Frequency ratio determines discrimination of concentric radial frequency patterns in the peripheral visual field

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Abstract

Using a radial frequency discrimination task that has not been tested in many previous studies, we examined the dependence of the pattern radius (4 to 16 deg) on the radial frequency thresholds of two different types of concentric radial frequency (RF) patterns: constant circular contour frequency (CCF) RF patterns with different radii, which have the constant physical length of modulation cycle in external real-world space, and constant radial frequency magnified RF patterns with different radii, which have the constant cortical length of modulation cycles. These two types RF patterns used as the reference stimuli had an equal maximum orientation difference from circularity regardless of change in radius. The discrimination threshold expressed by the frequency ratio between RF patterns of different frequencies vs. radius functions for the constant CCF RF patterns indicated different functional forms dependent on the modulation amplitude of the RF patterns. The thresholds increased with increasing pattern radius for small modulation amplitude RF patterns but were relatively flattened for large-amplitude RF patterns. This dependence was ascribed to the eccentricity effect wherein the deformation thresholds for discriminating the RF pattern from a circle increase with increasing stimulus eccentricity (Feng et al. 2020). The discrimination thresholds vs. radius functions for the magnified RF patterns were also flattened for different modulation amplitudes and frequencies. The thresholds (frequency ratio) were similar at all eccentricities. Cortical magnification neutralized the eccentricity effect observed for the constant CCF patterns.

Keywords Concentric radial frequency pattern · Eccentricity effect · Discriminability · Frequency ratio · Orientation ratio · Retinocortical mapping · Discrimination task

Introduction

Radial frequency (RF) patterns, which were introduced by Wilkinson, Wilson, and Habak (1998), have frequently been

used to investigate aspects of shape processing. The simple mathematical definition of RF patterns has made RF patterns a popular stimulus in psychophysical, physiological, and imaging studies (Wilkinson et al., 2000; Löffler, 2008; Löffler, 2015; Salmela et al., 2016). However, Wilkinson et al. (1998) explained the mathematical limitation of RF patterns and stressed their differences from Fourier shape descriptors, which can in principle be used to create any kind of closed two-dimensional shapes. Recently, Schmidtmann and Freund (2019) demonstrated that compound RF patterns can represent only a very small subset of natural and synthetic shapes, but the RF patterns are not suited as universal shape descriptors utilized by the visual system. Schmidtmann and Freund (2019) also noted that it is important to keep in mind that studies of RF patterns implicitly restrict them to only certain ranges of perceptually meaningful dimensions. Despite this limitation, there remains the possibility that accumulating structural properties of the visual performance of RF patterns further by using a wider range of stimulus condition and a different visual task and by exploring a possible model may open a new perspective and lead to deep insight into the shape encoding mechanism.

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Here, we provide an additional test on the discrimination performance of RF patterns presented in the peripheral visual field. We asked three questions regarding the visual performance to concentric RF patterns with a radius that varied from 4 to 16 deg, using a frequency discrimination task: What are the most important factors to determine the discrimination between different RF patterns? How does the discrimination performance vary with eccentricity? How can spatial scaling (the retinocortical transformation) account for the eccentric properties of the discrimination performance?

RF patterns are constructed by applying a radial sinusoidal modulation to the radius r_0 such that the radius of the deformed pattern at polar angle θ in radians is:

$$r = r_0(1 + A\cos(\omega\theta + \varphi)) \quad (1)$$

where r_0 is the mean radius, A is the radial modulation amplitude, and ω is the radial frequency. The primary parameters of visual performance for RF patterns are modulation amplitude (A) on the radial coordinate and radial frequency (ω) on the angular coordinate. These two parameters are in an orthogonal relationship with each other. They can be varied independently. However, they cannot affect the salience of the modulation independently. Properties such as local curvature and the maximum local orientation difference from circular vary with both of these parameters. Most previous studies have used a deformation threshold task in which the modulation amplitude threshold to discriminate the RF pattern from a circular pattern is measured by manipulating the modulation amplitude. In previous studies, the radial frequency of RF patterns was used as an important parameter that affects visual performance. Surprisingly, to our knowledge, radial frequency discrimination tasks, in which the discrimination threshold to detect the difference in the frequency between two RF patterns is measured by manipulating the radial frequency of the RF patterns, has rarely been studied; some exceptions, however, include psychophysical studies that have used radial frequency identification tasks (Wilkinson et al., 1998; Poirier and Wilson, 2006; Bell et al., 2007; Dickinson et al., 2013). The frequency discrimination threshold task used in this study may plausibly shed new light on the aspect of the visual performance of RF patterns. The radial frequency discrimination task may be useful to examine the sensitivity to changes in the angular coordinate of RF patterns and the frequency selectivity in the processing of RF patterns.

Achtman, Hess, and Wang (2000), reported that the performance of RF patterns presented in the peripheral retinal location decreased, and Achtman et al. (2000) Zolubak and Garcia-Suarez (2017, 2018), and Zolubak, Schmidtman and Garcia-Suarez (2018) showed that the eccentricity effect could be neutralized by scaling with respect to cortical magnification factors. In these last studies, the researchers used a deformation task. In the present study, we tested the eccentricity effect for

magnified and nonmagnified concentric presentations of RF patterns using the radial frequency discrimination task.

