta}^2_G = .00$. The operand signs did not significantly interact with horizontality, $F(2, 64) = 1.15$, $p = 0.33$, $\hat{\eta}^2_p = .04$, 90% CI for η^2_p [.00, .11], $\hat{\eta}^2_G = .01$, or verticality, $F(2, 64) = 1.14$, $p = 0.33$, $\hat{\eta}^2_p = .03$, 90% CI for η^2_p [.00, .11], $\hat{\eta}^2_G = .01$.

Discussion

The results from this study suggest that people's multiplication accuracy is independent of the location of the operands for a spatio-temporal asynchronous operand presentation. These results for multiplication differ markedly from our results for addition (Study 1a), which suggest that the operation type moderates how people combine spatial dimensions while performing mental arithmetic tasks with a spatio-temporal asynchronous operand presentation. To provide preliminary support for this hypothesis, we conducted a between-study analysis by combining the data from our Studies 1a and 2.

We ran a Bayesian mixed-design ANOVA with the mean frequency of correct response (i.e., accuracy) as dependent variable, horizontality and verticality as within-subject independent variables, and operation type (multiplication vs. addition) as between-group independent variable. According to the BMA across matched models, our data suggest that it is 6.96 times more likely that the operation type moderates Horizontality × Verticality interaction than it does not. This BF_{incl} can be interpreted as moderate relative evidence for the existence of an Operation Type × Horizontality × Verticality interaction effect. According to the frequentist mixed-design ANOVA, this interaction effect was statistically significant, $F(1, 83) = 7.78$, $p = .01$, $\hat{\eta}^2_p = .09$, 90% CI for η^2_p [.01, .19], $\hat{\eta}^2_G = .01$.

One could argue that our results are neither consistent with our predictions derived from the spatial polarity correspondence account nor with those derived from the integration information theory. However, this apparent inconsistency is solved when considering that spatio-temporal asynchronous presentation of elements promotes an additive integration of this element rather than a multiplicative integration (Rulence-Pâques & Mullet, 1998). In this way, our between-study analysis suggests that people combine the horizontal and vertical dimensions according to the spatial polarity correspondence account when performing addition but not when performing multiplication for spatio-temporal asynchronous operand presentation.

General discussion

The aim of our studies was to examine whether people can combine the horizontal and vertical dimensions when performing addition with spatio-temporal asynchronous operand presentation as when performing multiplications with spatio-temporal synchronous operand presentation (Verselder et al., 2017). More importantly, we wanted to decide between our predictions derived from the spatial polarity correspondence account and our predictions derived from the integration information theory, which differed depending on the level of spatio-temporal synchrony of operand presentation. According to the spatial polarity correspondence account, we expected a cross-over Horizontality × Verticality interaction effect without any main effects. If so, the participants should have been more accurate when performing operations when the second operand appeared in the lower-left and upper-right spaces than in the upper-left and lower-right spaces. According to the integration information theory, we expected main effects of horizontality and verticality without the Horizontality × Verticality interaction effect. If so, participants should have been more accurate when performing operations when the second operand appeared in the upper and the right spaces than in the lower and left spaces. In other words, our predictions from the spatial polarity correspondence account and the integration information theory consisted of a multiplicative pattern and an additive pattern, respectively, for spatio-temporal asynchronous operand presentation.

For addition, we found a cross-over Horizontality × Verticality interaction effect predicted by the spatial polarity correspondence account. In contrast, for multiplication, we found no effect of spatial dimensions, which was consistent with neither the spatial polarity correspondence account nor the integration information theory. These different results for addition and multiplication might stem from the fact that spatio-temporal asynchronous operand presentation facilitates addition rather than multiplication as suggested by Rulence-Pâques and Mullet's (1998) results on geometric judgment of rectangle area. This result is important because Verselder et al. (2017) observed the opposite pattern with a cross-over Horizontality × Verticality interaction effect for multiplication but not for addition (Fig. 4, lower-left and upper-left panels, respectively). The critical difference between the studies reported in the present paper and those reported by Verselder et al. (2017) is that they used a spatio-temporal synchronous operand presentation. Further studies will have to explore whether this variable moderates the Operation Type × Horizontality × Verticality reported here and suggested by Verselder et al.'s (2017) results.

To lend preliminary support to this hypothesis, we conducted a between-paper analysis including the data reported here and those reported by Verselder et al. (2017, Study 2a and 2b). All the data used are depicted on Fig. 4. We ran a Bayesian mixed-design ANOVA with the mean frequency of correct response (i.e., accuracy) as dependent variable, horizontality and verticality as within-subject independent variables, and operation type and level of spatio-temporal synchrony of operand presentation as between-group independent variables. According to the BMA across matched models, our data suggest that it is 34.16 times more likely that the level of spatio-temporal synchrony of operand presentation moderates the Operation Type × Horizontality × Verticality interaction effect than it does not. This BF_{incl} can be interpreted as very strong relative evidence for the existence of a Spatio-Temporal Synchrony × Operation type × Horizontality × Verticality interaction effect. According to the frequentist mixed-design ANOVA, this four-way interaction effect was statistically significant, $F(1, 146) = 11.49$, $p = .000$, $\hat{\eta}^2_p = .07$, 90% CI for η^2_p [.02, .15], $\hat{\eta}^2_G = .01$. Thus, this between-paper analysis is consistent with our hypothesis that the level of spatio-temporal synchrony of operand presentation defines a boundary condition of the predictive value of the spatial polarity correspondence account.

