Zöllner illusion as perceptual enlargement of acute angle*

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The magnitude of the Zöllner illusion varies according to the angle of intersect between the test and inducing lines and is maximal at about 20 deg of intersect angle. Increasing the density of the inducing lines multiplies the illusion by a constant amount. Obtained error functions agree well with error functions derived according to Piaget's law of relative centrations.

A feature of the geometrical illusions is the occurrence of a maximal effect when there is a certain separation between the test and inducing contours. In particularly, this characteristic has been described by Wallace & Crampin (1969) for the version of the Zöllner illusion shown in Fig. 1. For a given density (d) of the inducing lines, Wallace and Crampin found that the illusion, as a function of the angle (θ) between the inducing and test lines, was maximal at about 15 to 20 deg. They also determined the error function of two different values of d, and concluded that "patterns of different density yield functions of the same shape simply multiplied by a constant related to the pattern density [p. 174]." Thus the magnitude of illusion can be expressed as

$$P = abf(\theta) + c \qquad (1)$$

where a, b, and c are constants, and b is specifically related to d. When d is altered, b will change. This relationship will be examined with respect to the law of relative centrations formulated by Piaget (1969).

Piaget derived his centration law from a probabilistic consideration of the manner in which the S attends to different parts of a stimulus figure. The law essentially expresses the amount of illusion as a function of the size relationships of the stimulus figure and predicts relative values for the qualitative characteristics of illusion functions. One instance of the law concerns the perceptual enlargement or overestimation of an acute angle (Wundt, 1896; Berliner & Berliner, 1948; Fisher, 1969). The overestimation of an acute angle (θ) is given by

$$P = 2K \operatorname{Sin} \theta (\operatorname{Cos} \theta - \operatorname{Sin} \theta) \qquad (2)$$

where K is a constant (Piaget, 1969, p. 20).

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In the total figure, the effect of the angle will increase as the density of the background inducing lines increases. If d is the perpendicular distance between adjacent inducing lines and T is the length of a test line, then this increase will be inversely proportional to d and directly proportional to T. Therefore, the constant b of Eq. 1 can be replaced by kT/d. In addition, part of the test line is "confused" with the inducing lines, so that Eq. 1 applies to the proportion not confused. When the line width is w, this proportion is approximately $(1 - 2w/d)$ (Wallace & Crampin, 1969). Thus, $k = (1 - 2w/d)$. Putting K = ab and substituting for K in Eq. 2, the total illusion produced by the Zöllner figure becomes

$$P = 2a(T/d)(1 - 2w/d)$$
$$\operatorname{Sin} \theta (\operatorname{Cos} \theta - \operatorname{Sin} \theta) + c. \qquad (3)$$

The constant, c, is added according to Eq. 1 and a and c can be determined empirically.

The factor $\operatorname{Sin} \theta (\operatorname{Cos} \theta - \operatorname{Sin} \theta)$ is what Piaget (1969) refers to as the "difference coupling." According to Piaget, the coupling of correspondences between the visual system and one line and between the visual system and another line, is incomplete when one line exhibits a greater density of correspondences than the other. If there is a systematic reason for one of the lines to be more frequently centered (or fixated) than the other, then the couplings remain incomplete, and the line length is overestimated. Thus, the difference coupling is an expression of the "perceptual contrast" existing between two lines. In these terms, the perceptual enlargement of an acute angle is a consequence of the overestimation of the distance between the sides adjacent to the angle, this distance being more frequently fixated than any other.

In the present experiment, the magnitude of the Zöllner illusion was measured as a function of the angle between the test and inducing lines for a number of different background densities, and the obtained error functions were compared with the distributions of error generated by Eq. 3.

METHOD

The apparatus and stimuli were similar to those used by Wallace & Crampin (1969). The 50 stimulus figures were black ink drawings on white card. Each figure comprised a series of 1-mm-wide background lines spaced at one of five distances apart (d = 5, 15, 30, 50, 80 mm) and inclined at 1 of 10 angles to the horizontal (θ = 5 to 50 deg in 5-deg steps). The circular stimulus field was 15.5 cm in diam. (Thus, for example, the pattern with density of 80 mm consisted of a single pair of inducing lines.) Two 1-mm-wide test lines were superimposed on the background and could be varied in tilt. The test lines were 40 mm apart at the fixed left-hand end, and adjustment of a screw by S could effect their simultaneous convergence or divergence. The luminance of the stimulus field was 34 mL.

