Subjective probabilities for unions of events[1]

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Ss estimated probabilities of events and of the unions of those events in three different tasks. Probability estimates for the unions were approximately equal to the sum of the estimates for the component events, a relation demanded by probability theory.

Investigations of subjective probability customarily focus on the degree to which probability estimates (SPs) correspond to probabilities that are defined as limits of relative frequencies (Ps). Accuracy, however, is only one criterion for evaluating SPs. A second criterion is consistency; the degree to which relations among SPs correspond to the relations demanded by probability theory.

If SPs are accurate they are consistent by definition. Sometimes, however, insufficient understanding of the processes that determine Ps can lead to inaccurate SPs, and sometimes calculations appropriate for evaluating accuracy cannot be performed. Even under these circumstances SPs should represent Ss' opinions about Ps and, if opinions are constrained to be coherent and if this coherence is described by probability theory, even inaccurate SPs should be consistent.

Investigations of consistency have examined the correspondence between SPs for aggregates of events and the appropriate combinations of the SPs associated with each of the component events. The results indicate that, even when they are inaccurate, SPs for intersections of events approximate the product of the inaccurate SPs for the component events, and hence are consistent with probability theory (e.g., Beach, 1966; Peterson et al, 1965). For unions the results are less clear. For example, conflicting results are reported for whether or not the sum of SPs for the components of an exhaustive set of mutually exclusive events is equal to 1.00 (e.g., Lindman, 1965; Phillips, Hays, & Edwards, 1966). For nonexhaustive unions, Organist (1964) found a mean correlation of only .76 between SPs for unions and the sum of SPs for the components; a correlation constrained by a mean response reliability of only .71. This is not as high as might be expected on the basis of the studies on intersections of events. The purpose of the experiment reported here was to investigate consistency by examining the agreement between SPs for unions of events and the sum of SPs for the component events when responses were reliable but not necessarily accurate.

Method

Three experimental tasks were selected on the basis of pretests to insure reliability and inaccuracy. In each task Ss stated SPs for each event in an exhaustive set of mutually exclusive events (e.g., A, B, C, D, E, F, G). Then the array of events was partitioned into unions (e.g., three unions such as: A or B; C or D or E; F or G) and SPs for the unions were obtained. Finally, the Ss again stated SPs for the individual events.

Tasks.—Binomial: Ss were told to imagine an urn containing 80% red poker chips and 20% blue poker chips, from which samples of 11 chips were drawn repeatedly with replacement. Then they stated SPs for the occurrence of each of the 12 different kinds of samples (from 0 red chips and 11 blue ones through 11 red chips and 0 blue ones), and for 15 different unions of these samples. *Frequentistic:* A 7-outcome probability-learning task was used to teach Ss the relative frequencies of occurrence of 7 stimulus lights. Training was too short to permit perfect learning. Then Ss stated SPs for each of the lights and for 17 unions obtained by 7 different partitions. *Conceptual:* Ss stated SPs for each of 7 well-known Republicans receiving the 1968 presidential nomination; the list was to be assumed exhaustive. The SPs for 17 unions were obtained from 7 partitions of the list of candidates. For this task no relative frequencies exist and the question was whether SPs would be consistent when there was no criterion for accuracy.

Apparatus and subjects. The SPs were obtained by having Ss arrange 100 counters in calibrated troughs, one trough for each individual event or for each union of events. Ten male university students were Ss. Nine reported that their estimates were based on intuition and one said he consciously added. This one S is marked by an asterisk in Table 1.

Results

Results were analyzed by computing mean squared differences (MSD) and multiplying the resulting values

Table 1. Mean squared differences for accuracy and consistency for each S (all values multiplied by 10^3).

	Binomial		Frequentistic		Conceptual
Ss	Acc.	Const.	Acc.	Const.	Const.
1	16.23	3.36	13.48	.58	3.53
2	7.12	2.57	2.26	4.37	11.27
3	21.09	.57	1.58	1.27	1.06
4	26.80	3.40	12.38	1.68	4.23
5	9.24	5.19	15.32	8.55	23.25
6	.43	.03	10.72	.68	3.73
7	4.85	.19	5.72	.10	.30
8	7.44	1.56	2.00	.42	.16
9	33.43	4.08	7.69	3.75	12.91
10*	7.93	2.86	27.01	2.88	1.04
Median	8.58	2.97	9.20	1.48	3.63
Median Reliability	.80			.32	1.12
Median Resp. Variance	64.96			43.54	43.98

* *S who said he added.*

by 10^3 in order to reduce the number of zeros preceding the digits. The MSD between each S's first and second set of SPs for the individual events served to measure reliability. Median reliability for the group is reported at the bottom of Table 1 and is high. Accuracy was evaluated for each S by the MSD between his SPs for unions of events and the corresponding Ps for the same unions. For consistency the MSD was computed between each S's SPs for unions and the sum of his SPs for the individual events that comprised the union. Both accuracy and consistency are reported in Table 1 for each S. In all but one case (S No. 2, frequentistic task) the MSDs for consistency are lower than they are for accuracy. The median response variability (also multiplied by 10^3), at the bottom of Table 1, is the overall variability in Ss' response for unions. The MSDs for accuracy and consistency represent the degree to which this overall response variability is reduced by taking into account either the sums of the Ps or the sums of the SPs for the component events. The sums of the SPs account for the greater variability. Even though MSDs for accuracy vary from S to S, MSDs for consistency remain low.

Clearly, Ss' subjective probability estimates for unions of events closely approximate the sums of their estimates for the events that comprise the unions. These results support the hypothesis that for unions of events the relationships among subjective probabilities are consistent with those relationships required by probability theory and that this consistency persists even when the subjective probabilities are inaccurate.

References

Beach, L. R. Accuracy and consistency in the revision of subjective probabilities. *I.E.E.E. Transactions in human factors in electronics*, 1966, HFE-7, 29-37.

Lindman, H. R. The simultaneous measurement of utilities and subjective probabilities. Unpublished Ph.D. dissertation, University of Michigan, 1965.

Organist, W. E. Use of subjective probabilities as a response technique in multiple-choice behavior. Paper read at meetings of the Psychometric Society, 1964.

Peterson, C. R., Ulehla, Z. J., Miller, A. J., Bourne, L. E., & Stilson, D. W. Internal consistency of subjective probabilities. *J. exp. Psychol.*, 1965, 70, 526-533.

Phillips, L. D., Hays, W. L., & Edwards, W. Conservatism in complex probability inference. *I.E.E.E. Transactions in human factors in electronics*, 1966, HFE-7, 7-18.

Notes

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* The method employed in composition makes it prohibitive to use Greek letters frequently in the text. Accordingly the English alternative SP has been used for ψ throughout this article. — Editor