Perception & Psychophysics
1985, 37 (4), 323-334

On size, distance, and visual angle perception

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Standard descriptions of visual spatial experiences, especially illusions, create destructive paradoxes because, along with the perceived distance variable (D′), they use only one "perceived size" variable (S′) in the equation S′/D′ = V rad, to describe perception of a target's linear size, S m, its distance, D m, from the eye, and the visual angle, V deg, its outer edges subtend at the eye. Simple paradoxes vanish in descriptions using the different equation, S′/D′ = V′ rad, which adds the perceived visual angle variable, V′ rad. Redefining classic illusions as illustrations primarily of misperceived direction difference (V) values removes the pseudoparadoxes that have made extant explanations of illusions seem unsatisfactory.

Kilpatrick and Ittelson (1953) noted that prevalent theories of visual spatial perception unfortunately make observers' normal reports about their "size" and distance experiences seem paradoxical. Classic examples are the "paradox of converging parallels" (Boring, 1952; Gibson, 1952) and the size-distance paradox (Epstein, Park, & Casey, 1961). After the brief review below, I elaborate an approach that removes paradoxes (McCready, 1965, 1983).

REVIEW

The SDIH

Standard theories of perception (cf. Epstein, 1977a) yield the well-known size-distance invariance hypothesis (SDIH) (Gilinsky, 1951; Schlosberg, 1950), written as,

$$S'/D' = kV = kS/D \text{ rad} \qquad \text{(the SDIH).}$$

As shown in Figure 1A, S m is a target's frontal linear size value, say its width or height, D m is its optical distance from the eye, and V rad is the visual angle, which is the optical direction difference of the target's edges from the eye. The rule for small angles, V = S/D rad, customarily is the analog for theory statements. The response value D′ m is the perceived distance, as indicated in Figure 1C. The response value S′ usually is called simply "perceived (or apparent) size," but the SDIH requires that it be the perceived linear size value, S′ m. The observer constant, k, nominally is 1.0 (Gogel, 1963, 1977; Oyama, 1977a).

Emmert's law. For a target subtending a constant visual angle (V rad), the SDIH predicts that the perceived linear size (S′) will be a direct function of the perceived distance (D′). That general rule is Emmert's law, which originated for afterimages (cf. Weintraub & Gardner, 1970).

Perceived Visual Angle

Some writers attribute the traditional difficulties to the use of only one "perceived size" variable instead of two (Baird, 1968, 1970; Joynson, 1949, 1958a, 1958b; Joynson & Kirk, 1960; McCready, 1965; Ono, 1970; Rock, 1977; Rock & McDermott, 1964). They claim we perceive not only linear size (S) values but also direction difference (V) values. Thus, the reason "textbook" descriptions are confused is because they omit the perceived visual angle value, V′ rad, more descriptively called the visually perceived direction difference. Problems created by ignoring V′ deg are clearly revealed when comparisons of two targets are described with the relative SDIH, which is,

$$(S_2'/D_2')/(S_1'/D_1') = V_2/V_1 \qquad \text{(the SDIH).}$$

For example, consider Figure 2.

Ebbinghaus illusion. Viewed normally, the two central circles subtend equal visual angles, so the SDIH is,

$$S_2'/S_1' = D_2'/D_1'.$$

The Ebbinghaus illusion is that circle 2, surrounded by small circles, looks as much as 20% wider than circle 1 surrounded by large circles (Coren & Girgus, 1977, 1978; Jaeger, 1978; Massaro & Anderson, 1971; Weintraub, 1979). I will use the large value of 20% for examples.

According to the SDIH and Emmert's law, the report "looks 20% wider" can be recorded only as $S_2' = 1.2\ S_1'$, which requires that $D_2' = 1.2\ D_1'$; circle 2 should look 20% farther than circle 1. That outcome is

I thank my colleagues for absorbing the added teaching load imposed upon them because I had the sabbatical semester, Spring 1980, to begin this project. I am grateful to Richard Kelley and I. Ning Huang for reading early manuscripts, listening critically, and giving encouragement for so long. I thank J. C. Baird and nine other referees (anonymous) for their advice on preceding manuscripts. I thank Lisa Horton for preparing the figures. Finally, I especially thank my wife, Sally, for tolerating my long devotion to obtaining an acceptable draft.

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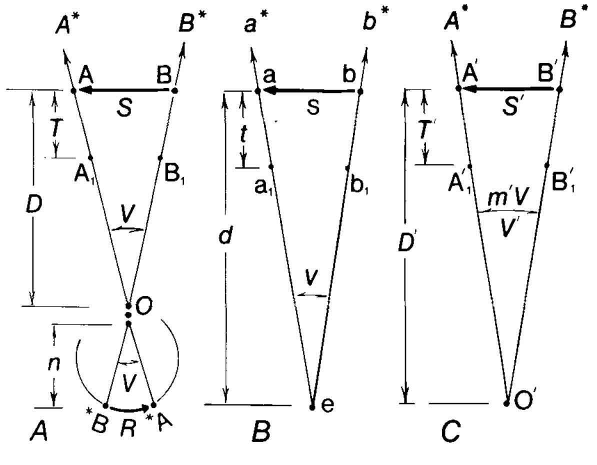

Figure 1. Stimulus values are shown in A as a top view of one's eye aimed at an extent of linear size, S m, at the optical distance D m, so endpoints (A and B) subtend the visual angle, V deg, at eye point O. The target's retinal image (*A,*B) is R mm wide. Phenomenal correlates of the stimulus values are shown in B, which diagrams one's visual world. Response values, shown in C, are the reports one gives about the subjective values indicated in B.

described in Figure 3 by the top views, with circle 1 (Figure 3A) and circle 2 (Figure 3B) at the same viewing distance, D m. For the convenience of later discussions, the comparative value of 20% is divided equally, so circle 1 looks 10% narrower and closer than it is ($S_1' = 0.9$ S; $D_1' = 0.9$ D) and circle 2 looks 10% wider and farther than it is ($S_2' = 1.1$ S; $D_2' = 1.1$ D). Also, the visual angles look equal ($V_2' = V_1'$). But to understand standard arguments, one has to ignore the V′ symbols in Figure 3 and use only the equal V values.

Size-Distance Paradox

The common report that circle 2 looks larger and on the same page as circle 1 (so $D_2' = D_1'$) clearly unbalances the SDIH and cannot be illustrated by Figure 3. A seemingly more paradoxical report (fostered by asking which central circle looks closer) is that circle 2 looks larger and *closer* than circle 1. Those size-distance paradoxes occur for many illusions (Coren & Girgus, 1977, 1978; Epstein et al., 1961; Woodworth & Schlosberg, 1954).