In the present study, using constant circular contour frequency (CCF) patterns with different radii, which have the constant physical length of modulation cycle in external real-world space, and constant radial frequency magnified RF patterns with different radii, which have the constant cortical length of modulation cycles, we examined the effect of the RF pattern radius (between 4 and 16 deg) on radial frequency discrimination thresholds. As a consequence, we found that the discrimination thresholds between different frequency-magnified patterns were determined by the frequency ratio of the two patterns, regardless of different pattern radii and reference frequencies.

General methods

Participants

The participants included one of the authors and four healthy students (three males, 27.7 ± 3.8 years old). All participants had normal vision or corrected-to-normal vision and provided written informed consent for the procedure, which was previously approved by the ethics committee of Okayama University and was organized to ensure conformity to the Declaration of Helsinki.

Apparatus

MATLAB (MathWorks, Inc.) and Psychophysics Toolbox (Brainard, 1997), which provides high-level access to the C-language Video Toolbox (Pelli, 1997), were used to generate the custom stimuli used in this experiment. The program was run on a ThinkPad T540p notebook (Lenovo). Stimuli were presented on a SHARP-PN455 (SHARP) display with a 60-Hz frame rate, 8-bit colors and 1920×1080 pixels. The gamma nonlinearity of the screen was corrected using a look-up table that was obtained via calibration through a KONICA-MINOLTA CS-100A (Konica Minolta Japan, Inc.) spectroradiometer. The color of the stimuli was set to CIE 1931, $x = 0.34$, $y = 0.33$, and the mean luminance of the screen was 175 cd/m^2 . The stimulus luminance was modulated around a mean luminance of 175 cd/m^2 about each eccentricity, while the remainder of the screen was maintained at the mean luminance.

The participants viewed the display monocularly under dim room illumination at a distance of 80 cm. Each display subtended 63.3 deg horizontally and 38.2 deg vertically. The participants were instructed to sit on a chair, put their chin on a chin-stand to ensure that the head did not move and maintained the distance from the screen, and keep their fixation at the fixation point presented in

the center of the display during the presentation of the target stimuli.

Base stimuli

The base pattern used in this study was a circular contour with a cross-sectional luminance profile defined by a radial fourth derivative of a Gaussian (D4). The pattern used in this study was the same as that used by Wilkinson et al. (1998).

The equation for the circle is

$$D4(r) = C \times \exp\left(-\frac{(r-r_0)^2}{\sigma^2}\right) \times \left(1 - 4\frac{(r-r_0)^2}{\sigma^2} + \frac{4}{3}\frac{(r-r_0)^4}{\sigma^4}\right) \quad (2)$$

where C is the pattern contrast, which was 100% through all current experiments; r_0 is the mean radius of the stimulus, the center of which was located at the fixation point; and σ determines the thickness of the cross-sectional profile.

The base circle was deformed by applying a radial sinusoidal modulation to the radius r_0 in Eq. (2) such that the radius of the deformed pattern at polar angle θ in radians was as shown in Eq. (1).

The angular phase of the pattern is determined by φ . The radial modulation amplitude (A) was not permitted to exceed 1.0, and the radial frequency was always an integer value.

Peripheral sensitivity for radial frequency discrimination

Schmidtman, Logan, Keendy, Gordon, and Loffler (2015) tested the hypothesis of enhanced sensitivity to a range of stimuli within the lower visual field by measuring discrimination sensitivity for orientation, curvature, shape (radial frequency patterns), and faces at various para-central locations and eccentricities. Peripheral sensitivity was isotropic for orientation and curvature. By contrast, observers were significantly better at discriminating shapes throughout the lower visual field compared to elsewhere. For faces, peak sensitivity was found in the left visual field. Note here that the visual field preference was observed for small visual stimuli. To our knowledge, no study for large RF stimuli such as our experimental stimuli (8–32-diameter stimuli) has been examined. No one knows whether the low field preference is also observed or not for our large RF stimuli. This problem is interesting, but beyond the scope of the present study.

Our radial frequency discrimination which is closely related to orientation or curvature discrimination is isotropic in sensitivity in the peripheral visual field.

Procedure

The thresholds to discriminate the target radial frequency from the reference frequency were measured using a

three-alternative forced-choice procedure and the method of constant stimulus. In the frequency discrimination experiments, there were seven radial frequencies for the target stimuli with constant modulation amplitude: one frequency was the same as the reference frequency (for example 9 cycles/360 deg), three frequencies were higher (10, 11, and 13 cycles/360 deg for the reference frequency of 9 cycles/360 deg), and three frequencies were lower (6, 7, and 8 cycles/360 deg). They were arranged at nearly equal intervals on a logarithmic scale. When the target and reference have the same radial frequency, the participants may feel substantial difficulty in making a binary decision, e.g., “target has a higher radial frequency than the reference”, or “target has a lower radial frequency than the reference”. To circumvent this problem, we used the three-alternative forced-choice procedure rather than the two-alternative forced-choice procedure used by the majority of previous RF pattern research, in which the participants may select a third choice “target and reference have the same radial frequency”. Additionally, we used the three-alternative forced-choice procedure rather than the two forced-choice procedure because the latter is known to present with a potential difficulty: at low signal intensities (small differences in frequency between target and reference), the participant (particularly a naïve observer) is often truly uncertain about the correct answer from a two-alternative forced-choice task. Being forced to make a choice even when one does not have the slightest idea regarding which response is correct may introduce an uncomfortable aspect to this task. This may be particularly important in situations in which the participant is confronted for the first time with this task or where it is infeasible to dispel the reluctance of the participants to act randomly because it would require too much explanation and might reduce the confidence of the participants in the seriousness of the investigation (Kaernbach, 2001). The feasibility of using a “guess-button” in addition to the conventional response keys defined in 2FAC tasks has also been proved to yield a more precise result (García-Pérez, 2010).

