Titchener's $\perp$ in context 1—delimited, discrete monomotif patterns, line arrangements, and branching patterns

Klaus Landwehr¹

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Abstract Three experiments tested the effects of the presence of nontarget $\perp$ s on Titchener's (1901) $\perp$ -illusion. Experiment 1 used patterns of four separate $\perp$ s, Experiment 2 used branching patterns in which four $\perp$ s were stuck together, and Experiment 3 used patterns of four triangles or four beehive forms for which the $\perp$ could be seen as a skeleton. Three independent samples of 12 observers each had to haptically indicate the lengths of target lines and verbally judge the relative lengths of the two lines of target $\perp$ s. The illusion to judge or indicate the $\perp$'s undivided line as longer than its divided line survived throughout except for the branching patterns: here, haptic indications did not differ between the two types of lines. Specific features of these patterns and of the $\perp$ itself may be responsible for these effects.

Keywords Visual illusion · Haptic signaling · Monomotif patterns · Shape skeletons · Symmetry

Titchener's (1901) $\perp$ is arguably one of the most effective visual illusion figures: if both of its constituent lines are equally long, the undivided one usually appears to be longer (cf. Hamburger & Hansen, 2010; Taylor, 2001, for qualifications). According to Pastore (1971), the earliest observation of this phenomenon dates back at least to Malebranche (1638–1715) who had observed it with an upright, written T (also cf. Johannsen, 1971). Although often referred to as an instance of the (would-be) horizontal-vertical illusion, first described by Fick (1851), who had noted an overestimation of the ver-

tical extents of rectangles (cf. Robinson, 1972, pp. 96–98), the $\perp$ -effect is due mainly to the bisection of one line by another—as demonstrated convincingly by Künnapas (1955) by simply rotating the $\perp$ around $\pi/2$. Another way to show the effectiveness of the division is to gradually transform the $\perp$ into an L (ibid.; cf. Sanford, 1898). The division effect, in turn, is not the familiar Oppel-Kundt illusion in which an extent, marked (or “filled”) with small dividers, is typically seen as *longer* than an unfilled one (Kundt, 1863; Oppel, 1860–1861; although number, size, and distribution of the dividers exert moderating effects; Robinson, 1972, pp. 49–51). In the $\perp$ -figure or similar arrangements of dots and lines, it seems to be the length of the dividing line that determines the apparent width of a spatial interval. While Obonai (1954) had shown that as a line that symmetrically bisected the distance between pairs of dots lengthened, that distance was at first overestimated and then consistently underestimated, I demonstrated a similar, if opposite, “induction effect” in the haptic modality with Titchener's $\perp$: haptic indications of the $\perp$'s divided line were scaled according to the length of the undivided line [Landwehr, 2009, although the effect did not replicate in a second study (Landwehr, 2014); however, the basic $\perp$ -illusion occurred reliably with both verbal judgments and haptic length indications].¹ Yet, in vision, a minor effect of rotation sometimes remains (e.g., Charras & Lupiáñez, 2010; Mamassian & de Montalembert, 2010), which is bound to the presence of two lines in one's field of view, though: for

✉ Klaus Landwehr
landweh@uni-mainz.de

¹ Psychologisches Institut, Universität Mainz, 55099 Mainz, Germany

¹ The notion of haptics is confronted here with touch: while the former mode of perception is based on the output of mechanoreceptors at the joints and muscles, the latter is based on the activity of receptive structures in the skin (Gardner & Johnson, 2013; Lederman & Klatzky, 2009). The $\perp$ may also elicit a haptic-tactile illusion when a model or a raised-lines drawing is explored blindfoldedly by sliding one's finger tips across the figure (Day & Avery, 1970; Heller, Bracket, Salik, Scroggs, & Green, 2003), but that variant of the illusion is not dealt with here.

individual lines seen in complete darkness, there is no horizontal–vertical illusion (Verrillo & Irvin, 1979).

Still, attempts to explain the $\perp$ -illusion have often focused on a putative, perceptual horizontal–vertical anisotropy, and thus, as far as the $\perp$ -figure is concerned, may have missed their target. Early optical and ophthalmological accounts of the illusion (Fick, 1852; Valentine, 1912) could only be upheld with reservations (Pearce & Matin, 1969), and cognitive explanations, although still popular (e.g., Williams & Enns, 1996), did not fare much better. Künnapas’ (1957) “framing theory”, according to which the oval shape of our binocular field of view causes vertical extensions to be overestimated, was not supported by Houck, Mefferd, and Greenstein (1972; but cf. Prinzmetal & Gettleman, 1993) and, with regard to possible effects of phenomenal perspective (von Collani, 1985), Ward, Porac, Coren, and Girgus (1977) could not find substantial evidence for a three-dimensional (3D) interpretation of the $\perp$ -figure (cf. Wolfe, Maloney, & Tam, 2005, for a formal experimentum crucis). Although there have been several proposals for a general, neurophysiological account of visual illusions (e.g., Fermüller & Malm, 2004: neural noise; Ganz, 1964: lateral inhibition), none has picked Titchener’s $\perp$ as its prime example, or offered a detailed explanation of the $\perp$ -illusion.

In previous work—apart from the framing studies cited in the preceding paragraph—the $\perp$ has always been presented in isolation. For individual lines, it is known that the presence of other, especially differently oriented, lines affects length estimates of target lines: vertical lines were more likely to be judged longer than horizontal ones in the presence of long horizontals and short verticals, and shorter in inverse configurations (Armstrong & Marks, 1997). In order to distinguish this kind of context from the one of a frame, I shall refer to it as a *self-similar context* because lines of different lengths and orientations can be obtained from given lines by means of similarity transformations (Coxeter, 1969; Grünbaum & Shephard, 1987, pp. 216–217). In my own work, I have always been at pains to downgrade the effects of framing by presenting the $\perp$ -stimulus within a circular window on an otherwise dark computer screen in a dimly lit room (Landwehr, 2009, 2014, 2015). Here, I report three experiments in which target $\perp$ s were surrounded by other, nontarget $\perp$ s. The question was whether the presence of such nontarget $\perp$ s, in analogy to the case of length estimates of individual lines, would affect the amount of illusion concerning the target $\perp$. The results of my earlier experiments with individual $\perp$ s will be used as benchmarks for the findings of the present series of experiments with $\perp$ s put in contexts (see Figs. 3 and 4).