Paradox Lost

The paradoxes vanish, however, when Figure 2 and other illusions are redefined as examples of misperception of visual angle (V) values (McCready, 1964, 1965, 1983). That is, the equivocal report "looks 20% wider" is properly recorded *first* as $V_2' = 1.2\ V_1'$, which of course cannot be entered into the SDIH and is not illustrated by Figure 3. The illusion thus mimics a veridical perception for a visual angle ratio of $V_2/V_1 = 1.2$, obtained with either a pair of equidistant circles, with $S_2 = 1.2\ S_1$, or a pair of equal linear sizes ($S_2 = S_1$), with $D_2 = 0.83\ D_1$.

Alternative

Some alternative approaches that use V′ rad along with S′ m and D′ m (Baird, 1968, 1970, 1982; Rock, 1977, 1983; Rock & McDermott, 1964) nevertheless have tended to follow tradition by treating the linear and angular size experiences for a target as mutually exclusive values on the *same* "perceptual size" dimension: The standard assumption has been that a target's "perceived size" (S′ in the SDIH) may be, at one time, a "distal" value that approximately equals S m and, at another time, a "proximal" value that somehow correlates with V deg (see Epstein, 1977a, 1977b). To experience one value or the other, an observer supposedly switches between two "attitudes" (Boring, 1952), or two "perceptual modes" (Rock, 1977, 1983): The observer's choice of mode is determined by the observer instructions (Baird, 1970; Carlson, 1977) and by the relative abundance of distance cues (because if D′=D, then S′=S). Alternatively, the observer is said to switch between experiencing the target's "size" as an extent in the three-dimensional visual world and as one in a visual field (Gibson, 1950, 1952). Or, finally, S′ might be valuated now by a "law of size constancy" and then by a "law of the visual angle" (Holway & Boring, 1941).

That characterization of standard views may seem unfair because it addresses the problem of defining "absolute size" experiences. The dichotomies mentioned above were created to describe comparisons observers make under different conditions when judging two targets at different distances: Ideally, the "size" comparison made in the "distal" (or "objective," or "constancy") condition agrees with the S values, and in the "proximal" (or "analytic," "retinal," "perspective," or "reduced") condition it instead agrees with the V values. Use of the distal/proximal distinction might aid such descriptions, but it causes serious problems when it leads to the assump-

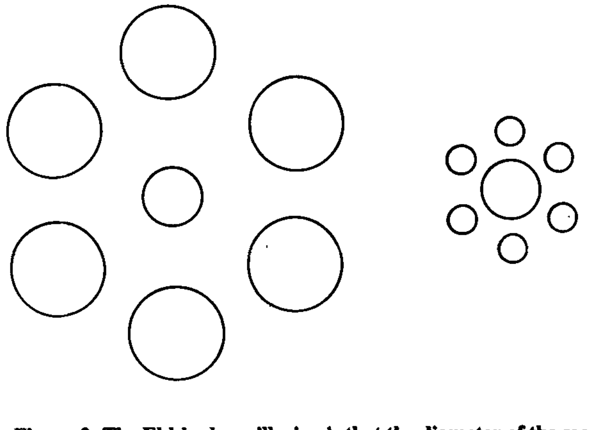

Figure 2. The Ebbinghaus illusion is that the diameter of the second circle, surrounded by smaller circles, appears to subtend a greater visual angle than the equal diameter of the first circle, surrounded by larger circles.

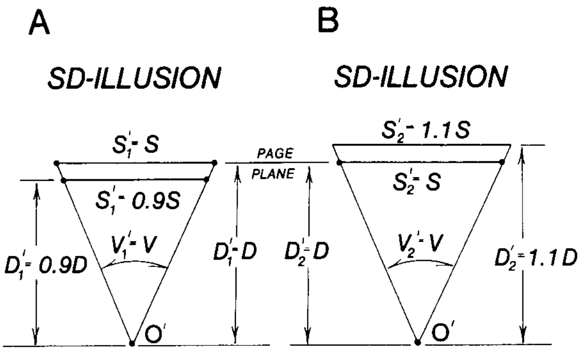

Figure 3. Top views show some likely responses to the central circles in Figure 2 for a rare observer who perceives the visual angle values accurately ($V_2' = V_1' = V$ deg). In true perceptions, circle 1 (in A) and circle 2 (in B) both appear at the page distance D m and look their actual linear size, S m. Diagram A also describes an SD illusion in which circle 1 looks 10% linearly narrower and closer than it is; diagram B describes an SD illusion in which circle 2 looks 10% linearly wider and farther than it is. Together, A and B describe a relative SD illusion in which circle 2 looks 20% wider and farther than circle 1.

tion that those two different "absolute size" experiences for *each* target are different values of a single "size" experience.

Simultaneity. The present assumption is that not only are the linear size and angular experiences (s and v) qualitatively different, but, as Figure 1B indicates, they are also simultaneous perceptual experiences which, along with the distance experience, d, relate in accord with the equation s/d = v subjective radians (McCready, 1965).

In the present view, the "switches" between conditions mentioned above do not educe different values of a single "size" experience; instead, they shift attention between the concurrent perceptual values s and v. That is, while attention is upon s or v, the other does not disappear from perception, as was assumed in classic theories (Helmholtz, 1910/1962; Koffka, 1935). By the same token, neither s nor v is obliterated when attention shifts to d.

In what follows, this alternative viewpoint, also occasionally suggested by Rock (1977, pp. 341-342), is elaborated with improved symbols and equations (McCready, 1983). It may be called the s=dv theory of linear size, distance, and visual angle perception and is reviewed below.

THE s=dv THEORY

For the sake of clarity, definitions of variables shown in Figure 1 are extended below.

Stimulus Values

In Figure 1A, two points, A and B, equidistant from eye point O (the center of the entrance pupil) define a frontal separation of S m at the optical distance D m. Lines AO and BO are chief rays. In reverse, they indicate the optical directions A* and B*, which differ by V deg.

The depth, or thickness, value, T m, is both a distance difference and a former linear size value viewed end-on. Perception of T will not be discussed in detail.

Retinal size. The retinal image size, R mm, is the distance between points *A and *B, the images of A and B. It is given by R/n = S/D rad, in which n is the constant nodal distance, about 17 mm in an average eye (Bennett & Francis, 1962, p. 105). (For present purposes, point O and both nodal points may be treated as one point.) A satisfactory optical rule thus is, R/n = V rad.