Participants were presented with a 5-min dark adaptation to the dim room and a 3-min light adaptation to the mean luminance of the display. This procedure was performed to acclimate the participant’s eyes to quickly adapt to the mean luminance of the experimental display. Each trial subsequently consisted of two 200-ms stimulus presentation intervals with a 300-ms ISI and a response interval; one presentation interval contained the reference stimulus, while the other presentation interval contained one of the target stimuli. The presentation order of the reference and target were randomized within a session. The phase of the stimuli was varied randomly between trials. This random-phase setting rendered it impossible

for the subjects to predict the exact locations at which the maximum and minimum radial deviations would occur. The participants were instructed to indicate whether, compared to the first stimulus (S1), the frequency of the second stimulus (S2) was higher, lower or same using one of three buttons: one button for the S2 frequency being higher than the S1 frequency, another button for the S2 frequency being lower than the S1 frequency, and a third button for S2 and S1 having the same frequency (Fig. 1). With the participant's self-paced starting signal, the next session began. Within a session, the seven radial frequency targets were randomly presented, with each presented ten times, while the other stimulus parameters of the target and the reference were held constant. The four stimulus radii were randomly presented across sessions.

Analysis

We used a three-alternative forced-choice procedure. The responses of the participants were composed of “higher”, “lower” and “same” responses. Based on the “higher” and “lower” responses, we obtained the following logistic curve equation:

$$y = (1 + e^{\alpha + \beta x})^{-1} \quad (3)$$

where the responses of the participants are expressed from 0 to 100% on the ordinate. The abscissa corresponds to the target radial frequency from 6 cycles/360 deg to 13 cycles/360 deg, and 9 cycles/360 deg is the reference radial frequency (ω_R). From the logistic curve, we obtained

two key points on the abscissa: ω_{TH} , the frequency where the participants achieved 75% on the ordinate, and ω_{TL} , the frequency where the participants achieved 25% on the ordinate, as shown in Fig. 2.

We calculated the radial frequency discrimination thresholds by using the following equation:

$$\begin{aligned} \text{Discrimination thresholds} &= \frac{\ln(\omega_{TH}) - \ln(\omega_{TL})}{2} \\ &= \ln \left(\left(\frac{\omega_{TH}}{\omega_{TL}} \right)^{\frac{1}{2}} \right) \end{aligned} \quad (4)$$

Dickinson, McGinty, Webster, and Badcock (2012) showed that the quantity that was equated at thresholds for detection of modulation on RF patterns of different frequencies but with the same number of cycles of modulation was the maximum angular deviation from circularity. This was for patterns with relatively low frequencies of modulation that have been shown to exhibit integration of information across cycles, but not for patterns with higher frequency. It was consistent with the results of other study (Loffler et al., 2003). It is suggested that an important cue for the discrimination between RF different stimuli, regardless of information integration, may be the maximum angular deviation from circularity.

The maximum angular deviation (orientation difference) between an RF and a circle occurs where an RF and a circle intersect each other and given by the following equation (Dickinson et al., 2012; Schmidtman and Kingdom, 2017).

Fig. 1 Time course of the radial frequency discrimination experiment. Each trial consisted of two 200-ms stimulus presentation intervals with a 300-ms ISI and a response interval; one presentation interval contained a reference stimulus, while the other presentation interval contained a target stimulus. The participants were instructed to indicate whether, compared

to the first stimulus (S1), the frequency (ω) of the second stimulus (S2) was higher, lower or the same using one of three buttons: one button for the S2 frequency being higher than the S1 frequency, another button for the S2 frequency being lower than the S1 frequency, and a third button for the S2 frequency being the same as the S1 frequency

Fig. 2 Derivation of radial frequency discrimination thresholds. We used a three-alternative forced-choice procedure. The responses of the participants were composed of the “higher”, “lower” and “same” responses. Based on the proportion of correct “higher” and “lower” responses versus the target radial frequency, we obtained a logistic curve equation, where the relative proportions of the “lower” and the “higher” responses are expressed from 0 to 100% on the ordinate. The abscissa indicates the target radial frequency from 6 to 13 cycles/360 deg, and 9 cycles/360 deg is the reference radial frequency (ω_R). From the logistic curve, we obtained two key points on the abscissa, ω_{TH} , the frequency where the participants achieved 75% on the ordinate, and ω_{TL} , the frequency where the participants achieved 25% on the ordinate. We calculated the radial frequency discrimination thresholds by using the following equation:

$$\text{Discrimination threshold} = \frac{\ln(\omega_{TH}) - \ln(\omega_{TL})}{2} = \ln\left(\left(\frac{\omega_{TH}}{\omega_{TL}}\right)^{\frac{1}{2}}\right)$$

$$\text{Maximum orientation differences } (\tan(\alpha)) = A\omega \quad (5)$$

Where α represents the angle between the tangential directions of an RF pattern and the circle at the crossing point. A and ω represent the modulation amplitude and the radial frequency of the RF pattern, respectively. The maximum orientation difference is abbreviated as MOD.