While Armstrong and Marks (1997, p. 1201) interpreted their findings of context effects on length estimates of individual lines in terms of “a high-level process of stimulus-specific perceptual attenuation”, following Caelli (1977), I considered more basic mechanisms—namely, local interactions between

orientation-sensitive neurons—to be at work in Titchener’s $\perp$ (Landwehr, 2009, 2014). However, recent experiments with modified $\perp$ s (Landwehr, 2015) clearly falsified this conjecture: the common illusion was still present in $\perp$ s composed from dashed lines or dots, and even when the $\perp$ had been torn apart (“dissected”) into two separate lines. I concluded that neural interactions, if relevant at all, would have to be of a more global nature. Alternatively, a higher-order, figural mechanism (a “T-schema”) might be invoked (cf. Neisser, 1976). The present series of experiments focused initially on global interactions. As Experiments 1 and 2 yielded only scant evidence for such interactions, Experiment 3 combined my two research strategies—of using modified $\perp$ s and of putting them in context—by using patterns of plane figures in which the T-junction was not optically specified (see Fig. 6). Evidence will favor the idea of a T-schema, necessitating its biological and physiological specification.

Experiment 1

Experiment 1 used the stimuli illustrated in Fig. 1. In the following, the pattern shown in Fig. 1a will be referred to as the type-1 4- $\perp$ pattern, and the one shown in Fig. 1b as the type-2 4- $\perp$ pattern. Variants, also used in the experiment, included mirror images and rotations of the patterns. Since both pattern types are of the cyclic symmetry group c_4 , rotations around $45^\circ \pm 90^\circ$ give the unique orientation at which all lines in these patterns are inclined at this angle. The $\perp$ s were scaled in size, and patterns were scaled with regard to density (i.e., the $\perp$ s were composed of lines of different lengths, and they were shifted closer together or farther apart; see *Methods* for details). Observers, after a short visual inspection of the stimuli, had to indicate haptically the lengths of selected, individual lines, and afterwards, judge verbally the lengths of selected lines relative to the second, attached line of the $\perp$ to which the target line belonged.

Although I could not base my research on a full-fledged theory, there were several reasons why I started out with delimited 4- $\perp$ patterns. Four is the generally accepted “magic number” for immediate apprehension (Cowan, 2000).² That is, four discrete items can usually be recognized and remembered from a short glance (cf. Fukuda, Awh, & Vogel, 2010, for recent, more advanced theoretical perspectives). With regard to the $\perp$, four rotations suffice to show the figure at four cardinal orientations with its undivided line pointing either upwards, downwards, or to the left or to the right. The $\perp$ -illusion has often been found to be attenuated in the last two

² The reason why Ebbinghaus (1885) estimated the immediate memory span to amount to 7 ± 2 units was—as he himself noted—that he had rehearsed verbal materials (consonant-vowel-consonant trigrams) aloud, thereby generating rhythmic patterns (cf. Miller, 1956, on “chunking”).

Fig. 1 The default versions of the two types of 4- $\perp$ patterns used in Experiment 1 with all lines having the same length

cases—as well as at oblique orientations of the $\perp$ (Hamburger & Hansen, 2010; Künnapas, 1955; Landwehr, 2009, 2014; Mamassian & de Montalembert, 2010). In the default variants of the 4- $\perp$ patterns shown in Fig. 1, there are two $\perp$ s (the upright and the inverted ones) for which, if they were presented in isolation, the standard illusion would be expected, and two $\perp$ s (the laterally oriented ones), for which a reduced illusion should occur. Now, context effects can be of three different kinds at least: (1) balance: each $\perp$ interacts with every other one, so that a compromise amount of illusion results for all; (b) dominance: either the upright $\perp$ s or the lateral ones take the lead, so that either the standard illusion or a reduced one results for all; (c) nil: every $\perp$ is appreciated in its own right, so that the standard illusion results for the upright $\perp$ s, and a reduced illusion for the lateral ones. The materialization of these alternatives will depend on whether the patterns are apprehended as wholes or in a piecemeal fashion. The variation of pattern density was intended to support these different kinds of processing (dense: wholistic; scattered: piecemeal). For rotated 4- $\perp$ patterns, only alternatives (a) and (c) seem relevant, which lead to the same prediction of a null effect of context.

Method

Participants

Twelve psychology undergraduates participated in partial fulfilment of a class requirement. All participants save two were right-handed, all had normal or corrected-to-normal vision, and none was physically handicapped. The left-handed persons encountered no difficulties with the haptic task, and their data did not deviate from those of the others. One participant missed the second, visual part of the experiment, and was replaced by a comparable person drawn from the same population. Written, informed consent was obtained from all participants, and persons were treated according to the World Medical Association's Declaration of Helsinki (1963/2013).

Apparatus and haptic baseline trials

A touch-sensitive computer screen (size: 50.9×28.6 cm; resolution: 1920×1080 pixels; response time: 5 ms) was used for both stimulus presentation and response registration. The monitor was set flush into a table, and its center was at a distance of 44 cm from the observer's cyclopean center of view. Stimuli were presented within a circular, light grey window (diameter: 28.5 cm; plane visual angle: $35.9 \times 31.3^\circ$; luminance: 228 cd m^{-2}). For haptic baseline trials, the stimulus was a single black line (length: 6.5, 7, 7.5 cm; width: 0.6 mm), shown at two orientations (vertical and oblique). The six different stimuli were repeated five times in random order. For a first set of 30 baseline measurements, a line was shown for 3 s, and participants had to respond within this time window, during which the stimulus was visible. In a second set of another 30 baseline measurements, a nonverbal cue was introduced, later to be used to instruct participants which line of a pattern we wanted them to treat as target: a line was shown for 2 s and then it started to "blink" (physically, it changed its color from black to red for 200 ms, and then back to black for another 200 ms). Now participants had to respond immediately after stimulus wipe-out. In the Results section, the difference between the two sets of baseline measurements will be referred to as "stimulus visibility". In order to equate trial durations, the interstimulus interval was 2.4 s in the first set, and 3 s in the second set. The purpose of the baseline measurements was the determination of constant errors related to the specific conditions of the main experiment, notably, the orientation of the stimuli and their being turned off prior to participants' responses (Landwehr, 2009). For further details of the measurement of haptic responses, see the Responses section and Landwehr (2014, p. 1154).