Phenomenal Values

Figure 1B diagrams one's visual world in which one's immediate perceptual values are phenomenal linear size, s, phenomenal distance, d, and the phenomenal visual angle, v, more descriptively called the visual direction difference (McCready, 1965). Point e is the center of visual directions, or visual egocenter (Roelofs, 1959). Line ea denotes point a's visual direction, a* (Hering, 1879/1942; Ono, 1979, 1981), and b* is point b's visual direction.

For example, when one sees a charging bull head-on, a* and b* predict the directions from oneself to its right and left horns, thus v predicts the value of an orienting response that will aim one's eye, head, or hand from one horn to the other; d predicts the distance to its head, and s predicts its head width. (Let the phenomenal depth value, t, predict a horn's length.)

Other spaces. Figure 1B can also illustrate auditory space, haptic space, or a common perceptual space (Auerbach & Sperling, 1974; Taylor, 1962). Thus, a* could denote an auditory direction or a haptic direction as well as a visual one. We may suppose that those three a* directions normally agree and that the three b* directions agree, so the auditory, haptic, and visual direction difference (v) values would be equal for points a and b. Each of those three v values undoubtedly can gauge the magnitude of a ballistic reorientation of a body part from one seen, heard, or felt direction to another, if that is one's aim.

Presumably, v also gauges covert reorientations of attention (Remington, 1980; Shulman, Remington, & McLean, 1979). Evidently one's visual attention turns to a different point, v phenomenal degrees away, just before one rotates one's eyes to examine it more carefully (Posner, 1980). Indeed, v has been defined as the visual correlate of an oculomotor efference readiness to perform a saccade (Festinger, Burnham, Ono, & Bamber, 1967; Taylor, 1962).

In order to shorten the text at this point, a supportive, but peripheral, discussion of memory variables appears in the Appendix, along with a discussion of visual processing summarized below.

Visual Processing

As Rock (1977, 1983) also advocates, the visual direction difference, v (which he calls a "pure extensity" experience), is considered to be the primary perceptual value, whereas the linear experiences, s and d, are secondary perceptual values in the relation s/d=v. The value v becomes a direct function of the visual angle value, V deg, by way of the retinal image size value, R mm. As already implied, illusions such as Figure 2 demonstrate that v is not an invariant function of V rad (and R mm), so the ratio s/d also is a noninvariant function of V rad.

Valuations. In the present model, the value R mm for a target first valuates v more or less accurately in a "preprocessing" step. Then, as standard models of visual processing suggest, either d or s becomes *scaled* and the other becomes *computed*, in accord with the rule s/d=v (Epstein, 1977b; Oyama, 1977a; Rock, 1977). Standard models, of course, use the rule s/d = V rad instead.)

Scaling of d may occur by distance cuing, and that of s by linear size cuing. Scaling also may occur by hypothesizing, in which d or s acquires a suggested or assumed value (see Singer, Tyer, & Pasnak, 1982). The computing step keeps the s/d ratio always equal to v. (See Appendix.)

Response Values

To fully communicate one's immediate frontal extent experience, one thus must furnish at least the three response values, S′ m, D′ m, and V′ deg. The linear values, S′ m and D′ m, may be verbal estimates or haptic ones, such as the distance between two indicators that *feel* as far apart as the target value S or D looks (Gogel, 1977; Ono, 1970).

The perceived visual angle value, V′ deg, could be a verbal estimate (Foley, 1965; Saltzman & Garner, 1950), but most of us cannot skillfully estimate direction differences verbally, so haptic estimates are used, such as the initial angle through which the head, an eye, or a pointer is turned when aimed ballistically from one viewed point to another (Attneave & Pierce, 1978; Komoda & Ono, 1974; Ono, 1970).

Directions. In Figure 1C, point O′ denotes a response-measured locus of the cyclopean eye (Barbeito & Ono, 1979; Ono & Barbeito, 1982). Procedures for locating O′ generate aiming-response lines, such as O′A′ and O′B′, which specify the visually perceived directions A# and B# for points A and B. The angle between those response lines thus provides another V′ value. The synonym "visually perceived direction difference" clearly identifies V′ deg as the response value common to "size" and distance research and research on direction perception (see Shebilske, 1977).

As defined above, the response values are the data values invariably published as the "perceived," "apparent," "judged," and "estimated" values.[1] Veridical or not, the momentary response values for a target should relate as shown in Figure 1C and as stated by Equation 1:

$$S'/D' = V' \text{ rad}. \qquad (1)$$

Equation 1 is the basic testable hypothesis of the s=dv theory. [For targets subtending large angles, the equation to use instead is $S'/D' = 2\tan(V'/2)$.]

Psychophysical Equation

The further hypothesis that both V′ and the ratio S′/D′ rad are noninvariant functions of V is stated by Equation 2 below, which, left to right, links response values (Figure 1C) to stimulus values (Figure 1A):

$$S'/D' = V' = m'V = m'(S/D) \text{ rad}. \qquad (2)$$

The variable m′ is response magnification, the ratio of the perceived visual angle to the visual angle. It nominally equals 1.0 and is calculated from an observer's response data using both equations, $m' = V'/V$ and $m' = (S'/S)(D/D')$. Those two m′ values of course should agree.

To complete the s=dv theory, it will be necessary to have specific equations that relate m′ to variables known to cause V′ deg to deviate from V deg. Some are mentioned in the final section. (As may be anticipated, the most relevant variables are also known as "cues" to distance.)

Mediation by R mm. Equations stating that the perceived visual angle value, V′ deg, becomes a direct function of V deg by way of the retinal size value, R mm, are obtained by inserting the optical rule, R/n = V rad into Equation 2 to yield both $V' = m'R/n$ rad and $S'/D' = m'R/n$ rad. For example, if m′ is 1.0, a retinal separation of 1.7 mm specifies both an angular response of V′ = 5.73 deg and a linear response relation of D′ = 10 S′ m.