Thus, the ratio of MODs between two RF patterns of equal radius and modulation with ω_1 and ω_2 is equal to the radial frequency ratio of the two patterns, implying that the ratio of MODs may be a critical cue to discriminate two different RF patterns as well. The discrimination thresholds in the Eq. (4) is expressed also by the ratio of MODs of the two patterns.

Experiment 1

Experimental stimuli: Constant circular contour frequency (CCF) RF pattern stimuli

The CCF is defined by the number of radial cycles per deg of unmodulated contour length measured in deg of viewing angle. This is an important factor in determining the deformation thresholds of RF patterns because regardless of the RF or

radius, the radial deformation thresholds are closely matched across the entire range of circular contour frequencies, suggesting that the patterns with the same circular contour frequency should have the same radial deformation threshold (Jeffrey et al., 2002).

To examine the dependence of the radial frequency discrimination thresholds on the radius (eccentricity), we used the RF reference patterns with constant CCF ($r_0/\omega = \text{constant}$) and constant absolute modulation amplitude ($A r_0 = \text{constant}$) for changes in radius (4 ~ 16 deg). The constant CCF RF patterns were created by covarying the radius and radial frequency such that they have an identical CCF regardless of the change in the radius. Figure 3a shows a schematic of an example of the constant CCF RF pair. The two patterns with different radii and RFs have the same CCF. Moreover, the σ of the two patterns remains constant. These two patterns' cycles of modulation travel the same length in physical space. We used three series of constant CCF (CCF = 0.358 cycles/cl-degree) patterns with different radii (4, 8, 12, and 16 deg) but the same thickness ($\sigma = 1$ deg) as the reference stimuli. Figure 3b shows the experimental reference stimuli used in this experiment.

The absolute modulations ($r_0 A$) of the reference RF patterns were held constant (0.12, 0.24, 0.48 deg) within a session but were different across sessions. The product of $r_0 A$ and the contour length reflects the size of a single cycle of the RF patterns. This indicates that the size of the stimuli was held constant within a session.

Results

In Experiment 1, the discrimination thresholds were measured as a function of the radius over a range from 4 to 16 deg (14 deg instead of 16 deg when $r_0 A = 0.12$ deg, because of the impossibility of measurement otherwise) for the five participants. In Fig. 4, the mean of the discrimination thresholds for the five participants is plotted against the radius. The thresholds vs. radius functions were strongly dependent on the stimulus conditions ($r_0 A$) of the target and reference stimuli. The thresholds substantially increased with increasing radius for the condition $r_0 A = 0.12$ deg, i.e., the small stimulus size and slightly increased for the condition $r_0 A = 0.24$ deg, while they remained constant for the condition $r_0 A = 0.48$ deg, i.e., the large stimulus size. In this instance, the increase in the threshold implied a decrease in the discrimination performance, which indicates a radius effect of the RF patterns. The radius effect was observed for the condition $r_0 A = 0.12$ deg, but not for the condition $r_0 A = 0.48$ deg. To test for significant differences in the group data, we used three one-way ANOVA. Based on the results, a main effect for the condition $r_0 A = 0.12$ deg, $F(3, 12) = 9.627$, $p < 0.05$;

(a). Schematic Examples of Constant CCF Patterns with Different Radii (CCF=0.159 cycles/cl-deg)

(b). Constant CCF Patterns with Different Radii Used in Experiment 1 (CCF=0.358 cycles/cl-deg; $\sigma=1 \text{ deg}$)

(1). $r_0A=0.12 \text{ deg}; A\omega_R=0.27$

(2). $r_0A=0.24 \text{ deg}; A\omega_R=0.54$

(3). $r_0A=0.48 \text{ deg}; A\omega_R=1.08$

and the condition $r_0A = 0.24 \text{ deg}$, $F(3, 12) = 23.606$, $p < 0.05$ has been found. However, no main effect for the

condition $r_0A = 0.48 \text{ deg}$, $F(3, 12) = 0.197$, $p = 0.77$ has been found.

