Logical and empirical thinking in a problem solving task[1]

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Subjects can base a solution on either logical or empirical properties of a problem solving task. In this study, the difficulty of making a solution on empirical grounds and of making a solution on logical grounds were both varied. It appeared that as the strength of the empirical properties diminished, Ss utilized logical properties to a greater extent. Further, logical properties took precedence over empirical properties as the former became easier to explicate.

In most problem-solving tasks, S must overcome a tendency to use the problem materials in a common way, and find instead a novel, or uncommon, use for them, in order to achieve a solution. The common ways of using the materials may be called the empirical properties of the materials. They presumably arise from the knowledge, beliefs and opinions about the materials which S brings to the problem situation. In contrast, the problem materials may also possess logical properties. Logical properties consist of information from which S may deduce conclusions about the materials. These conclusions are not immediately obvious to him when first faced with the problem situation. What will S do when the response he would make on the basis of the empirical properties of the materials contradicts that which he would make on the basis of the implications of the logical properties?

This question is related to one which arises from the study of syllogistic reasoning. Henle (1962) has pointed out that Ss often make errors by basing conclusions on the empirical veracity of an argument, rather than on logical grounds. How are errors of this sort overcome? Under what conditions will S reason, instead of responding on the basis of an empirical judgment?

Length of inference has been shown to be a variable determining whether or not Ss will draw a conclusion from information they are given (Reid, 1951). A distinction can be made between specific and nonspecific logical properties—the former requiring relatively few deductive steps to reach a conclusion, the latter requiring relatively many.

It is hypothesized that specific logical properties should decrease the likelihood of S responding to the empirical properties of the problem materials. Conversely, nonspecific logical properties should increase the influence of the empirical properties. A corollary of this point of view is that when the problem materials do not have definite empirical properties, Ss should explicate the logical properties to a greater extent than when the problem materials do have definite empirical properties.

SUBJECTS

Subjects were 48 paid volunteers from Brandeis undergraduate psychology courses.

TASK

In order to test the above hypotheses, it was necessary to construct problem materials the empirical properties of which lead S to make an erroneous response. In this experiment a geometrical illusion, the Sander parallelogram (Woodworth & Schlosberg, 1954, p. 420), was used for this purpose. (See Fig. 1). The angle ADF = 50 deg; the height of the figure was 9/16 in., and the width was 1-11/16 in.

Now consider the following task. The S is asked to determine whether the sum of Lines PAEF is equal to, greater than, or less than the sum of Lines PCEF, where P is the midpoint of AC. (In the experiment, P was not shown; Ss were required to imagine the midpoint P). Because all components

look equal (PA = PC, EF is common) with the exception of AE, which appears longer than CE, although in fact they are equal, Ss will invariably report that AE is longer than CE, and thus that PAEF is longer than PCEF.

If S is told that angle CAE = angle ACE, he has two ways of making a judgment. Either he can rely on inspection (as illustrated above) in which case PAEF is longer than PCEF, or he can deduce that AE equals EC since they are opposite sides of an isoceles triangle. If he does the latter, he must conclude that PAEF = PCEF.

In the present experiment the strength of the illusion, and thus Ss' tendency to judge AE greater then CE, was varied by changing the angle ADF from 50 deg (the strong illusion—SI) to 85 deg (the weak illusion—WI).

The variable specific logical properties (Sp) vs nonspecific logical properties (NSp) was manipulated by varying the number of inferences necessary to conclude from the given information that AE = CE. Thus, S was told either that angle CAE = angle ACE (Sp), in which case he could infer that AE equals CE in two steps, or he was told that angle CAE = angle CEF (NSp). The latter case requires three inferences to draw the proper conclusion.

PROCEDURE

The two variables were cast in a 2 by 2 design. Six men and six women served as Ss in each of the four conditions.

Subjects were presented with either the strong or the weak illusion and asked: "Is the sum of PA plus AE plus EF equal to, greater than, or less than the sum of PC plus CE plus EF?" In addition, they were told either that angle CAE = angle ACE or that angle CAE = angle CEF. Ss were asked to "think aloud." From the protocols E determined whether each S's judgments were correct or not, and also how he made it—by inspection of the figure, or by deduction.