Distal/proximal. All response values are "distally focused" (Brunswik, 1944; Gibson, 1950, 1952, 1979; Hochberg, 1974; Koffka, 1935). As Gogel (1969, 1971) has insisted, there is no "proximal" response value that would be called "perceived retinal size," R′ mm. But the universal rejection of use of R′ mm does not apply to V′ deg, which properly replaces those perceived linear size (S′) concepts sometimes called "perceived proximal size" or "an extent in the visual field."[2]

V channels. The classic assumption that V deg has no perceptual correlate (Gogel, 1969, 1971; Helmholtz, 1910/1962; Koffka, 1935) is also being rejected in the current literature by use of hypothetical constructs called "size detectors," "spatial frequency detectors," or "size channels" in the visual system (Blakemore, Nachmias, & Sutton, 1970; Campbell & Maffei, 1970). To promote clarity, such constructs could be called "visual angle detectors" or "V channels" in order to avoid any implication that there might be "linear-size detectors" or "S channels."

Limits to m′

Although the visual-angle misperceptions in classic illusions have created great problems for theories, it is very important to keep in mind that they are quite small and apparently limited in two ways, as follows.

Difference limit. The difference between V′ and V rarely exceeds ½ deg for a target in the central field of view: Indeed, in Ebbinghaus illusion data, even the difference between V_2' and V_1' usually is less than 0.4 deg.

Magnification limits. Moreover, for the equal target extents in geometrical illusions such as Figure 2, even the "relative" ratio value V_2'/V_1' typically lies between only 0.7 and 1.3 (Rock, Shallo, & Schwartz, 1978; Wenderoth, 1976). Direct measures of the response magnification value, $m' = V'/V$, are lacking; but it might be as small as 0.5 in accommodation-convergence micropsia (Biersdorf, 1966; Heinemann, Tulving, & Nachmias, 1959; Komoda & Ono, 1974; McCready, 1965; Ono, Muter, & Mitson, 1974). On the other hand, the largest m' values (perhaps up to 1.8) might exist for very distant targets in the horizon direction outdoors, as suggested by moon illusions (McCready, 1983; Restle, 1970; Rock & Kaufman, 1962): At any rate, an m' value as great as 2.0 is quite unlikely.[3]

DESCRIPTIONS

Descriptions using Equation 2 are clearer if the equation is split into Equations 3A and 3B, which *together* describe an immediate experience:

$$V' = m'V \text{ rad} \quad (3A)$$

$$S'/D' = m'(S/D) \text{ rad} \quad (3B)$$

Two Types of Illusions

All response values might be veridical, but illusions are ubiquitous. The present approach reveals two types: SD illusions, in which $m' = 1.0$, and VSD illusions, in which m' is reliably greater or less than 1.0.

SD illusions. In SD illusions, because $V' = V$ deg, the ratio value S'/D' rad also is veridical, but both S' and D' are nonveridical. For example, assume the diameter (S) of this printed circle, O, is 2 mm, and let the viewing distance (D) be 40 cm, so that $V = 0.005$ rad. If $V' = V$, then S'/D' also is 0.005 rad. Now, to illustrate pictorial perception (Gibson, 1971, 1979; Haber, 1980; Hagen, 1974), this circle O may appear to be a pictured baseball, so S' is 74 mm and D' becomes 14.8 m.

Figure 3 describes some possible SD illusions for the rare observers whose V' values are accurate for Figure 2.

VSD illusions. In VSD illusions, V' does not equal V, so the ratio S'/D' does not equal S/D. For most readers, the V' value for at least one of the central circles in Figure 2 does not equal the V value, so, as presently discussed, the ratio S'/D' rad must be incorrect. Although most classic illusions are VSD illusions, the V' values have rarely been obtained, so the m' values generally remain unknown. Research has focused instead on target comparisons, as discussed below.

RELATIVE OUTCOMES

Equation 4 describes comparisons of two targets or of one target seen at times 1 and 2.

$$(S_2'/D_2')/(S_1'/D_1') = V_2'/V_1' = M'(V_2/V_1). \quad (4)$$

The variable M' can be seen to be the ratio of the two response magnifications $[M' = m_2'/m_1' = (V_2'/V_2)(V_1/V_1')]$. It may be called perceptual magnification.[4]

For describing a momentary comparison, it helps to divide Equation 4 into Equations 5A and 5B:

$$V_2'/V_1' = M' (V_2/V_1) \quad (5A)$$

$$(S_2'/S_1')(D_1'/D_2') = M'(V_2/V_1) \quad (5B)$$

Equality Outcomes

Examples often emphasize the perceptually economical equidistance and equisize (size-constancy) outcomes.

Equidistance. Two targets may look equidistant, or a target may appear to be the same distance away at times 1 and 2. In models of "relative" visual processing (Gogel, 1977), an *initial* scaling of $D_2'=D_1'$ may be attributed to equidistance cuing, to equidistance hypothesizing, or to an "equidistance tendency" (Gogel, 1965). A corollary was Gibson's "visual field" concept, which he virtually abandoned (Gibson, 1979, p. 286) because it became misinterpreted to be an illusory surface at an unspecified distance (see Note 2). It was an unnecessary substitute for use of projective geometry to describe v values in the visual world (Johansson, 1977; Taylor, 1962).

Equisize or size constancy. Two extents may appear to be the same linear size. Or, to unequivocally define perceptual size constancy, an extent may appear to have the same linear size at times 1 and 2. Initial scaling of $S_2' = S_1'$ may be attributed to equisize cuing, to equisize hypothesizing (Ittelson, 1960), or to an equisize tendency. As a corollary of the adult assumption of identity constancy (Day & McKenzie, 1977), the size-constancy assumption, rather than the equidistance assumption, dominates everyday perception of objects when their movements or observer movements alter the viewing distances and visual angles. But, in general, this size-constancy tendency complements the equidistance tendency, as the following examples indicate.

Relative SD Illusions

For these two circles, oO, let the visual angle ratio be $V_2/V_1 = 2.0$, and assume it is accurately perceived, so $M' = 1.0$. True relative perceptions and relative SD illusions thus are described by Equations 6A and 6B:

$$V_2'/V_1' = 2.0 \quad (6A)$$

$$S_2'/S_1' = 2.0(D_2'/D_1') \quad (6B)$$

For an equidistance example, let circle o appear to be a pictured ping-pong ball ($S' = 37$ mm) with circle O again a pictured baseball ($S' = 74$ mm). The complete report is that the second ball looks twice wider angularly, twice wider linearly, and at the same distance as the first one.

For an equisize example, let both o and O appear to be pictured baseballs, so because the second one looks twice wider angularly, it looks half as far away as the first one.[5]

Now reconsider conventional descriptions and problems.