Fig. 3 Constant circular contour frequency (CCF) patterns. To examine the dependence of radial frequency discrimination thresholds on the radius (eccentricity), we use a series of CCF constant patterns with different radii (4, 8, 12, and 16 deg). The CCF is considered to be an important factor in the visual performance of RF patterns regardless of radial frequency or radius (see text). (a) shows schematic examples explaining how two patterns with different radii (r_0) and radial frequencies (ω) have the same circular contour frequency (CCF = 0.159 cycles/cl-deg). CCF is defined by the number of radial cycles per deg of unmodulated contour length measured in deg of viewing angle and calculated by $\omega/2\pi r_0$. (b) shows three series of reference patterns with a constant CCF of 0.358 cycles/cl-deg and a constant MOD ($A\omega_R$) used to examine the dependence of the pattern radius in experiment 1. Here, MOD is the Maximum orientation difference between the RF pattern and the unmodulated circle. The reference radial frequency (ω_R) increases with increasing radius to keep the CCF constant (0.358 cycles/cl-deg); meanwhile, the σ of the patterns remains constant ($\sigma = 1$ deg). The absolute modulations (r_0A) of the target and the reference RF patterns hold constant (0.12, 0.24, 0.48 deg as shown in (1), (2) and (3)) within a session but are different across sessions. Also, MODs of the reference stimuli holds constant ($A\omega_R = 0.27, 0.54, 1.08$) within a session. We use seven radial frequencies for the target stimuli (ω_T): the same frequency as the reference, and three higher and three lower than that of the reference. We use 14 deg instead of 16 deg when $r_0A = 0.12$ deg because of the impossibility of measurement otherwise. Meanwhile, we use 31 cycles/360 deg as the reference stimulus to keep the CCF constant. Note here that four constant CCF reference patterns with different radii within a session have the same orientation difference between RFs and a circle at crossing point

Discussion

As shown in Fig. 4, the discrimination thresholds between different-frequency RF patterns vs. radius functions for the constant CCF RF patterns indicated different functional forms dependent on the modulation amplitude of the RF patterns. The thresholds showed an increasing functional form for small modulation amplitude RF patterns ($r_0A = 0.12$ and 0.24 deg) but a flattened function for large amplitude RF patterns ($r_0A = 0.48$ deg).

The effect of the pattern contrast (100 %)

We used the pattern contrast of 100 % through all present experiments. The relationship between the physical variables and radial frequency discriminations does not always involve a simple one-to-one mapping between the two variables. The other stimulus characteristics which are likely to modify the relationship include the contrast and spatial frequency of the base stimulus. We used the stimulus contrast of 100 % and the peak spatial frequency of 0.45 c/deg in the Experiment 1. The perceived contrast of the Gabor patches decreases with increasing eccentricity (0 ~ 13 deg) even partially compensating by scaling stimulus size in the periphery (Vanstons and Crognale, 2018). The shape discrimination threshold of RF pattern of 4 cycles/360 deg from a circle decreases with increasing stimulus contrast from about 20 ~ 100 % for the condition of spatial frequency = 1 cpd and radius = 2.5 deg,

Fig. 4 Results of radial frequency discrimination for Experiment 1. Mean discrimination thresholds for the five participants plotted against the radius (4, 8, 12, and 16 deg). Thresholds (frequency ratio) are expressed as the square root of the higher frequency/lower frequency on the ordinate. The absolute amplitude (r_0A) was held constant in each session but was varied across sessions; discrimination thresholds are shown for the small stimulus size: 0.12 deg (1), the middle stimulus size: 0.24 deg (2), and the large stimulus size: 0.48 deg (3). When $r_0A = 0.12$ deg, the threshold for 16 deg is estimated from the thresholds for 12 and 14 deg that were measured in the experiment. Error bars = 1 S.E.

but not for the condition of spatial frequency 5 ~ 10 cpd and radius = 2.5 or 1 deg (Ivanov and Mullen, 2012). The shape discrimination threshold of RF pattern decreases with increasing stimulus contrast from 12.5 ~ 100% for the condition of radial frequency of 5 ~ 12 cycles, but not for the condition of radial frequency of 2 ~ 4 cycles/360 deg (Wilkinson et al., 1998). This implies that the reduction in the perceived contrast with increasing eccentricity may result in increase of discrimination thresholds observed at eccentricity of 16 deg for the condition of $r_0A = 0.12$ and 0.24 deg in Fig. 4. However, only the reduction in the perceived contrast with increasing

eccentricity cannot explain our result indicating the constant discrimination thresholds for the condition of $r_0A = 0.48$ deg regardless of eccentricity change.

Pattern radius effects of radial frequency discrimination thresholds for constant CCF RF patterns

The path of the RF patterns is less salient as the radius of the RF pattern increases. It might be the case that for the patterns with $r_0A = 0.12$ (and to a lesser extent 0.24) the larger receptive fields (lower spatial frequency preference) span cycles of modulation and smooth the percept of the path. This might result in an increase in the threshold.

It is well established that the performance of visual discrimination, such as the frequency, orientation, temporal frequency, vernier acuity, blur, displacement, and separation, is dependent on the signal strength, such as the contrast and size of a stimulus, with performance improving as the signal increases (Bowne, 1990; Reisbeck and Gegenfurtner, 1998; Skottun et al., 1987; Webster et al., 1990; Westheimer et al., 1999; Marechal and Shapley, 2004).

If radial frequency discrimination is processed by the orientation selective mechanism, our results may show common behavior with previous studies concerning orientation discrimination performance.

Specifically, in experiments where the stimuli (grating, Gabor element) were small, increasing the contrast lowered the orientation threshold (Regan and Beverley, 1985; Skottun et al. 1987; Reisbeck and Gegenfurtner, 1998; McIlhagga and Mullen, 1996), whereas in the experiments when the stimuli were large, orientation discrimination thresholds were fairly contrast independent (Bowne, 1990; Skottun et al., 1987).