However, we would like to prompt readers to treat our between-paper and between-study analyses with caution. Even if these analyses provide preliminary statistical support for some of our hypotheses, it remains to directly assess the causal nature of the observed effects in a randomized experimental design. Indeed, without an appropriate methodological control (e.g., random assignment of participants to various operation types), any differences in the participants from the different studies included in these analyses could explain the results (e.g., differences in arithmetic skills or age). Despite this limitation regarding causal inference, these analyses remain interesting as they support the existence of correlations, which is one of the three necessary conditions for a valid causal inference (e.g., Mill, 1973). The two other necessary conditions are the temporal precedence of the independent variable over the dependent variable and the absence of confounds. Whereas the temporal precedence is met according to the experimental design of each individual study included in the analyses, the absence of confounds is not, according to the non-random assignment of participants to the various studies.

Thus, further studies are needed to confirm the causal role of the level of spatio-temporal synchrony of operand presentation and the operation type as moderator of the Horizontality × Verticality interaction effect on arithmetic accuracy. This seems to be of theoretical importance as such a four-way interaction effect suggests how to articulate the spatial polarity correspondence account and the integration information theory, despite what we considered initially as two different accounts with divergent predictions. Indeed, the spatial polarity correspondence account might predict that people integrate horizontal and vertical dimensions in a multiplicative way while performing arithmetic operations. The integration information theory might predict when people will use this multiplicative integration according to the operation type and the level of spatio-temporal synchrony of operand presentation. This would mean that the spatial polarity correspondence account might simply be a part of the integration information theory (for a first examination of this idea, see Mullet et al., 2014). However, more theoretical studies are needed to develop a more integrative view of these accounts by bridging the gap between them.

Our studies present two limitations that we need to consider when interpreting our results and planning for new studies. First, our results for subtraction are neither consistent with our predictions derived from the spatial polarity correspondence account nor with those derived from the integration information theory. We encourage other researchers to replicate our studies to estimate whether this apparent discrepancy comes from the particularities of our own participants or if it can be generalized to other samples. We advocate manipulating the operation type within-subject with a counterbalanced block-wise variation. Doing this will allow a better control on some potential confounding variables (e.g., participants' arithmetic skills) and an increase of statistical power for testing higher-order interaction while minimizing the potential cost of switching from one operation type to another. Second, in the present studies, we manipulated the horizontal location of the first operand while keeping its vertical location constant. Then, the second operand appeared at the same horizontal location as those of the first operand but we manipulated its vertical location. Manipulating the spatial dimensions of the operands in this way could have limited the generalizability of our conclusions. For instance, our conclusion might not generalize to situations in which verticality is manipulated before horizontality or in which the location of the second operand is independent of those of the first one. As previous research found a similar SNARC effect for horizontality and verticality (Dehaene et al., 1993; Ito & Hatta, 2004), we do not expect different results for such situations.

In this perspective, an interesting way to investigate the boundary conditions of the spatial polarity correspondence effect observed in Study 1a could be to estimate the relative contribution of the level of spatial and temporal synchrony of operand presentation. This could be done by manipulating independently the spatial inter-stimulus interval (SISI) and the temporal inter-stimulus interval (TISI) by increasing or decreasing, respectively, the distance and the delay between the presentation of the two operands. Landy and Goldstone (2010) have made a first step in this direction. They observed that people tend to favor additive integration of the operands when they are located far from each other (i.e., a large SISI) rather than close to each other (i.e., a small SISI). According to this result, we expect that increasing the spatial or temporal asynchrony by increasing the SISI and the TISI, respectively, will increase the spatial polarity correspondence effect for addition.

Another promising direction for future studies is to investigate more deeply what governs the effect of combined spatial dimensions on mental arithmetic. First, there were no control conditions in the studies reported here or in those reported by Verselder et al. (2017). This observation does not decrease the merit of these studies, but it limits the conclusions that one can draw from them. Indeed, performing operations displayed with compatible spatial polarities (i.e., lower-left and upper-right spaces) might improve arithmetic performances. On the contrary, performing operations displayed with incompatible spatial polarities (i.e., upper-left and lower-right spaces) might decrease arithmetic performances. Thus, further studies will have to explore whether spatial polarities have a facilitating or interfering effect (or both) on arithmetic performances depending on their compatibility.

Second, further research is needed to disentangle the role of the magnitude of operands, as cognitive polarities, from those of spatial polarities in the effect of combined spatial dimensions on mental arithmetic. Indeed, the processing of single numbers seems to be easier when their cognitive polarity, determined by their magnitude, matches their spatial polarity, determined by their location (e.g., Santens & Gervers, 2008). Following this idea, we expect that people should be more accurate in solving arithmetic operations when the spatial polarities of the operands are compatible than when they are not, but this effect should be larger when the cognitive and spatial polarities of each operand correspond. For instance, it might be easier to solve "7 + 8," two large numbers (i.e., two positive cognitive polarities), when the operands appear in the right, upper, lower-left, or upper-right spaces (i.e., positive spatial polarity) than in the left, lower, upper-left, or lower-right spaces (i.e., negative spatial polarity). In contrast, it might be easier to solve "7 + 2," a large and a small number (i.e., one positive and one negative cognitive polarities) when "7" appears in right, upper, lower-left, or upper-right spaces and "2" appears in the left, lower, upper-left, or lower-right spaces rather than the reverse.

In summary, our results support an embodied approach for high-order conceptual functioning, such as mathematical reasoning, and they are of importance for learning considerations. Mathematics is usually learned through conceptual and amodal strategies, whereas it would be useful for teachers to use modal strategies situating mathematical learning within concrete perceptual and motor experiences. In this direction, this article establishes that teachers might go beyond the examination of one spatial dimension, either horizontal or vertical, for the processing of arithmetic operations (Anelli et al., 2014; Lugli et al., 2013). It would be relevant to consider several spatial dimensions depending on how they are processed (spatio-temporal synchrony vs. asynchrony) to facilitate different types of operations.

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