Conventional Treatments

Standard descriptions use the SDIH as Equation 7.

$$(S_2'/S_1')(D_1'/D_2') = V_2/V_1 \quad \text{(the SDIH)}. \quad (7)$$

It can be recognized to be Equation 5B with $M' = 1.0$. Thus, conventional approaches assume that all visual angle ratios are correctly perceived and all illusions are SD illusions. That error sets the stage for paradoxes.

Basic paradox. When asked for a "size" comparison of two targets, observers often discover both the S comparison and the V comparison (Joynson, 1949).[6] A complete comparative report of course includes two incomplete "size" and distance reports, the "S&D" report and the "V&D" report. If the V&D report is illogically entered into Equation 7, as if it were an S&D report, it creates the paradox Kilpatrick and Ittelson (1953) exposed and which I will call the "basic paradox" of the SDIH.

For example, SDIH descriptions for the pattern oO are limited to the rule, $S_2'/S_1' = 2.0\ D_2'/D_1'$. Thus, for all pictorial size-constancy outcomes, if the correct V&D report that O "looks twice wider and half as far away" as o is mistakenly entered, it unbalances the equation. That equation also applies to an object that approaches halfway toward the eye; so the "looming" experience (cf. Hershenson, 1982), which often *includes* size constancy, seems paradoxical if the researcher misinterprets the correct V&D report "looks larger and closer" to be an S&D report.

The quite different size-distance paradox occurs with relative VSD illusions, as discussed below.

RELATIVE VSD ILLUSIONS

Ebbinghaus Illusion Again

The Figure 2 illusion example with $M' = 1.2$ is described by Equations 8A and 8B:

$$V_2'/V_1' = 1.2 \quad (8A)$$

$$(S_2'/S_1')(D_1'/D_2') = 1.2 \quad (8B)$$

Equation 8A suggests that the angle of a ballistic eye saccade across a diameter of circle 2 will be about 20% greater than it will be for circle 1. To my knowledge, saccades have not been measured for the Ebbinghaus illusion; but they were measured for the Müller-Lyer illusion by Yarbus (1967) and by Festinger, White, and Allyn (1968), who obtained M' values as large as 1.3. (They also found an illusion decrement, which, if it occurred for Figure 2, means that repeated glances among the many contours would make the central circles look more alike: an M' value of 1.2 would drift toward 1.0, but not reach it.)

Outcomes. As mentioned earlier, in equidistance outcomes (say both circles appear on the page), the second circle looks 20% linearly wider because it looks 20% angularly wider. And in a size-constancy pictorial outcome ($S_2' = S_1'$), because the second one looks 20% angularly wider, it looks closer, at about 8/10ths the distance of the first one. Many other outcomes, of course, can jointly fit Equations 8A and 8B.

Absolute VSD Illusions

An absolute VSD illusion is occurring for one or both central circles in Figure 2. For the sake of an example, let the perceptual magnification value, $M' = m_2'/m_1' = 1.2$, be divided equally between the two m' values, so $m_1' = 0.9$ and $m_2' = 1.1$, as described in Figure 4.

For circle 1, with $V_1' = 0.9\ V$, Figure 4A illustrates two simple outcomes: In a veridical distance outcome ($D_1' = D$), circle 1 looks 10% linearly narrower than its actual size ($S_1' = 0.9\ S$). In a veridical linear size outcome ($S_1' = S$), it looks 10% farther than it really is ($D_1' = 1.1\ D$). For circle 2, with $V_2' = 1.1\ V$, as shown in Figure 4B, a veridical distance outcome ($D_2' = D$) yields $S_2' = 1.1\ S$, and a veridical linear size outcome ($S_2' = S$) gives $D_2' = 0.9\ D$.

It is important to recognize the enormous difference between Figures 3 and 4.

Illustrations. Very few published diagrams show perceived visual angle (V') values, as do Figures 3 and 4. Even rarer are diagrams like Figure 4 which show V' values unequal to the V values (McCready, 1965): Dia-

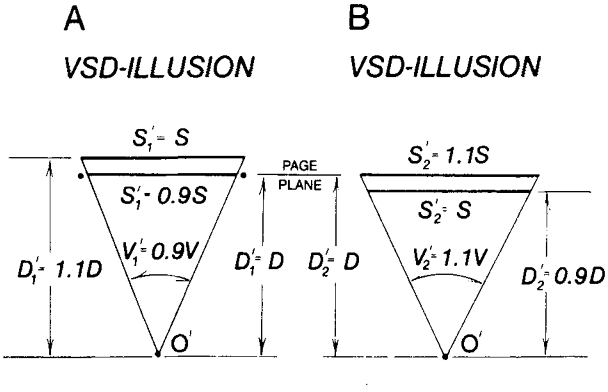

Figure 4. Top views show possible responses to the central circles in Figure 2 for typical observers who misperceive circle 2 to be angularly wider than circle 1, say by 20%, so $V_2' = 1.2\ V_1'$ deg. Diagram A describes two VSD illusions for circle 1, with V_1' arbitrarily 10% less than V; and diagram B describes two VSD illusions for circle 2 with V_2' 10% greater than V. Together, A and B describe several relative VSD illusions. The most common outcome probably is that both circles appear on the page plane ($D_2' = D_1'$), so circle 2 looks 20% linearly wider than circle 1 ($S_2' = 1.2\ S_1'$) because it looks 20% angularly wider. Note the great difference between this figure and Figure 3.

grams like Figure 4 for other classic illusions (McCready, 1983) remain unpublished.[7] Meanwhile, the widely published plan-view diagrams, which purport to "explain" those VSD illusions, instead resemble only Figure 3, so they obviously fail to describe most observers' experiences and thus propagate the size-distance paradoxes.

Size-Distance Paradoxes

A size-distance paradox (SD paradox) always occurs when an attempt is made to describe a relative VSD illusion with Equation 7 or with a diagram like Figure 3. For example, for the Ebbinghaus illusion, Equation 7 is Equation 9:

$$S_2'/S_1' = D_2'/D_1' \quad \text{(the SDIH).} \tag{9}$$

Simple SD paradox. As previously noted, most observers' S&D reports for Figure 2 unbalance Equation 9. The samples are: "looks larger [linearly] and equidistant" and "looks the same size [linearly] and closer."

Complex SD paradox. Also, the majority V&D reports unbalance Equation 9 if illogically entered. Examples are: "looks larger [angularly] and equidistant" and "looks larger [angularly] and closer."