Stimuli

The stimuli used for the main experiment were the delimited, discrete monomotif patterns shown in Fig. 1 (cf. Grünbaum and Shephard, 1987, pp. 204–205, for a precise mathematical definition). Patterns were doubly scaled in size as follows: first, three different lengths (6.5, 7, and 7.5 cm) were crossed factorially to yield nine different $\perp$ s. In order not to destroy the patterns' symmetry, all $\perp$ s of a given pattern were always modified in the same way. Next, patterns were enlarged or reduced by dragging the $\perp$ s outwards or by pushing them inwards. In accordance with the terminology used in the introduction to this experiment, in the following, these modifications will be referred to as variations of pattern density. Gap sizes between orthogonally facing undivided lines in the type-1 pattern varied between 1.75 and 5.25 cm (2.3 – 6.8° visual angle) and gap sizes between pairs of the central, facing tips of the divided lines in the type-2 pattern varied between 1.75 and 3.5 cm (2.3 – 4.6° visual angle). In order to keep patterns

comparable with regard to their being stretched or shrunk, they were built up from the inside out. That is, the four innermost endpoints of the $\perp$ s' undivided (type-1 stimulus) or divided (type-2 stimulus) lines were used as starting points for the generation of the $\perp$ s. While this ensured constant sizes and boundary markings of an inner, empty square across variations of the $\perp$ s, it also entailed that lines could extend beyond the outer, virtual boundaries of the patterns when lines were of unequal lengths. This provided a subtle cue that line length was being varied, and that lines were of the same length on some trials. Reducing the basic variant of the type-2 4- $\perp$ stimulus as shown in Fig. 1b eventually leads to a figure in which endpoints of lines coincide (Fig. 5c). That figure looks qualitatively different from all the others used here; it was therefore excluded from Experiment 1 (but used in Experiment 2). Conversely, a further enlargement of the type-2 pattern was not possible because the stimulus no longer fitted into the presentation window.³

Responses

Haptic responses consisted in reproducing the length of a target line by spreading one's thumb and index finger and leaving a two-point trace on the touch-sensitive computer screen. Although several authors (e.g., Franz, 2003) regard this kind of—visually mediated—haptic response, when directed at an illusion-inducing figure, as just another measure of the visual illusion, it is regarded here as a case of intermodal coordination (Gibson, 1966: “haptic signaling”), and likened to the motor activity of grasping. In fact, participants were instructed to place their fingers in such a way that the space between those points on their finger tips, that would make contact with the substrate if a real bar was to be grasped, corresponded to the target line's length. In order to make participants aware of the critical finger-tip areas, the response was exercised before the experiment by having participants pick up a small wooden bar in a pincer-grip manner. As implicated by Smeets and Brenner's (1999, 2001) theory of grasping, the equalization of grasping and signaling responses is adequate if grasping is considered as a case of two-finger pointing (cf. Smeets, Martin, & Brenner, 2010, vs. van de Kamp & Zaal, 2007, for conflicting empirical evidence). Eventually, between trials, subjects were asked to put thumb and index finger onto resting positions to the right or to the left of the stimulus presentation window, the positions of which were marked by two yellow dots, spaced vertically 8 cm apart, and 25 cm away from the center of the presentation window.

³ Incidentally, an interesting pattern-size illusion obtains: although the basic variant of the type-2 4- $\perp$ pattern (Fig. 1b) occupies a larger area than the corresponding variant of the type-1 pattern (Fig. 1a), it appears to be appreciably smaller.

Haptic responses were directed only at those lines that afforded comfortable grasps, or haptic indications. For the sake of comparability, this limitation was maintained for the visual trials, too. For every stimulus, there were four target lines: two divided and two undivided ones. For example, for the stimulus shown in Fig. 1a, the undivided lines of the upright and the inverted $\perp$ s were targets, as were the divided lines of the two laterally oriented $\perp$ s. Target positions corresponded between default and rotated variants of the stimuli so that every target line was grasped (and later judged) at both vertical and oblique orientation. Limitations imposed by the size of the available computer screen prevented rotation of the patterns around the midpoints of target lines, as had been done in my earlier experiments (Landwehr, 2009, 2014). Hence, haptic responses had to be directed to slightly different places on the screen, both with regard to top-bottom and left-right.

“Visual” ($\equiv$ verbal) responses consisted in judging a target line to be shorter, longer, or equally long, relative to the other, nontarget line of a selected $\perp$. For visual trials, the yellow dots from the haptic trials were replaced by the sentence “The blinking line was ...”, and three grey buttons, arranged in a line, labeled according to the available response alternatives. From the descriptions of the different response measures (haptic vs. verbal), it should be clear that the related tasks and prospective amounts of illusion cannot immediately be compared. The haptic response had always been directed at a singular object (a line) and, in this sense, it was “absolute” (Vishton, Rea, Cutting, & Nuñez, 1999). Even if considered as two-finger pointing, the response is not assumed to be based on a comparison (e.g., between the positions of the line's endpoints; Smeets & Brenner, 2001). The verbal response, on the other hand, was expressly “relative”, in that the length of one line had to be compared to the length of another line. The relevant “illusion measures” with regard to the $\perp$ are, in the first case, the differences in haptic (mis-)indications of the two lines of the $\perp$ and, in the second case, the biases in preferring one response alternative over the others when in truth all three apply equally often. In agreement with Morgan, Hole, and Glennerster (1990), in the following, the notion of “illusion” will be mostly avoided. As we shall see, despite the differences in the haptic indications of the $\perp$'s two lines, observers can be shown to be quite sensitive to the different lengths of the lines. Similarly, analyses of verbal responses by means of psychometric functions yield independent measures of sensitivity and bias (slopes and displacements of the function graphs). For this reason, I shall prefer to speak of haptic over- and under-shooting, and of verbal over- and under-estimation. Also note that speaking of a “target line” for visual trials is a shorthand expression concerning the direction of comparison, that is, whether the undivided line had to be compared to the divided line or vice versa. In analogy to haptic trials, the blinking line will always be called “target”, and the nonblinking line(s) “nontarget(s)”. The issue how haptic and verbal responses

relate to one another will be taken up again in the Discussion of Experiment 2.

Design

In order to keep the number of trials manageable, the main experiment was split up into four parts, which, conforming to a Latin square, were run in four different sequences so that each part occupied every position in the sequence once. Part 1 used the zero degree, default variant of the type-1 4- $\perp$ stimulus (Fig. 1a) and its mirror image, Part 2 used the same stimuli inclined at 45°, Part 3 used the 0 deg, type-2 4- $\perp$ stimulus (Fig. 1b) and its mirror image, and Part 4 used the 45° inclined stimuli of that type. Due to constraints concerning the variation of pattern density (cf. the preceding section), Parts 1 and 2 comprised the two mirror images of the type-1 4- $\perp$ stimulus at three different densities, whereas Parts 3 and 4 comprised the two mirror images of the type-2 4- $\perp$ stimulus at two different densities only. Factorially crossing stimuli and pattern densities with the sizes of individual $\perp$ s and selected target lines yielded, for Parts 1 and 2, 2 (mirror images) $\times$ 3 (pattern densities) $\times$ 3 (lengths of the $\perp$'s undivided lines) $\times$ 3 (lengths of the $\perp$'s divided lines) $\times$ 2 (types of target lines) $\times$ 2 (positions of target lines) = 216 trials that were ordered randomly. Parts 3 and 4 yielded 2 (mirror images) $\times$ 2 (pattern densities) $\times$ 3 (lengths of the $\perp$'s undivided lines) $\times$ 3 (lengths of the $\perp$'s divided lines) $\times$ 2 (types of target lines) $\times$ 2 (positions of target lines) = 144 trials, again ordered randomly. After every 72 trials, short breaks were inserted. Participants did haptic trials of Parts 1 and 4 (or Parts 4 and 1) on one day, and these trials of Parts 3 and 2 (or Parts 2 and 3) on another day about 1 week later, or vice versa. Complete sessions of 360 trials lasted about 60 min. The haptic part of the experiment was preceded by the haptic baseline measurements which made for another 60 trials (2 conditions $\times$ 3 lengths $\times$ 2 orientations $\times$ 5 repetitions). About 1 month later, the whole experiment (save the baseline measurements) was repeated with the same participants, following the same scheme (with sequences interchanged between subjects), to secure visual trials.