RESULTS

Proficiency

The proportions of Ss in each group who made the correct judgment of relative lengths of lines is given in Table 1. The statistical significance of the obtained differences was tested for the distribution as a whole by Cohen's (1967) test for the significance of the differences between proportions ($U'_0 = 14.76$, df = 3, p < .005) and for all pairs as shown in Table 2. These results show that a significantly greater proportion of Ss in Group SpWI solved the problem correctly than in Group NSpSI. No other differences were statistically significant.

Type of Judgment

Each S's judgments were scored as either perceptually or geometrically (logically) determined. This assessment was made without regard to the correctness or incorrectness of the judgment. A logically determined judgment was defined as one in which S gave geometrical reasons for his judgment, e.g.,
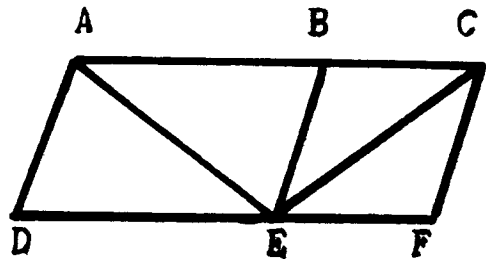

Fig. 1. Sander Parallelogram

Table 1
Proportion of Subjects in Each Group Making Correct Judgments

Group	NSpSI	NSpWI	SpSI	SpWI
Proportion Judging Correctly	.25	.58	.75	.92
N	12	12	12	12

Table 2
Difference Between Arcsin Transformed Proportions of Table 1

Group	NSpWI	SpSI	SpWI
NSpSI	.63	1.04	1.52*
NSpWI		.36	.84
SpSI			.48

*$p < .05$

Table 3
Proportion of Subjects in Each Group Judging Deductively

Group	NSpSI	NSpWI	SpSI	SpWI
Proportion Judging Deductively	.17	.75	.75	.83
N	12	12	12	12

Table 4
Differences Between Arcsin Transformed Proportions of Table 3

Group	NSpWI	SpSI	SpWI
NSpSI	1.24*	1.24*	1.44*
NSpWI		.00	.20
SpSI			.20

*$p < .05$

"Since those angles aren't equal, those two triangles can't be congruent, so the lines can't be equal." As the example shows, S could give geometric (logical) reasons for an incorrect judgment. The details are given in Table 3. The statistical significance of the obtained differences for this distribution ($U'_0 = 15.72$, df = 3, $p < .005$) and for all pairs are shown in

Table 4. These results allow us to assert that a significantly greater proportion of Ss in all of Groups SpSI, SpWI, and NSpWI solved the problem logically than did Ss in Group NSpSI.

DISCUSSION

Although Group NSpWI Ss did not make a significantly greater proportion of correct judgments than Group NSpSI Ss, they did derive their conclusions logically to a greater extent than Group NSpSI Ss. This finding is consistent with the hypothesis that as the strength of the empirical properties of the materials diminishes, the tendency to explicate the logical properties of the materials increases.

The results also show that well-specified logical properties are more likely to be explicated than not well specified logical properties, at least when the materials have pronounced empirical properties. This difference is not present when the materials *do not* strongly evoke a particularly strong response tendency. That SpSI Ss did not make more correct judgments than NSpSI Ss indicates that well-specified logical properties were not enough to overcome the influence of a strong illusion in determining the *kind* of judgment (right or wrong) Ss made, although they did determine *how* he made the judgment (i.e., logically vs perceptually).

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category correctly, and it was impossible to compute probabilities for the R− condition. The results of the analysis showed no significant differences. While this might be attributed to the loss of df, it is more likely, in view of the small differences obtained, that it represents the true state of affairs. This is perhaps the result of some continued learning of the coordinate stimulus word during the presentation of the words alone. Certainly it was possible to have a certain amount of consolidation in the case where the LD member of the pair was recalled to the HD member following failure to recall the HD member to the LD.

The results of this study support the previous finding of Jacobus & Leonard (in press) that distinctiveness of words in a compound stimulus provides the basis for cue selection. The

results also indicate that integration of the stimulus components assists recall by the capacity of the less distinctive stimulus to evoke the more distinctive stimulus.

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