For many such illusions with equidistant equiangular targets, the reports that the "larger looking" target 2 looks the same distance away or closer than the "smaller looking" target 1 captured attention because they seemed to contradict the perspective explanation, which uses Equation 9 and proposes that S_2' exceeds S_1' *because* target 2 either appears farther than target 1 or "unconsciously registers" as farther (Coren & Girgus, 1977, 1978; Day, 1972; Gregory, 1963, 1970, 1975; O'Leary & Wallach, 1980; Rock & Kaufman, 1962; Woodworth & Schlosberg, 1954). That proposal does not describe the slightly unequal V′ values, but it does explain the very unequal S′ values in the important pictorial illusions discussed below.

PERSPECTIVE ILLUSION

Figure 2 fairly represents all illusions in which extent 2 looks larger than angularly equal extent 1, *and* the context patterns logically can be said to indicate that pictured extent 2 is more distant than pictured extent 1 (Rock et al., 1978). For example, assume that the six large satellite circles correctly look three times angularly wider than the six small ones. A linear perspective outcome begins with all 12 satellite circles' appearing to be pictured disks of the same linear diameter so that the second set of six looks three times farther away than the first set, to illustrate "misapplied (or inappropriate) size-constancy scaling" (Day, 1972; Gregory, 1963, 1970). If each central disk now looks in the same plane as its satellites, then disk 2 looks three times farther than disk 1, and because their visual angles look approximately equal, disk 2 looks about three times linearly wider than disk 1. However, as discussed below, that large linear size illusion has not been the one studied.

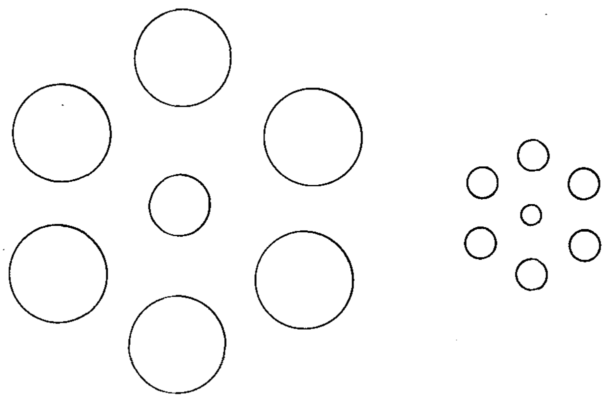

Figure 5. A "size"-matching-response pattern expected from a research participant who suffers a full-perspective outcome for Figure 2 and sees pictured disks with central disk 2 looking about three times farther away than central disk 1 ($D_2' = 3\ D_1'$). Thus, if the instruction to make the two targets "look the same size" is interpreted to mean that those pictured disks should look the same linear width ($S_2' = S_1'$), then circle 2 necessarily is adjusted to make the visual angle of disk 2 look about one-third the visual angle of disk 1 ($V_2' = V_1'/3$).

Linear size illusion. First consider the relatively few observers who correctly perceive that the central disks in Figure 2 subtend equal visual angles. Their perspective illusion is a relative SD illusion, described as $V_2' = V_1'$, $D_2' = 3.0\ D_1'$, and $S_2' = 3.0\ S_1'$, for a 300% illusion. However, for most observers, the perspective illusion instead is a relative VSD illusion: For the example with $M' = 1.2$, the description begins with $V_2' = 1.2\ V_1'$, so, if $D_2' = 3.0\ D_1'$, then $S_2' = 3.6\ S_1'$ for 360% illusion.

In either case, if research participants thought the instruction to adjust circle 2 to appear the same "size" as circle 1 meant that the two pictured disks in the perspective outcome should look linearly equal, then they would reduce circle 2 to about one-third its present diameter, as shown in Figure 5, and the illusion magnitude would be recorded as about 300%. (This size-constancy match is approximately described as $V_2' = V_1'/3$, $D_2' = 3.0\ D_1'$, and $S_2' = S_1'$.)

Participants in studies of the perspective illusions have rarely produced results like Figure 5: They, instead, reduced the "larger looking" extent by only about 20%. The full perspective illusion thus has not been the interesting illusion.

To deal with this matter, Gregory (1963), using the SDIH (Equation 9), proposed that the perspective "cues" not only cause (as "secondary scaling") the large S′ illusion (300%) for the target extents, but also cause a "primary scaling" of S′, which is the smaller illusion (20%). But Equation 9 and Figure 3 demand that if "primary scaling" makes S_2' 20% greater than S_1', then target 2 also must look 20% farther away than target 1. But it does not. An SD paradox remains, and the misperceptions of V values still need to be explained.

EXPLAINING VSD ILLUSIONS

A thorough discussion of explanations for shifts of V′ deg away from V deg is beyond the scope of this article. It is useful, however, to mention the two types of explanation that remain after possible optical distortions of retinal images have been either ruled out or taken into account. The most familiar one appeals to contour (neural) interactions; the other appeals to oculomotor factors concerned with accommodation and convergence responses.

Contour (Neural) Interactions

Over (1968) concluded that geometrical illusions, such as Figure 2, which illustrate "size contrast" (Restle, 1971, 1978) and "size assimilation" (Pressey, 1972), are best *described* as contour interaction illusions, which, in turn, are being *explained* in terms of mutually interacting activities of neighboring neurons in the visual system (Fisher, 1971; Ganz, 1966a, 1966b; Oyama, 1977b). This category also includes the successive "size-contrast" illusions currently being described in terms of *adaptation* of "spatial frequency detectors" (V channels) in the visual system (Anstis, 1974; Blakemore et al., 1970; Blakemore & Sutton, 1969). Finally, this category also encompasses "perspective" illusions, such as Figure 2, in which the interacting contours coincidentally form linear-perspective and texture-gradient patterns.

Equations that specify V′ distortions for the target extent as a function of the V values in the texture gradient pattern near it include those offered by Restle (1971, 1978; Restle & Merryman, 1968) based upon Helson's adaptation-level theory. Some other equations are based, instead, on presumed interactions among neurons (Ganz, 1966b; Walker, 1973).

In many two-dimensional illusion patterns, such as Figure 2, the texture gradient pattern of larger V values near target 1 than near target 2 of course correlates with the texture gradient pattern present in natural viewing when object 1 is closer than object 2. Therefore, the result that m_1' is slightly less than m_2' for those targets in two-dimensional patterns parallels the common finding that, for three-dimensional arrangements, the value m′ is slightly less for a near object than it is for a far one (Joynson, 1949). This natural VSD illusion is the basis of the second type of explanation.