Results

Haptic baseline trials⁴

Original line lengths were reproduced remarkably well with a nearly constant, mean overshoot of 0.5 cm during the visibility condition, which seems to be due to the touch methodology (cf. the explication of participants' instruction in the Apparatus

section). Lines were indicated a bit longer when invisible during responding; variability also increased slightly ($SD_V = 0.64$; $SD_{NV} = 0.91$; Fig. 2). Linear regressions of indicated lengths to the differences of presented lengths from mean line length yielded slopes close to 1: $y_V = 0.940 \delta_x + 7.503$, $R^2 = .466$; $y_{NV} = 1.129 \delta_x + 7.692$, $R^2 = .346$. In terms of t -tests, the slopes did not differ significantly from 1 or between each other ($d = 0.58$), and neither did the intercepts ($d = 0.33$). A Greenhouse-Geisser corrected, repeated-measures analysis of variance (rmANOVA), however, yielded a main effect of line length, $F(1.726, 18.986)_B$; $\tau = 183.987$, $P < .001$, $\eta_p^2 = .944$, and an interaction effect between line length and stimulus visibility, $F(1.932, 21.255)_B$; $\tau \times v = 4.092$, $P < .033$, $\eta_p^2 = .271$, both of which are apparent in Fig. 2.⁵

Haptic experimental trials

Again, the different lengths of target lines were reproduced well, with an even smaller, but nonconstant, continuously decreasing overshoot (pattern type 1, undivided line: 0.56, 0.44, 0.22 cm, divided line: 0.45, 0.27, and 0.01 cm; pattern type 2, undivided line: 0.52, 0.52, 0.37 cm, divided line: 0.37, 0.30, and 0.17 cm, for 6.5, 7, and 7.5 cm, respectively). Hence, linear regressions yielded somewhat shallower slopes, ranging between 0.750 and 0.821, when computed separately for the different pattern types and target lines. While the nonconstant overshoots would normally have called for the specific constant-error corrections of data that I had applied throughout my earlier experiments (Landwehr, 2009, 2014), a closer inspection of the baseline measurements suggested that this might not be appropriate this time. Although participants were excellent with regard to average performance, individual standard errors and 95 % confidence intervals of mean haptic indications were often excessively large. This made correction factors unreliable, and their application would have distorted rather than smoothed the data. In fact, original data of the main experiment were inherently consistent for all participants in that target lines were always indicated in the correct, transitive order. Correction factors destroyed this order for seven participants. Hence, specific corrections were dismissed, and further statistical analyses were performed with the uncorrected measures.

A rmANOVA showed main effects of length, type, and position of the target line, $F(1.196, 13.156)_T = 397.377$, $P < .001$, $\eta_p^2 = .973$; $F(1, 11)_{Un/Di} = 17.658$, $P < .001$, $\eta_p^2 = .616$; $F(1, 11)_P = 7.309$, $P < .021$, $\eta_p^2 = .399$. Although, as already emphasized, target line lengths were discriminated well, undivided

⁴ To avoid potential confusion, means, standard deviations, regression equations, and F ratios have been indexed as follows: B baseline, Di divided line of the $\perp$, M mirror image of the pattern, Mo pattern motif, NT length of the nontarget line, NV nonvisibility of the target line during responding, P position of the target line, R rotation of the pattern, $S1$ type-1 stimulus, $S2$ type-2 stimulus, T length of the target line, Un undivided line of the $\perp$, V visibility of the target line during responding.

⁵ At the request of a reviewer, statistical power of all F and t tests was calculated post-hoc by compromise analyses, using G*Power 3, setting $\beta/\alpha = 1$ (Faul, Erdfelder, Lang, & Buchner, 2007). Unless noted otherwise, for all effects reported, $1 - \beta$ was .950 at least. Note that in general I shall not attempt to interpret effects with $\eta_p^2 < .500$; across my experiments on the $\perp$ -illusion, such effects seldom replicated (Landwehr, 2009, 2014, 2015, manuscript in preparation)

Fig. 2 Mean haptic length indications (in centimeters) during baseline trials. Bars 95 % confidence intervals

lines were almost always indicated longer than corresponding divided ones (Fig. 3: the four bars on the left). Target lines in the lower halves of the patterns were generally indicated longer than those in the upper halves. The other factors of the design either did not show any effects, or effects were bound up in interactions, only one of which attained sufficient power: rotation of the patterns interacted with type of target line, $F(1, 11)_{R \times Un/Di} = 11.839$, $P < .006$, $\eta_p^2 = .518$. Except for the divided line in pattern type 1, haptic misindications with regard to both of the $\perp$'s lines were somewhat enhanced when the patterns were rotated to a 45° orientation.

Visual trials

Similar to my earlier experiments (Landwehr, 2009, 2014, 2015), visual data were analyzed in terms of psychometric functions. However, in order to be able to test effects of the experiment's variables statistically, this time functions were fitted separately per participant. Response categories were dichotomized into longer versus same or shorter judgments and shorter versus same or longer judgments. Parameters were estimated from the binary logistic regression routine of SPSSTM software. Taking the intersection points of the functions for longer and shorter judgments as measures of the points of subjective equality (PSE; Urban, 1908), yielded the amounts of bias displayed in Fig. 4 (the four bars on the left;

averages from nine participants). Essentially the same results were obtained when same responses had been deselected prior to the analyses. Paired t -tests showed that the difference in bias related to target line was significant for the type-1 pattern, $t(8) = 2.849$, $P < .022$, $d = 0.95$, but not for the type-2 pattern, although power was not satisfactory, $1 - \beta = 0.910$. No fits were attempted concerning the other variables of the experiment's design because binomial and chi-square tests revealed that frequency distributions of response categories did not differ significantly between levels. In particular, the position of the target $\perp$ in the upper or lower halves of the patterns was ineffective in vision, as was the density of the patterns.