Ten males and 10 females, with a mean age of 20 years, served as Ss. The seated S monocularly viewed the stimulus figure placed in the frontoparallel plane at a distance of 60 cm. Prior to each judgment, the test lines being obviously divergent, S was instructed to position the two lines so that they looked as though they were parallel. Following each

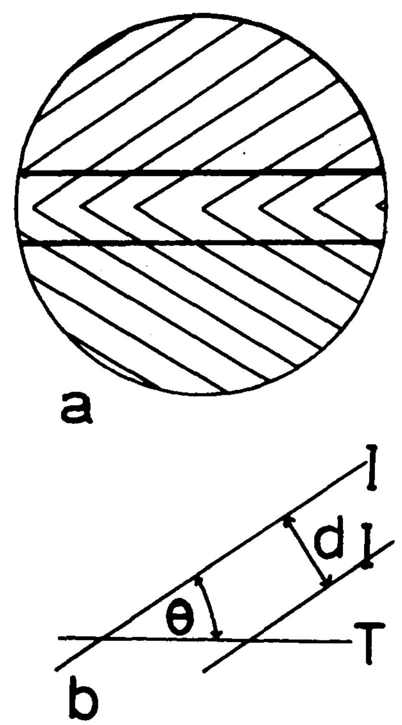

Fig. 1. Zöllner illusion (a), and its geometrical features (b). I and T are inducing and test lines, respectively.

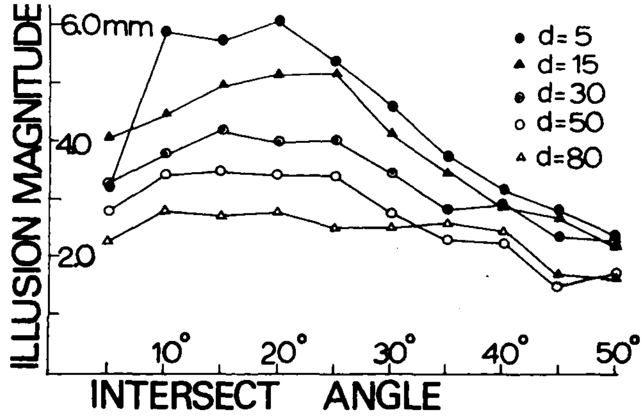

Fig. 2. Magnitude of the Zöllner illusion as a function of intersect angle and background density. Standard deviations average 0.43 of mean illusion magnitudes shown.

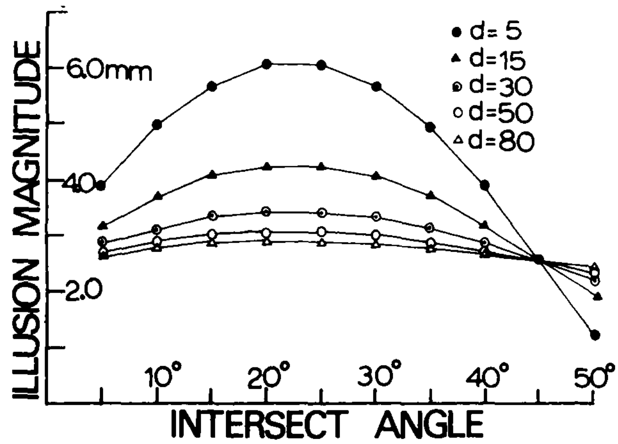

Fig. 3. Error functions for the Zöllner illusion derived according to Piaget's law of relative centrations (see Eq. 4).

judgment, the test lines were returned to the divergent position and a new background pattern was inserted. Each S made only one adjustment for each figure (50 judgments per S) and each adjustment required 8-10 sec. The order of presentation of the 50 stimulus displays was randomized for 10 Ss, these orders being reversed for the remaining 10. The magnitude of illusion was measured as the deviation of the upper test line from the position of parallel at the right of the figure.

RESULTS

Mean illusion magnitudes and their standard deviations were determined from the 20 estimates obtained for each background density, d, and each intersect angle, θ. These means, expressed in millimeters, are plotted in Fig. 2 as a function of d and θ. The illusion increases as d increases, and the error functions rise convexly from 5 to about 20 deg and then fall concavely to 50 deg. The data were submitted to an analysis of variance which showed the effects of θ [$F(9,950) = 35.27$, $p < .001$], d [$F(4,950) = 64.75$, $p < .001$], and their interaction [$F(36,950) = 2.18$, $p < .01$] to be significant.

In general, the functions have maxima at about 15 to 20 deg. When the frequencies of individual maxima were obtained for each θ at each d, a chi-square analysis (angles at which maxima occurred, by densities) indicated that the position of the maximum illusion did not shift as a function of the density of the inducing lines.

Predicted Functions

Since the present experiment measured only half of the illusion, the expression for illusion magnitude derived from the law of relative centrations is, as it applies to the present situation,

$$P = a(T/d)(1 - 2w/d) \sin\theta(\cos\theta - \sin\theta) + c \quad (4)$$

When a = 1, c can be found by using a single data point from Fig. 2. By substituting the value for illusion magnitude at d = 5 and θ = 20 deg for P in Eq. 4, c was calculated as 2.54. For a = 1, c = 2.54, w = 1, and T = 145, values of illusion magnitude as a function of intersect angle were obtained for each of d = 5, 15, 30, 50, and 80 mm. These error functions, derived according to Eq. 4, are shown in Fig. 3. When the obtained values for P (Fig. 2) were compared with the derived values (Fig. 3) using a chi-square goodness of fit test, nonsignificant test statistics ($p > .95$) indicated that in all cases the respective curves for the different pattern densities fitted very closely.