Accommodation-Convergence Minification

A ubiquitous VSD illusion is accommodation-convergence minification (AC minification), in which the value m′ = V′/V decreases when the oculomotor efference (AC efference) pattern directs the eyes to focus and converge to a closer point (Biersdorf, 1966; Heinemann, et al., 1959; Komoda & Ono, 1974; McCready, 1965, 1983; Ono, 1970; Ono et al., 1974; Richards, 1967, 1971). Conversely, as AC magnification, m′ increases with the changes in AC efference that will adjust the eyes to a farther point. In other words, for an object subtending a constant visual angle, V, an attempt to converge the eyes to a lesser (greater) distance slightly decreases (increases) the perceived visual angle value, V′ deg, whether or not that AC efference effects eye movements. The resultant VSD illusions generate S&D and V&D reports which, of course, have been classic examples of size-distance paradoxes.

Purpose of AC minification. I have proposed that for most observers AC minification is a purposeful *perceptual adaptation*, which in natural viewing makes the visual direction difference value, v, a more accurate predictor of the head-rotation angle than of the eye-rotation angle needed to orient from one nearby point to another. Some type of perceptual-motor correction is necessary because various head-turning axes lie from 10 to 20 cm posterior to axes for eye rotations (Roelofs, 1959). Consider an example.

For an average interaxis distance of 15 cm, a frontal target at D = 15 cm from the eye is about 30 cm from a head-rotation axis: Therefore, although the eye saccade that will shift fixation from one target edge to the other essentially equals the visual angle value, V deg, the head-rotation angle that will aim the nose, hence both eyes and both ears, squarely from one edge to the other must be half the visual-angle value. Consequently, if the visual-direction-difference value, v, equals V deg, it will gauge a head rotation of V′ = V deg, twice as great as needed. However, if, instead, v has been perceptually reduced to equal V/2, then it will gauge an accurate head-rotation value of V′ = V/2 deg. And, to be consistent, that minified v value also will gauge an eye rotation of V′ = V/2 deg, which will be too small. In either case, the response value, V′ = V/2, of course defines the perceived visual angle and an illusion with m′ = 0.5. Presumably, after several erroneous saccades and consequent corrective saccades among contours at that target distance of 15 cm, V′ values will become more accurate: that is, as the illusion decrement, m′ will approach 1.0. Then, if AC efference directs the eyes to a great distance, m′ increases (as AC magnification) and might momentarily exceed 1.0, until saccades among contours at the great distance restore m′ to 1.0.

For an average interaxis distance of 15 cm, the basic general equation (McCready, 1965) may be stated as,

$$m' = V'/V = D_c'/(D_c' + 15 \text{ cm}),$$

in which D_c' cm is the distance to which the eyes have been accommodated and converged. I still find that this rationally derived equation fits much published data better than do empirically derived equations. (It must be modified to take the illusion decrement into account.)

In later papers, I hope to present those data analyses and also to elaborate the additional proposal (McCready, 1983) that although the V′ distortions in flat patterns, such as Figure 2, may be caused by neural (contour) interactions, they also may be a *conditioned* AC-minification response evoked by the texture-gradient patterns of V values acting as conditioned stimuli.

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NOTES

1. The adjective "apparent" may still confuse terminology because in astronomy it denotes an angular measure—the "apparent diameter" of the moon is V = ½ deg, and its linear diameter is S = 3,475 km—but in psychology "apparent" means "perceived." In present terms, a report that the moon looks 100 m wide is recorded as a perceived (or apparent) linear size value, S′ = 100 m, and an additional report that its outer edges appear directionally different by ½ deg is recorded as a perceived (or apparent) visual angle value, V′ = 0.5 deg. Technically speaking, an astronomer could call V′ the "perceived apparent size" or even the "*apparent* apparent size" without being redundant.

Although the visual-angle symbol usually is a Greek letter, the letter V is more consistent with use of the letters S and D, and, as a mnemonic aid, V itself is an angle symbol.

I now use primed symbols in the accepted manner, rather than in my earlier confusing way (McCready, 1965).

2. It is instructive to notice that, with m′ close to 1.0, the rule S′/D′ = m′R/n rad specifies that the perceived linear size of any target will equal its retinal image's size (S′ = R mm) if that target illusorily looks only about 17 mm in front of the eye, so that D′=n. (The SDIH predicts the same thing.) Perhaps terms such as "perceived proximal (retinal) size" and "extent in the visual field" have been referring to that illusory value, S′ = R mm: If so, no mention has been made of the very short "perceived proximal distance" of the target and visual field.

3. An m′ value as large as 2.0 is illogical (at least for V values larger than ½ deg in the central field of view). For example, consider what would happen if m′ were 2.0 when one tried to look from a point, A, to another point, B (as indicated in Figure 1). The visual angle, V deg, from point A to point B would look twice its actual value (V′=2V); therefore one's saccade to the right would overshoot point B by V deg. Consequently, with m′ still 2.0, point B would now appear 2V deg to the left of the fixation point. So, in the renewed attempt to look at point B, one's "corrective" saccade to the left would be an eye rotation of 2V deg, which would aim the eye back upon point A. Obviously, m′ must be less than 2.0 if one is going to look from any point to another quickly.

4. The ratio of the visual angles, V_2/V_1, may be called ocular magnification, M (which term appropriately fits typical designations of magnification for optical devices). The ratio of the perceived visual angles, V_2'/V_1', may be called perceived magnification, M″. The perceptual magnification value, M′, thus expresses the degree to which, as an illusion, the perceived magnification (M″) differs from the ocular magnification (M). The three angle ratios in Equation 4 thus relate as M″ = M′M.

5. Size-constancy scaling for oO initiates what traditionally has been called "cuing" of perceived distance by "relative size," linear perspective, or a texture gradient (Gibson, 1950, 1979; Gogel, 1977). That is, as the equation $(S_2'/S_1')(D_1'/D_2') = V_2'/V_1'$ indicates, if S_2' equals S_1', then the ratio value D_1'/D_2' is specified by the ratio value V_2'/V_1'.

It is important to notice that the equivocal term "relative size" here obviously cannot be referring to the equal perceived linear size (S′) values. It somehow refers to the ratio of the V′ values. Indeed, in the final analysis, "relative size" clearly refers to the *absolute* perceived visual angle value (V_2' deg) for target 2 *in units of* the absolute perceived visual angle value (V_1' deg) for target 1. In other words, in standard discussions, the undefined variable "relative size" has been substituted for the *perceptual* visual angle variable, v (or V′ deg), which in those same discussions is said not to exist.