Discussion

Regarding my hypotheses for the main experiment, no clear picture emerged: there was no effect of pattern density, so that we did not get a clue how participants apprehended the patterns (as wholes or in a piecemeal fashion). Neither was there a general attenuation of the $\perp$ -illusion for rotated patterns nor a specific attenuation for lateral $\perp$ s. The effect of target position on haptic indications did not concern specifically oriented $\perp$ s but their positions in the upper or lower halves of the patterns. An attempt to explain this unexpected finding might read as follows: first, there is now ample evidence for differences in resolution and sensitivity between the upper and lower retinal

Fig. 3 Average amounts of haptic misindications (in millimeters) in Experiments 1–3. *Black bars* Undivided lines of Titchener’s $\perp$ or the heights of the plane figures used in Experiment 3, *gray bars* divided lines of the $\perp$ or the bases of the figures. *Lines* on top of the bars indicate standard

deviations. *Horizontal line* Mean misindication of the undivided line as seen in Landwehr (2014; uncorrected data, horizontal orientation of the stimulus display). The *benchmark line* related to the $\perp$ ’s divided line coincides with the abscissa. Stimuli are labelled according to Figs. 1, 5, and 6

hemifields (Levine & McAnany, 2005; Previc, 1990); furthermore, since observers looked at the stimuli with an inclined line of gaze, gradients of perspective and foreshortening applied (Gibson, 1950); finally, the different lengths and curvatures of the trajectories of arm and hand to reach the target—albeit small—may have exerted an influence. Additional experiments with proper controls for these motor effects and with different orientations of the stimulus presentation surface are needed to disentangle these factors (see Experiment 2, Method). As the effect of target position did not occur during visual trials, it may be taken as evidence for a difference in

information processing that depends on the nature of the task: if a motor response is involved, observers may attend to egocentrically defined spatial information more than for a verbal response (Bridgeman, Lewis, Heit, & Nagle, 1979; Creem & Proffitt, 2001; Goodale, 2011; Milner & Goodale, 2006).

An excessively large bias in verbal responses was obtained with the type-1 4- $\perp$ pattern when the undivided line was target (Fig. 4, leftmost bar). While it may be related to the horizontal orientation of the stimulus presentation display (Landwehr, 2014, p. 1156), a neurophysiological concept to explain this observation in the present case is excitatory long-range

Fig. 4 Average amounts of judgmental bias (“visual illusion”) in millimeters in Experiments 1–3. *Black bars* Undivided lines of Titchener’s $\perp$ to their divided lines, or, for Experiment 3, comparisons of the heights of the plane figures, used in that experiment, to their bases;

gray bars inverted comparisons. *Horizontal lines* Mean amounts of bias as seen in Landwehr (2014; horizontal orientation of the stimulus display; *upper line*: comparisons of undivided to divided lines; *lower line*: inverted comparisons). Stimuli are labelled according to Figs. 1, 5, and 6

interactions between orientation-sensitive mechanisms that are activated by parallel lines between which no other lines are interspersed (cf. Chavane et al., 2011; Gilbert & Wiesel, 1985; Joo, Boynton, & Murray, 2012). In the type-1 4- $\perp$ pattern (Fig. 1a), between pairs of undivided lines, there is just empty space, whereas in the type-2 pattern (Fig. 1b), that space is occupied by pairs of divided lines. In the first case, orientation signals can be supposed to win in gain, in the second case, they are supposed to lose in gain. Future experiments with modified patterns will have to further test this conjecture.

A possible reason why baseline measurements were unreliable may reside in historical changes. As I suggested in one of my earlier publications (Landwehr, 2014), the advent of small-scale touch- and gesture-sensitive communication devices may have led to improved dexterity so that participants found the task of indicating haptically the length of individual lines trivial and boring. The task is much more challenging in the main experiment as participants do not know in advance which line will be signaled as target. In terms of 95 % confidence intervals of means, during the main experiment, performance was quite good for all individual participants.

Experiment 2

The finding from Experiment 1, that both haptic indications and verbal judgments, referred to the lengths of the $\perp$ -figure's two lines, remained unaffected by the density of the 4- $\perp$ patterns, suggests that the context of nontarget $\perp$ s affected responses to a target $\perp$ only to a small degree, so that illusory responses seem to be tied mainly to the $\perp$ -figure as such. I therefore transformed the discrete patterns of Fig. 1 into nondiscrete ones that still contain four $\perp$ s but look quite different (cf. Experiment 1, Stimuli): continuously shrinking the patterns used in the preceding experiment, eventually leads to patterns in which endpoints of lines coincide (Fig. 5a, c) or intersect (Fig. 5b). In accordance with mathematical terminology, such patterns will be referred to as branching patterns and line arrangements, respectively (Grünbaum, 1972; Stevens, 1974). The internal coherence of these patterns, in which the original pattern motif is now firmly integrated or even perceptually hidden, leads to the prediction that, in general, an attenuation of the $\perp$ -illusions will occur. In Fig. 5a, all lines of all $\perp$ s are divided lines – hence, no illusion should occur. In Fig. 5c, four undivided lines remain and the divided ones are interconnected, forming a cross—a figure that by itself is hardly illusion-prone (Charras & Lupiáñez, 2009, 2010; Cormack & Cormack, 1974; Mamassian & de Montalembert, 2010). Furthermore, the pairwise nearness and parallelism of undivided and divided lines in this pattern may invite comparisons of lines that do not belong to the same, original $\perp$. Although geometrically equivalent (cf. the description of the 4- $\perp$ patterns' construction in Experiment 1), observers may exploit

Fig. 5 a–c The three types of stimuli used in Experiment 2, rotated 45°. a and b are derived from the type-1 4- $\perp$ stimulus (Fig. 1a), c from the type-2 4- $\perp$ stimulus (Fig. 1b), by shrinking the central square areas of the patterns. In the specimen of line arrangement b shown, the simply divided lines are longer than the originally undivided lines

the disruption of the whole pattern's virtual, square boundary when the two types of lines are differently long; this should support performance. A prediction for Fig. 5b, in which lines intersect, is less obvious. Referring back to the General Introduction, the Oppel-Kundt illusion might intervene and produce a perceptual lengthening of the formerly undivided lines of Titchener's $\perp$, thus enhancing rather than eliminating both the haptic and the visual illusions.

Method

The apparatus was the same as in Experiment 1. The stimuli as shown in Fig. 5, having been derived from those of the preceding experiment, are still of symmetry group c_4 . Thence, there is only one unique mirror image and one unique rotation image to each of them. In order to reduce the number of trials, mirror images of the patterns were not used because they had not shown an effect in Experiment 1. This leaves 3 stimulus variants $\times$ 2 different orientations $\times$ 3 lengths of the $\perp$ s' first lines $\times$ 3 lengths of the $\perp$ s' second lines $\times$ 4 target lines, making for 216 trials. Two breaks were inserted after one and two thirds of trials, respectively. Twelve new participants were recruited from the psychology undergraduate population at the University of Mainz. Haptic and visual conditions were run in two separate sessions, scheduled on different days. Since psychometric functions could not be fitted in many

cases, another ten participants, drawn from the same population, were called upon to provide more data. These additional participants did the visual part of Experiment 2 twice, and they were paid € 8 per hour for their service.