In the experiments performed by manipulating stimulus size, threshold estimates were lowest (acuity highest) for the largest size patch and increased as the stimulus size was reduced (Henrie and Shapley, 2001); the differential thresholds or JNDs in the orientation of a line stimulus at a given reference orientation (horizontal and oblique) decreased up to a stimulus length of 1 deg; over 1 deg, discrimination was not improved for the oblique orientation while for the horizontal orientation it improved gradually up to a length of 8 deg (Orban et al., 1984). Mareschal and Shapley (2004), by varying systematically both contrast and size of a circular patch of grating, found that orientation discrimination thresholds decreased as contrast was increased for stimuli roughly smaller than $0.5 \sim 0.8$ deg in diameter, and as size was increased for stimuli with less than $4 \sim 20\%$ grating contrast. The thresholds were saturated at constant level for larger stimuli than 1 deg in diameter and with greater than 20 % contrast.

The general finding was that as the stimulus signals (contrast and/or size) increased, the orientation discrimination

thresholds for grating patches decreased rapidly at first, then gradually and finally asymptotically approached a saturated level.

We asked whether the above context concerning the signal strength effect on orientation discrimination can explain the results from Experiment 1.

We hypothesized that under strong signal strength (large modulation amplitude) conditions, frequency discrimination thresholds might not be as affected by the modulation amplitude and would remain constant at the saturation level whereas they would be increased under low signal strength (small modulation amplitude) condition.

To test this hypothesis, we estimated the perceptual strength of the stimuli in Experiment 1, which is considered to correspond to the signal strength of the stimuli such as the contrast of the grating stimuli in orientation discrimination, and examined the dependence of radial frequency discrimination on perceptual strength.

These estimations were carried out by dividing the stimulus modulation parameter r_0A of the RF pattern by the amplitude thresholds to detect the deformation of the base circles which were reported in our other study (Feng et al., 2020).

Table 1 lists the amplitude detection thresholds reported and the estimations represented as the multiples of the deformation threshold. The perceptual strength increases as the eccentricity decreases, from 1.8 to 7.3 for $r_0A = 0.12$, from 3.5 to 14.5 for $r_0A = 0.24$ and from 7.1 to 29.1 for $r_0A = 0.48$. If the asymptote at which the signal strength effect saturates is approximately seven times the detection threshold, our hypothesis might suggest that above this level, the radial frequency discrimination thresholds may hold constant, whereas they may decrease with increasing signal strength up to the abovementioned asymptote.

The exception to this suggestion is the lack of a decreasing in threshold for the RF patterns when the radius decreases from 8 deg to 4 deg and the signal strength increases from 3.6 to 7.3 in the case of $r_0A = 0.12$ deg. However, the hypothetical suggestion is true in all the other conditions. It is not unreasonable that the radius size effect of the radial frequency discrimination seen in experiment 1 can be well explained by the context concerning the signal strength effect on orientation discrimination.

As a result, our finding that the radial frequency discrimination thresholds as a function of radius were dependent on the modulation amplitudes of the RF patterns is ascribed to the eccentricity effect reported in our other study (Feng et al., 2020), wherein the deformation thresholds required to discriminate the RF pattern from a circle increased with increasing stimulus eccentricity. Our results showed similar properties to previous studies concerning orientation discrimination performances, suggesting that the local contour orientation may be a critical cue for discriminating different frequency RF patterns.

Table 1 The deformation thresholds in our other study and the stimulus modulation amplitudes (A) in the present study

Radius (deg)	Results of our other study (Feng, et.al. 2020)	Stimulus parameters for the radial frequency discrimination experiment		
	Amplitude	$r_0A = 0.12$ deg	$r_0A = 0.24$ deg	$r_0A = 0.48$ deg
r_0	Threshold amplitude	Perceptual amplitude (A /threshold)		
4	0.0041	7.3	14.5	29.1
8	0.0042	3.6	7.2	14.4
12	0.0041	2.5	4.9	9.8
16	0.0042	1.8	3.5	7.1

We mention here that the effect of number of cycles of modulation of RF patterns should be examined to investigate locality or globality (global integration) of information processing of RF pattern in the peripheral field.

Dickinson, McGinty, Webster, and Badcock (2012) showed that the detection thresholds from circle described by gradient at a zero crossing decrease with increasing the number of cycles of modulation, being coincident at each value for number of cycles of modulation regardless of RF. In a future work, we will examine the effect of number of number of cycles of modulation on frequency discrimination threshold between different RF patterns.

Experiment 2

Magnified RF pattern

In Experiment 1, using a constant CCF RF pattern, the radial frequency discrimination thresholds as a function of radius were influenced by the eccentricity effect, wherein the deformation thresholds required to discriminate the RF pattern from a circle increased with increasing stimulus eccentricity. Experiment 2 was designed to test whether spatial scaling (the retinocortical transformation) can neutralize the eccentricity effect observed in Experiment 1.

There exist many magnification factors reported (Anstis, 1974; Farrell and Desmarais, 1990; Higgins et al., 1996; Levi et al., 1985; Rovamo and Virsu, 1979; Melmoth and Rovamo, 2003; Schwartz, 1977; Watson, 1987; Whitaker et al., 1992). In the present experiment, we used Schwartz's retinocortical mapping function.

We constructed a series of magnified stimuli with different radii that had the same form in the V1 field and examined the dependence of the radial frequency discrimination thresholds for the magnified patterns on stimulus radius.