6. Most observers have not learned appropriate adjectives with which to distinguish between their "angular size" (direction difference) and "linear size" comparisons, so they usually use different verbs instead. For example, it is easy to verify that observers comparing a distant house to the nearby window through which it is seen typically say either "the house *looks* smaller than the window, but I know it is larger" (with vocal emphasis on "looks"), or "it *looks* smaller, but if I take distance into account, it appears larger" (cf. Joynson, 1949; Rock, 1977). The popular choice of the verb "look" for reporting about the visual direction difference, v, indicates its perceptual primacy; and the choice of "know" (or a reference to "taking d into account") for reporting the linear size experience, s, reflects its status as a secondary perceptual value.

7. Outside of my classes, illustrations like Figure 4 for moon illusions and other illusions were used in lectures titled "The moon illusion problem" presented to colloquia at the University of Chicago (1964), Marquette University (1968), Lawrence University (1970), and the University of Wisconsin-Whitewater (1970, 1981). They also appear in unpublished manuscripts (and revisions) submitted for publication in 1965, 1981, 1982a, 1982b, 1983, and 1984.

APPENDIX

Memory Values and Visual Processing

The list of s=dv theory concepts is completed below, and then visual processing is discussed.

Memory Values

Two sets of memory variables are: the remembered values and cognitive values (McCready, 1965, 1983).

Remembered values. The remembered values, s′, d′, and v′, evidently may be coded both visually and verbally. Retrieval of visually coded values exemplifies visual imagery (Kosslyn, Ball, & Reiser, 1978; Pinker, 1980). Verbal coding of the linear values, s and d, is relatively easy, but most of us have difficulty coding a visual-direction difference value, v, verbally, presumably because we usually communicate v values haptically, with sweeps of our eyes, head, or hands (Ono, 1966, 1970). Komoda, Festinger, and Sherry (1977) used eye saccades to measure v′ values for points that had just been removed from view. Atteneave and Pierce's (1978) observers rotated an unseen pointer to indicate v′ values for objects they had just seen; they produced V′ values as accurate as those given for targets in view. Shebilske (1977) has reviewed studies of remembered visual directions (which instead usually are v′ values).

Cognitive values. Frequently remembered values may evolve into the cognitive values, s″, d″, and v″, usually called "familiar" or "known" values. The cognitive linear values may be "cues," which means that a present s or d value may be valuated (scaled) by acquiring a cognitive value.

For mobile people, few earthly extents subtend visual angles constant enough to remember. But, for a target that happens to subtend a constant visual angle, such as the moon, sun, or a constellation, its "familiar width" undoubtedly is the cognitive visual angle value, v″.

Consider now the valuation of phenomenal values.

Visual Processing

Visual processing traditionally is described as being analogous to solving the theory equation, s/d = R/n = V rad, from which the SDIH derives (Epstein, 1977a, 1977b; Gogel, 1977; Oyama, 1977a). In logical two-step processing models, the stimulus value V (actually as R, of course) for a target is given to "visual processing." Then either d or s becomes *scaled*, as described below, and, next, by an "unconscious calculation" (Helmholtz, 1910/1962; Rock 1977), or else by a "taking-into-account" process (Epstein, 1973, 1977a), the other value is *computed*.

Scaling. Scaling of d may occur by cuing, which involves "absolute distance cues" (Gogel, 1977). Also, d may be scaled by hypothesizing, a form of visual imagery in which the target appears at an assumed, presumed, or suggested distance (Gibson, 1950, 1952; Ittelson, 1960; Singer et al., 1982), either in the absence of distance cues or in spite of them. Once d is scaled, s becomes computed to be the value required by s=dV.

Scaling of s may occur by linear size cuing, which involves cues such as "familiar or known size" (Bolles & Bailey, 1956; Fitzpatrick, Pasnak, & Tyer, 1982; O'Leary & Wallach, 1980; Ono, 1969), or cues such as a felt object's phenomenal haptic size (Coltheart, 1969). Or s may be scaled by hypothesizing, in which the target appears an assumed, presumed, or suggested linear size (Coltheart, 1970; Hastorf, 1950; Ittelson, 1960). Once s is scaled, d becomes computed to be the value required by $d=s/V$.

That is, although d or s can be scaled to be almost any value, the computing step (or its logical equivalent) prevents conflicting independent scalings of d and s from producing an s/d ratio inconsistent with V rad.

Direct perception. Advocates of direct perception insist there is no computing step: The values s and d are scaled concurrently *and* they conform to the rule $s/d=V$, which, in natural viewing, has always paralleled the optical law, $S/D = V$ rad (Gibson, 1950, 1979; Hochberg, 1974).

The s=dv Theory Model

The present model of processing (McCready, 1965, 1983) differs from standard models by using v in place of V rad: That is, the ratio s/d is kept equal not to the stimulus value V rad, but to the subjective value v. The appropriate analog is not $s/d = kR/n = kV$ rad, but $s/d = v = mR/n = mV$ phenomenal rad. The variable m here is the ratio value, v/V, and thus may be called "phenomenal magnification." The response measure of m is the response magnification value $m' = V'/V$, in the psychophysical equation, $S'/D' = V' = m'V$ rad, which is used for describing data.

Contrary to the size-distance invariance hypothesis (stated either as $s/d = kV$ or as $S'/D' = kV$), the present hypothesis is that the ratio s/d is an invariant function of v (stated as $s/d = v$), but it is a noninvariant function of V rad (stated as $s/d = mV$).

Preprocessing. In standard accounts, the term "visual processing" invariably refers only to scaling and computing of s and d. To avoid terminological confusion, the valuation of v by R mm therefore will be called a preprocessing step. That is to say, the valuation of v by R mm should not be called "scaling" because v is not "cued," and for targets forming retinal images, an observer cannot alter the v values by assuming they are otherwise. Nor is v "computed," as if an equation, $v=s/d$, were being solved with obtained s and d values. Of course, knowing the optical rule, $V = S/D$ rad, a geometrically astute research participant could covertly use his or her verbal linear response estimates, S′ m and D′ m, to calculate the angle value, V′ rad, which he or she presumes is the value the experimenter expects (Carlson, 1977). However, in terms of the s=dv theory, that learned analytic "cognitive" calculation of the verbal response value V′ deg is not an appropriate analog for the "preprocessing" valuation of v or V′ from R mm.

(Manuscript received June 16, 1983;
revision accepted for publication February 27, 1985.)