The haptic part of the main experiment was preceded by newly conceived baseline measurements (or practice trials). Horizontal lines were deleted from the upright type-2 4- $\perp$ pattern (Fig. 1b), so that only four vertical lines were left, each of which could be target or nontarget. The idea behind this modification of the stimulus was to make measurements more interesting and also more challenging, in that participants had to attend to which line would be signaled as target. At the same time, these trials also tested the target position effect that had emerged in Experiment 1. Only the default variant of the original pattern and its mirror image were used to eliminate possible effects of rotation. Hence, there were 2 stimulus variants $\times$ 3 lengths of the originally undivided lines $\times$ 3 lengths of the originally divided lines $\times$ 4 target lines, yielding 72 trials. There was no visual counterpart to this haptic experiment because the patterns no longer contained appropriate comparison lines.

Results and discussion

Haptic performance did not differ between practice and experimental trials; therefore, the latter data were not corrected. For practice trials, only an effect of target line length was seen, $F(1.513, 16.639)_{B; T} = 159.790$, $P < .001$, $\eta_p^2 = .936$. The absence of an effect of target position in this haptic control experiment with four isolated lines suggests that the effect observed in Experiment 1 was tied to the 4- $\perp$ patterns and cannot be explained in biomechanical or geometrical-optical terms alone. That is, different temporal demands and differences in elbow flexion as such do not seem to produce a target position effect for isolated lines in the range of distances between hand and target used here. Similarly, perspective and retinal factors, at this scale, do not seem to work independently of the structure of the stimulus.

During the main experiment, as predicted, differences in haptic indications vanished with the stimuli shown in Fig. 5a and c, and they also did so with the stimulus shown in Fig. 5b (Fig. 3: six bars in the middle). Responses to formerly undivided or divided lines differed only slightly, although both were overshoot, $y_{Un} = 0.772 \delta_x + 7.458$, $R^2 = .131$; $y_{Di} = 0.862 \delta_x + 7.498$, $R^2 = .152$. This means that for haptic responses to $\perp$ s embedded in delimited branching patterns or line arrangements, neither remaining undivided lines nor the intersection of lines seem to matter much. Discrimination of target line lengths was excellent, $F(1.614, 17.758)_T = 176.003$, $P < .001$, $\eta_p^2 = .941$, and only compromised slightly by a main effect of the patterns' rotation, $F(1, 11)_R = 6.402$, $P < .028$, $\eta_p^2 = .368$, and an interaction effect of these two variables, $F(1.912, 21.032)_{T \times R} = 5.383$, $P < .014$, $\eta_p^2 = .329$. Haptic indications of line lengths were shorter for

oblique lines, and this effect was attenuated at a line length of 7 cm. The origin of these effects that partly contradict those of the previous experiment are not clear to me.

Concerning the visual data, the additional participants as a group performed similarly to the original sample; therefore, data were combined. With the stimuli used in Experiment 2, verbal biases remained fairly strong (Fig. 4: six bars in the middle). A particularly gross relative bias occurred with the line-arrangement pattern from Fig. 5b, similar to the effect seen with the basic type-1 4- $\perp$ pattern in Experiment 1 and suggesting that the Oppel-Kundt illusion did not make an extra contribution, $t(9) = 2.819$, $P < .020$, $d = 0.89$, $1-\beta = 0.909$. There was no effect of the target line for the branching patterns shown in Fig. 5a and c.

A likely explanation for the discrepancy between the absence of a haptic illusion and the survival of a visual one in the present experiment is this: for the haptic task, participants had been instructed to indicate the length of the single, blinking target line, whereas for the visual task, they had been asked to compare the length of the target line to the length of another line to which it was orthogonally connected (cf. Franz, Fahle, Bühlhoff, & Gegenfurtner, 2001; Pavani, Boscagli, Benvenuti, Rabuffetti, & Farnè, 1999; and Experiment 1, Apparatus and haptic baseline trials). Participants thus may have apprehended the stimuli as coherent, homogeneous patterns for the haptic task, but may have attempted to perceptually dissect them in order to find the $\perp$ s to which target and comparison lines belonged. Had they compared lines that were not part of a $\perp$, they might have been able to avoid any bias. This hypothesis was tested with an independent sample of 12 new participants drawn from the same population at the University of Mainz.⁶ A subset of the original stimuli was used—Fig. 5a and c in upright orientation—and two of the vertical lines were blinking in succession. Participants had to say whether the first blinking line had apparently been longer or shorter than the second blinking one. Due to the combinatorics behind the construction of the stimuli, there were four comparisons between lines of different physical lengths and two comparisons between lines of the same length, hence, for two patterns and two times three lengths of lines, a total of 108 comparisons, with the correct answers being longer, shorter, and same, for one-third each. The task was framed as two alternative forced choice (2AFC), though, so that longer and shorter judgments were expected to be distributed randomly for objectively identical line lengths. After deselection of the last mentioned cases (for which distributions of answers to equal line lengths were in fact balanced), all but one prediction for binomial distributions of answers conformed to expectations, that is, with one

⁶ Before serving for this purely visual control study, these participants had taken part in a (successful) replication of my earlier experiments with single $\perp$ s (Landwehr, 2009, 2014) with stimuli scaled down in size to mean line length of 3.25 cm (Landwehr, manuscript in preparation).

exception, observers correctly identified physically longer lines to be longer, shorter ones to be shorter, and lines of the same lengths to be equal. This again shows that participants encountered little, if any, difficulty in discriminating line lengths. It also suggests that the visual $\perp$ -illusion does not reflect a general inability to compare the lengths of lines—at least, if these are unconnected and parallel (cf. Landwehr, 2015, Experiment 3, for the orthogonal case, in which the illusion persisted).