The magnified RF patterns with the CCF being inversely proportional to the radius of the stimuli, were created using Schwartz's retinocortical mapping function. Schwartz showed that the two-dimensional retinocortical mapping functions are represented in complex variables as in Eq. (5) (Schwartz, 1977):

$$W(Z) = k \times \log(Z + a) \quad (6)$$

where the complex variable Z represents a point in the retinal plane; the complex variable W represents a point on the cortical plane; a defines the extent of the foveal singularity; and k is a scaling constant. For the value of a , there is rough consensus that the primate visual cortex is, to a first approximation, a complex logarithmic structure characterized by a foveal constant ranging from $a = 0.3$ deg to $a = 0.9$ deg (Wilson et al., 1990). In this study, we used $a = 0$ for the retina cortex function because our experimental condition was focused on $z \geq 4$ deg.

If one changes r_0 and σ in proportion to the changing eccentricity while maintaining A and constant, then the changed patterns have a similar form; however, their radii are different and are dependent on eccentricity in a linear manner. If these changed patterns, as shown in Fig. 5a (the magnified RF patterns), are transformed from the retinal plane into the cortical plane using Eq. (6), the transformed patterns are different in position in the cortical plane; however, they have the same forms and radii, which implies that all transformed patterns activate the cortical area equally.

In this experiment, we used the magnified RF patterns with an identical reference RF. The reference RFs (9 and 18 cycles/360 deg) were held constant within a session but were different across sessions. The amplitude modulations (A) of the target and reference RF patterns were held constant within a session but were different across sessions (0.03 and 0.06), as shown in Fig. 5 (b).

Results

For the magnified patterns, the thresholds were measured as a function of the pattern radius over a range from 4 to 16 deg. In Fig. 6, the means of the thresholds for the five participants are plotted against the radius under the four conditions ($\omega_R = 9$ cycles/360 deg and $A = 0.03$; $\omega_R = 9$ cycles/360 deg and $A = 0.06$; $\omega_R = 18$ cycles/360 deg and $A = 0.03$; and $\omega_R = 18$ cycles/360 deg and $A = 0.06$). As shown in Fig. 6, the thresholds remained constant across all radii, approximately 1.1, and regardless of the other stimulus parameters. To test for significant differences in the group data, we used a three-way

(a). Schematic Examples of Magnified Patterns with Different Radii

(b). Magnified Patterns with Different Radii Used in Experiment 2

(1). $\omega_R = 9 \text{ cycles/360 deg}$; $A = 0.03$; ; $A\omega_R = 0.27$ (2). $\omega_R = 9 \text{ cycles/360 deg}$; $A = 0.06$; ; $A\omega_R = 0.54$ (3). $\omega_R = 18 \text{ cycles/360 deg}$; $A = 0.03$; ; $A\omega_R = 0.54$ (4). $\omega_R = 18 \text{ cycles/360 deg}$; $A = 0.06$; ; $A\omega_R = 1.08$

Fig. 5 Magnified patterns in Experiment 2. (a) shows schematic examples created by using Schwartz's retinocortical mapping function. These patterns are different in position in the cortical plane but have the same forms and radii, implying that all the transformed patterns activate the cortical area equally. (b) shows the four series of reference magnified patterns used in Experiment 2. σ increases with increasing radius, while the radial frequency (ω_R) and amplitude are constant ($\omega_R = 9$ cycles/360 deg, $A = 0.03$, $\omega_R = 9$ cycles/360 deg, $A = 0.06$, $\omega_R = 18$ cycles/360 deg, $A = 0.03$ and $\omega_R = 18$ cycles/360 deg, $A = 0.06$ as shown in (1), (2), (3) and (4)) within a session but are different across sessions. Note here that four magnified reference patterns with different radii within a session have the same orientation difference between RFs and a circle at crossing. Also, MODs of the reference stimuli holds constant ($A\omega_R = 0.27, 0.54, 0.54, 1.08$) within a session but are different across sessions

ANOVA. ANOVA with the factors of radius, A and ω_R , was used to test for main effects. Based on the results, no main effect for the three factors: radius, $F(3, 12) = 4.771, p = 0.086$; A , $F(1, 4) = 0.25, p = 0.643$; and ω_R , $F(1, 4) = 0.019, p = 0.896$ has been found.

The radial frequency discriminability of the target stimuli from the reference stimulus is represented by the difference between the response frequencies corresponding to the “higher” responses and the “lower” responses. Figure 7 shows the cumulative discriminability of the target from the reference stimuli versus the target radial frequency over all participants and the four radii for each magnified pattern ($\omega_R = 9$ cycles/360 deg and $A = 0.03$; $\omega_R = 9$ cycles/360 deg and $A = 0.06$; $\omega_R = 18$ cycles/360 deg and $A = 0.03$; and $\omega_R = 18$ cycles/360 deg and $A = 0.06$). A positive value on the ordinates indicates the magnitude of discrimination performance for a target that the participants judge to have a higher frequency than the reference frequency (9 cycles/360 deg),

whereas a negative value indicates the magnitude of discrimination performance for a target that the participants judge to have lower radial frequency than the reference frequency. Discriminability increases as the target radial frequency increases (or decreases) from the reference frequency of 9 cycles/360 deg. The discriminability function shows a linear function of target frequency on the logarithmic scale within the frequency domain being presented.