DISCUSSION

It is clear from the present experiment that three main features characterize the shape of the error functions for the Zöllner illusion. The magnitude of illusion increases with increasing pattern density, the maxima of the functions occur at about 20 deg, and the distributions of error are skewed towards the smaller values of θ. These data exhibit close similarity to those presented by Wallace & Crampin (1969).

The agreement between the obtained error functions and those predicted by Piaget's (1969) centrations law is close, despite its approximate nature. In particular, two qualitative predictions are confirmed. First, the law of relative centrations successfully predicts the occurrence of spatial maxima at about 20 deg angle of intersect. Second, the expression of the illusion primarily as a function of the angle between the inducing lines is confirmed by the present analysis. To this extent at least, the present data support a description of the Zöllner illusion in terms of the law of relative centrations.

The present analysis also reinforces the notion that Eq. 1 adequately describes the data and that the effect of background density is to multiply the overall illusion. Judgmental variables (such as the nature of instructions) which are known to affect the illusion magnitude would be accounted for by a change in the value given to the constants. The main variation in illusion can be related to variations in the size relationships of the stimulus display. Although the illusion in the Zöllner figure depends on the orientation of the total display, the effect of orientation might only alter the overall amount of illusion rather than change the general characteristics of the error function (Piaget, 1969). Unfortunately, there seems to be an absence of systematic investigation, of the orientation sensitivity of geometrical illusions.

It should be noted that the law of relative centrations, as applied to acute angles, essentially expresses a relation between lengths. According to Piaget (1969), the acute angle is overestimated as a consequence of the lengthened virtual median; presumably, the virtual median receives more centration or attention than other regions of the figure, but it is not clear why this should be so. The location of the virtual median along the bisector of the angle does not pose a particular problem for the centration law, since Eq. 3 predicts the illusion independently of the lengths of the sides adjacent to the angle. It can be predicted, therefore, that changing the length of the inducing lines should not alter the illusion magnitude. There is some support for this prediction (Wallace, 1969), although it seems that for very short lengths the illusion is reduced. (An obvious instance is the zero illusion when the inducing lines have zero length.) In the case of a single slanted line, perceived orientation is independent of the line length for lengths greater than about 1 deg of visual angle (Bouma & Andriessen, 1968).

The difficulty encountered by the centration theory in specifying the features of the stimuli which are responsible for controlling the S's attention illustrates the major weakness of the theory. Piaget (1969) does not elaborate on the possible "systematic reasons" for one region of a stimulus figure to receive more centration than another. As Over (1968) notes, the centration theory specifies attentional processes in terms of their consequences rather than in terms of their initiating conditions.

There is some evidence, however, that the centers of acute angles do act as a stimulus to fixation. Kaufman & Richards (1969) found a definite tendency for Ss to fixate the center of acute angles in relation to a 90-deg angle, especially for angles of 20 deg. Although the center of fixation may not always correspond to the focus of attention (Pritchard, 1958), the conventional view is that the perception determines the eye movement (Yarbus, 1967). It is likely that the scanning operations of the eye are related to central attentional processes and that the latter contribute to the perceptual distortion of acute angles.

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Word sorting and free recall*

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Using Mandler's word sorting task, the effect of a possible experimental artifact which might account for the relation between the number of categories used and recall was investigated. When Ss sort into more piles, more cards are exposed to view, giving Ss the opportunity to scan more words. If scanning is analogous to repetition of the individual words, increased recall could be due to increased exposure to the words rather than to the use of more categories. No difference in the category-recall function was obtained between Ss who were and who were not allowed to see the top cards in the piles. These results suggest that repetition per se is not the cause of increased recall with more categories.

Recently, Mandler (1967, 1968; Mandler & Pearlstone, 1966) introduced a new method for exploring the relationship between Ss' organization of a set of words and the number of those words they can then recall. Typically, S is required to sort cards, each bearing a single word, into piles (categories) of related items. He is allowed to sort the words into two to seven piles, using any category system he wishes other than such arbitrary criteria as the number of letters, initial letters of the words, etc. Successive trials are given until he reaches some criterion of consistency in his sorting. Experiments utilizing this task have consistently found strong positive linear relationships between the number of piles used in sorting (NC, or number of categories) and the number of words recalled (R).

Mandler's research appears to support the position that the way a person organizes information in learning influences his ability to recall it later. It also provides a method for further analysis of this relationship. However, it is important to make sure that the relationship obtained between NC and R is, in fact, caused by the effect of organization on recall and not due to some artifact of the experimental task. Mandler has examined several of these possible alternative explanations. He reports that Ss sorting into different numbers of piles do not differ in the number of trials or in sorting time required to reach criterion in the task and that the relationship is still obtained when number of trials and time are held statistically constant. He also obtains the relation between NC and R when Ss are assigned randomly the number of categories they are to use in sorting, thus excluding the possibility that individual differences produce this relation. In fact, a later study (Mandler, 1968) found the same relation in data obtained by repeated testing of individual Ss, varying the number of categories they were asked to use on different lists.

The present study was designed primarily to determine if another aspect of the sorting task, the number of words exposed to S at any one time during sorting, could account for the NC-R relationship. It also provides further data on the relation of sorting time to NC and R. In carrying out the sorting task, cards are placed face up on the piles. As S decides where to

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