Experiment 3

While the haptic $\perp$ -illusion vanished in Experiment 2, the visual illusion did not readily go away. Furthermore, as in Experiment 1, the amount of judgmental bias was often in the range seen with individual $\perp$ s (Fig. 4: the benchmark lines; Landwehr, 2014). Hence, it seems highly probable that some intrinsic characteristics of the $\perp$ -figure contribute to or may even be mainly responsible for the illusion. Besides the T-junction, the areas adjacent to the $\perp$'s undivided line could exert some influence. Psychologically speaking, the $\perp$ might be apprehended as the skeleton of a triangle (cf. Feldman & Singh, 2006; Ogniewicz & Kübler, 1995, for more general cases dealing with discrimination and recognition of 3D shapes and objects). In 2D, this idea would relate the $\perp$ -illusion to sensitivity for the sizes of plane angles (Heeley & Buchanan-Smith, 1996; Snippe & Koenderink, 1994). If the constituent lines of the $\perp$ are of equal length, and the endpoints of the two lines connected, the angle at the top of the resulting isosceles triangle is approximately 53° , and the base angles are approximately 63.5° . In contrast to an equilateral triangle, an isosceles triangle with these specifications can probably be said to look pointed or elongated (Fig. 6a, b). A form that looks less pointed is an oval- or egg-shaped form. I chose a famous, if familiar shape that is defined by a flattened catenary: the outer curve of the “Gateway to the West” Arch in St. Louis, Missouri, the height of which exactly matches the width of its base (Osserman, 2010). The resulting stimuli are shown in Fig. 6c, d. These thimble-like forms can probably be said to be intermediate between triangles and rectangles (or squares).

For Experiment 3, the contexts constituted by the type-1 and type-2 $4\text{-}\perp$ patterns used in Experiment 1 were kept in order to see whether the different effects observed with these two pattern types would replicate in related patterns of plane figures. However, with regard to procedure, a change of the nonverbal cue that signified the target was necessary. Since one type of target line, referring to the heights of the figures, was specified only implicitly in the stimulus, the blinking technique used to inform participants about the target had to be modified. Now, only the endpoints of the base line or, alternatively, its midpoint and the point at the top of the relevant figure (triangle or arch) would blink. In order to introduce

Fig. 6 The types of stimuli used in Experiment 3. Those in the left-hand column were derived from the type-1 $4\text{-}\perp$ stimulus (Fig. 1a), those on the right from the type-2 $4\text{-}\perp$ stimulus (Fig. 1b). For illustrative purposes, in c, the figures are higher than wide

the new blinking technique to observers, the whole experiment was preceded by practice trials which, save the kind of blinking, were identical to those of the practice measurements that had preceded Experiment 2. In this way, it was possible to directly compare performance with regard to the two methods of specifying the target distance.

Method

Apparatus, design, and participants

The apparatus remained the same as in Experiments 1 and 2. The four stimuli shown in Fig. 6, still being of symmetry group $c4$, were presented at their two canonical orientations (0° and 45°), and the aspect ratios of the figures were varied exactly as had been done with the $\perp$ s in the previous experiments. In total, this gave 2 (types of figures) $\times 2$ (pattern types) $\times 2$ (rotations) $\times 3$ (height of figures) $\times 3$ (width of figures) $\times 4$ (target distances) = 288 trials. For technical reasons that had to do with the generation of the figures, figure and pattern types could not be combined in a single file. Therefore, the experiment was split up into four parts, which, similar to the procedure implemented for Experiment 1, were run in four different, balanced sequences.

For visual trials, participants were first shown figures of single triangles and catenary forms, and the distinction between the

bases of the figures and their only implicitly specified heights was explained. Subjects were instructed to regard that distance, the endpoints of which were blinking as target, and compare it to the unsignaled nontarget. For example, when the midpoint of the base and the uppermost point of a figure were blinking, the height of the figure had to be compared to its base. Subjects were warned that individual figures containing the target distance might appear upside down or rotated 45° or 90°. Only two response alternatives—shorter vs. longer—were allowed. The reason for this restriction was the following: for eight of the nine size calibrations of triangles, base angles were larger than the angle at the top; this suggested that the triangles' heights would almost always appear greater than their bases, which means that few same judgments were to be expected (similar considerations apply for the catenary forms in terms of local curvature). Twelve new participants were recruited from the psychology undergraduate population at the University of Mainz. Haptic and visual conditions were run in two separate sessions, scheduled on different days.

Results and discussion

A comparison of the practice trials which preceded Experiment 3 with those that had preceded Experiment 2 ascertained that, on average, responses were both more precise and more accurate when two target dots instead of a whole line had been blinking; in terms of linear regressions: $y_{\text{Dots}} = 1.102 \delta_x + 7.328$, $R^2 = .268$; $y_{\text{Lines}} = 1.183 \delta_x + 7.799$, $R^2 = .217$. In terms of a rmANOVA, again, there was no effect of target position, but only one of target distance (corresponding to target line length in the other experiments), $F(1.493, 17.920)_T = 167.788$, $P < .001$, $\eta_p^2 = .933$.

For the main experiment, a Greenhouse-Geisser corrected rmANOVA revealed main effects of length and type of target distance, and of rotation of the figure patterns, $F(1.382, 15.202)_T = 82.616$, $P < .001$, $\eta_p^2 = .882$; $F(1, 11)_{\text{Un/Di}} = 25.195$, $P < .001$, $\eta_p^2 = .696$; $F(1, 11)_R = 6.631$, $P < .026$, $\eta_p^2 = .376$. Similar to the first experiment, target distances were reproduced well, but the distances corresponding to the 4- $\perp$ patterns' divided lines (here: the bases of the figures) were generally indicated shorter than those corresponding to the undivided lines (here: the heights of the figures; for details see Fig. 3). In addition to these main effects, there were simple interactions between pattern type and rotation, and between pattern type and height of the target distance in the field of view, $F(1, 11)_{S1/2 \times R} = 5.374$, $P < .041$, $\eta_p^2 = .328$; $F(1, 11)_{S1/2 \times P} = 8.197$, $P < .015$, $\eta_p^2 = .427$, as well as threefold interactions between pattern type, position of target distance, and pattern motif, $F(1, 11)_{S1/2 \times P \times \text{Mo}} = 6.248$, $P < .030$, $\eta_p^2 = .362$, pattern type, position of target distance, and type of target distance, $F(1, 11)_{S1/2 \times P \times \text{Un/Di}} = 7.900$, $P < .017$, $\eta_p^2 = .418$, and pattern type, motif, and type of target distance, $F(1, 11)_{S1/2 \times \text{Mo} \times \text{Un/Di}} = 6.334$, $P < .029$, $\eta_p^2 = .365$. The essence of this flurry of effects is shown in Fig. 7. For the

figures' heights (corresponding to the $\perp$'s undivided line), there was a disordinal interaction related to pattern type: with pattern type 1, haptic indications were larger when the target distance was in the lower half of the pattern, and with pattern type 2, the inverse was true. There was no such effect for the figures' bases (corresponding to the $\perp$'s divided line), and the effect was attenuated for the catenary motif. Although effect sizes were on the verge of impressiveness, an obvious interpretation suggests itself: in the type-1 pattern, the triangle or catenary forms are in upright orientation in the lower halves of the patterns, while in the type-2 pattern, they are upside down at this position (cf. Fig. 6). Although I never found a difference between an upright T and an inverted one with regard to haptic misindications or illusions (Landwehr, 2015), matters may be different for plane figures for which the $\perp$ acts as a skeleton. A control experiment with individual figures will be needed to substantiate this speculation.