Discussion

Comparison of our discrimination thresholds with the identification accuracies in previous studies

Wilkinson et al. (1998) found that participants were able to identify RF patterns of six or less cycles with an accuracy of over 90% and seven or more cycles with an accuracy of 60% or less. These identification accuracies were also observed by Poirier and Wilson (2006). The present study shows that the radial frequency discrimination thresholds (frequency ratio) are approximately 1.1 for RF patterns with a large stimulus size ($r_0A = 0.48$) and for the magnified stimuli, as shown in Fig. 4 (3) and Fig. 6, respectively.

Although these results cannot be compared with the identification accuracies in studies by Wilkinson et al. (1998) and Poirier and Wilson (2006) because of the different stimulus conditions, the logarithms of the ratios of 5 cycles to 6 cycles ($6/5 = 1.2$) and 6 cycles to 7 cycles ($7/6 = 1.16$) are larger than our radial frequency discrimination thresholds. This finding suggests that 5-cycle (or 6-cycle) RF patterns are easily

Fig. 6 Results of radial frequency discrimination Experiment 2. Mean discrimination thresholds for the five participants plotted against the radius (4, 8, 12, and 16 deg). Thresholds (frequency ratio) are expressed on the ordinate. The amplitude (A) and the radial frequency of the

reference stimulus (ω_R) were held constant within a session but varied across sessions: $A = 0.03$; $\omega_R = 9$ cycles/360 deg (1), $A = 0.03$; $\omega_R = 18$ cycles/360 deg (2), $A = 0.06$; $\omega_R = 9$ cycles/360 deg (3), and $A = 0.03$; $\omega_R = 9$ cycles/360 deg (4). Error bars = 1 S.E.

Fig. 7 Discriminability of the target from reference stimuli versus target radial frequency. The responses of the participants consist of their “higher”, “lower”, or “same” responses. The difference between the response frequencies of the “higher” responses and the “lower” responses is considered to represent the radial frequency discriminability of the target stimuli from the reference stimuli. Each panel shows the cumulative difference between the response frequencies of the “higher” responses and the “lower” responses over all participants and the four radii for each magnified pattern ($\omega_R = 9$

cycles/360 deg, $A = 0.03$; $\omega_R = 9$ cycles/360 deg, $A = 0.06$; $\omega_R = 18$ cycles/360 deg, $A = 0.03$; or $\omega_R = 18$ cycles/360 deg, $A = 0.06$). Error bars = 1 S.E. The observed small error bars imply little variance across the participants and radii. Discriminability increases as the target radial frequency increases (or decreases) from the reference frequency of 9 cycles/360 deg. The discriminability function shows a linear function of target frequency on the logarithmic scale within the frequency domain being presented

discriminated from 6-cycle (or 7-cycle) RF patterns. The results obtained for the RF patterns in this study are not inconsistent with the identification accuracies from Wilkinson et al. (1998) and Poirier and Wilson (2006).

Frequency ratio determines the radial frequency discrimination thresholds

Previous findings of radial frequency-selective effects, such as masking effects (Habak et al., 2006; Bell et al., 2007), adaptation effects (Bell et al., 2008; Bell et al., 2009), and sub-threshold summation effects (Bell and Badcock, 2009), suggest that detection of the RF pattern may be processed by several RF channels, each channel selectively sensitive to a limited band of radial frequencies. These channels may also be involved in the suprathreshold perception of radial frequency and of the waveforms of RF patterns. If this is so, the ability to discriminate between different radial frequency patterns as measured in the present study might be determined by the activity difference of the multiple channels for the different patterns.

Figure 6 indicates that the radial frequency discrimination thresholds for the magnified RF stimuli were the same for two different reference frequencies (9 and 18 cycles/360 deg),

implying that the ability to discriminate between the two different stimuli depends primarily on the ratio of their radial frequencies over a wide range of absolute radial frequencies. That we found equally discriminable radial frequencies to differ by nearly the same ratio may suggest that the spacing and bandwidth of the processing units of RF patterns may be more uniform if expressed on a logarithmic rather than a linear scale.

It should be mentioned here that the spatial frequency bandwidths of cortical cells turned to different spatial ranges have bandwidths that are roughly constant on a ratio, or logarithmic, scale (De Valois and De Valois 1990). Thus, equal logarithmic channel bandwidths have been reported for spatial frequency selectivity.

Our experimental results that the radial frequency discrimination thresholds were the same for the two different reference frequencies (9 and 18 cycles/360 deg) may be explained by a multiple channels model with equal logarithmic channel bandwidths of RF.

Conclusions

We found that the discrimination thresholds expressed by the frequency ratio between RF patterns of different frequencies

vs radius functions for the constant CCF RF patterns indicated different functional forms dependent on the modulation amplitude of the RF patterns. This dependence was ascribed to the eccentricity effect wherein the deformation thresholds for discriminating the RF pattern from a circle increase with increasing stimulus eccentricity (Feng, et al., 2020).

We also found that the discrimination thresholds vs. radius functions for the magnified RF patterns were flattened for different modulation amplitudes and frequencies. The thresholds were similar at all eccentricities. Cortical magnification neutralized the eccentricity effect observed for the constant CCF patterns.

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