Visual data, again, were analyzed by fitting psychometric functions. Results can be read off Fig. 4 (the eight bars to the right). According to paired t -tests, except for the type-1 catenary pattern, bias was not significantly greater when the figures' heights had to be compared to their bases rather than vice versa, $t(9) = 3.313$, $P < .009$, $d = 1.050$, $1 - \beta = 0.939$. However, when responses were directed to the figures' bases, biases differed significantly between pattern motifs, being smaller for the catenary forms than for the triangles, $F(1, 45)_{\text{Mo}} = 11.081$, $P < .002$, $\eta_p^2 = .201$, $1 - \beta = 0.963$. This accords with my prediction.

For both the visual and the haptic data of Experiment 3, it is important to note that the tasks did not wholly correspond to those posed for the line stimuli in Experiments 1 and 2: as the heights of the figures in Experiment 3 had not been marked by lines, observers always had to compare a line and an empty distance, or haptically indicate either the one or the other. This may have supported the asymmetries in the responses that go with the different targets (here: heights vs. widths). All haptic responses to figures were shifted towards shorter finger spans as compared to responses to line patterns. Hence, what had to be taken as an overshooting of the lengths of the undivided lines in 4- $\perp$ patterns in Experiment 1 must now be regarded an undershooting of the bases of the figures. The shift may have come about as follows: haptically indicating an empty distance which is marked by two dots seems easy, and in fact, participants' performance was close to being perfect. Given the effectiveness of the visual $\perp$ -illusion, the figures' bases most likely appeared shorter than the figures' heights. Haptic responses to the bases may then have been scaled down, resulting in undershooting.

General discussion

Effects of self-similar contexts on haptic and visual illusions related to Titchener's $\perp$ were less dramatic than expected but

Fig. 7 The disordinal interaction effect between pattern type and height of the target distance within the pattern and within observers' field of view when the figures' heights had to be haptically indicated. The

effect did not occur with the figures' bases as targets, and it was more pronounced with triangles than with catenary forms (the data shown have been aggregated across motifs)

were not wholly absent. With regard to haptic indications of the lengths of the $\perp$'s two lines, all illusion vanished when both a target $\perp$ and three distractor $\perp$ s were hidden in nondiscrete branching patterns and line arrangements (Experiment 2). With regard to verbal judgments of the relative lengths of the $\perp$'s two lines, the way that four $\perp$ s were arranged in discrete and nondiscrete patterns exerted significant effects: in the type-1 pattern, in which the $\perp$'s undivided lines were pointing towards each other (Fig. 1a), there was a greater decisional bias when these lines had to be compared to their counterparts than for the inverse comparison, whereas in the type-2 pattern, in which the $\perp$'s undivided lines were pointing away from each other (Fig. 1b), no such effect was seen (Experiments 1 and 2). Compared to the best comparable experiment with a singular $\perp$ (Landwehr, 2014), both the differences in haptic misindications concerning the two lines of the $\perp$ and the target line effect for verbal responses were apparently smaller in the context of other $\perp$ s than for an individual $\perp$ (see the benchmark lines in Figs. 3 and 4); in this sense, the $\perp$ -illusion can possibly be said to have been attenuated in 4- $\perp$ patterns.

Two major conclusions emerge: (1) unlike length estimates of individual lines (as in Armstrong & Marks', 1997, experiments), the haptic and visual illusions that commonly go with Titchener's $\perp$ were affected only weakly by discrete, self-similar contexts, suggesting that the illusions are tied mainly to the $\perp$ -figure itself. (2) As the haptic $\perp$ -illusion vanished with

the nondiscrete line-arrangement pattern and the branching patterns of Experiment 2, there seems to be an essential, perceptually relevant difference between such patterns and discrete ones. Naturally, both of these conclusions are preliminary. With regard to (1), we still have to test dissimilar contexts. Of particular interest would be patterns that preserve the overall organization of the patterns used in the present series of experiments but destroy it locally. For example, in the 4- $\perp$ patterns used in Experiment 1, the size variations of the $\perp$ s could be done for each $\perp$ independently. Similar to conclusion (1), substantiation of conclusion (2) requires additional tests with other branching patterns and line arrangements.

The first conclusion is supported strongly by the findings from Experiment 3. As a virtual $\perp$ elicited both the haptic and the visual $\perp$ -illusions, the mathematical properties that characterize the $\perp$ may be what causes the illusions. Either the dihedral symmetry group of the figure acts as a top-down organizing principle (Wertheimer, 1922), or the $\perp$ (inverted or not) is an acquired scheme that habitually guides perception (Landwehr, 2015). Animal or neuroscientific research may provide support for the first idea (cf. Shevelev et al., 2001), and cross-cultural studies may substantiate or falsify the second (cf. van der Kamp, Withagen, & de Wit, 2013). However, the existence of T-sensitive neurons or the demonstration of an acquired habit to draw the T's upstroke longer than its cross-stroke would not yet answer the ultimate question why such

mechanisms or habits developed in the first place. Here, evolutionary arguments (Howe & Purves, 2002) and biomechanical constraints will have to be taken into account. In either case, it is important to note that the earlier characterization of the $\perp$ -illusion in terms of the bisectioning of one line by another one is not sufficiently precise. As demonstrated by Cormack and Cormack (1974), and recently partly replicated in our laboratory (Doerfel, 2015), a $\perp$ with a tilted undivided line may lead to an even greater visual illusion than a $\perp$ with orthogonal lines, unless the angle between the two lines becomes strongly acute (cf. Appelle, 1972; Hansen & Essock, 2004; Westheimer, 2003, for other, pervasive “oblique effects”). This variation of the $\perp$ seems of particular interest to further investigate the specific, haptic and visual effects that emerged in the presently reported series of experiments.

With regard to conclusion (2), more complex and variegated discrete patterns, line arrangements, and branching patterns will have to be tried before we can be sure about the relevance of the underlying mathematical distinctions. In experiments with potentially infinite, periodic $\perp$ -patterns, I found fairly strong haptic and visual illusions, but also significant effects of the patterns’ rotations (Landwehr, manuscript in preparation), only partly supporting conclusion (2). I did not yet test discrete random patterns. For line arrangements and branching patterns, there are constraints if patterns are to be composed entirely from $\perp$ s (Grünbaum, 1972; Stevens, 1974), so that stimulus construction is far from trivial. For all multiple- $\perp$ patterns, an important control still to be done is to have observers compare lines that together form a $\perp$ across a distance. These would be “dissected $\perp$ s in context”, and from my previous work (Landwehr, 2015) as well as from Verrillo and Irvin’s (1979) study, I should predict that the visual $\perp$ -illusion, like the haptic one in the present Experiment 2, will eventually vanish.

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