Sample size, confidence, and optimality: Reinterpretation of a previous study

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In a recent article it was concluded that *S*'s confidence in a decision and his tendency to make the optimal decision decrease with increasing sample size. It is shown here that this conclusion is incorrect since it is based on an improper definition of likelihood. When likelihood is properly defined, the same data show that *S*'s confidence and tendency to optimize increase with increasing sample size.

In a recent article an experiment was described in which *Ss* made *N* independent, random observations to arrive at a decision (Pitz, 1967). Initially a bingo basket contained three red and three blue balls, but one ball was removed without *S* seeing it. After drawing *N* balls one at a time with replacement, *S* guessed the color of the ball which was removed and rated his confidence in the guess (*N* = 3, 9, or 19).

A rather startling result was obtained: as *N* increased for a constant "likelihood," *S*'s confidence in his decision decreased as did his tendency to make the optimal decision. This would seem to indicate that the more evidence a person gets the less confident he becomes and the less likely he is to use the evidence wisely.

It is our contention that this result is an artifact resulting from an improper definition of "likelihood." When likelihood is properly defined, the same data indicate that *Ss* are reasonable (if not perfect) statisticians, i.e., their confidence and tendency to make the optimal decision increase with increasing sample size.

Pitz used the absolute difference between the number of red and blue balls, $D = |R - B|$, as a measure of likelihood (*R* = the number of red and *B* = the number of blue balls drawn in the *N* trials). For constant *D* it was assumed that the likelihood for the two possible hypotheses (red was removed vs blue was removed) was constant as *N* varied. For example, drawing two red and one blue (*N* = 3) gave the same evidence as drawing five red and four blue (*N* = 9) since *D* was the same for each case.

When confidence and tendency to optimize were plotted as a function of *D* for each value of *N*, it was found that the curve for *N* equal to 3 was above that for *N* equal to 9, which was above the curve for *N* equal to 19. This led to the conclusion that confidence and tendency to optimize are inversely related to sample size.

However, a statistician drawing balls from the basket would not use *D* as a measure of likelihood. He would use $P = R/N$ or equivalently B/N (Mood & Graybill, 1963, pp. 178-184). For $P = R/N$ this is the maximum likelihood estimator for the proportion of red balls in the basket. It can also be used to test the

Table 1
Proportions (*P*) for Combinations of Sample Size (*N*) and Absolute Difference (*D*)

	<i>N</i>		
	3	9	19
<i>D</i>	1	.667	.556
	3	1.000	.667
	5	.778	.632
	7	.889	.684
	9		.737

hypothesis that there are more red than blue balls. That is, if *P* were greater than $\frac{1}{2}$, the statistician would conclude that there were more red than blue indicating that a blue ball was removed. Equivalently, if *P* were less than $\frac{1}{2}$, he would conclude that there were more blue than red indicating that a red ball was removed. For fixed *N* his confidence would increase as *P* deviates from $\frac{1}{2}$. For fixed *P* his confidence would increase as *N* increases.

The data were re-analyzed using *P* as a measure of likelihood. In Table 1 the value of *P* is shown for each combination of *D* and *N* (in this case *P* is equal to the proportion for the predominant color in the sample so that it corresponds with the use of *D* as an absolute difference). Notice that for constant *D*, the value of *P* changes as *N* changes. This means that constant *D* does not imply constant likelihood in the statistical sense.

The values of average confidence and per cent optimal responses for each value of *P* and *N* were determined graphically using Figs. 1 and 2 of the Pitz article. They are replotted here as a function of *P* in Figs. 1 and 2 respectively. It can be seen for a given likelihood (*P*) that confidence and per cent optimal responses increase with *N*. This is the exact opposite of the result in the Pitz article.

When likelihood is properly defined, it can be concluded that *Ss* are reasonable statisticians. For constant likelihood, increasing sample size leads to greater confidence and a greater tendency to make the optimal decision.

REFERENCES

- MOOD, A. M., & GRAYBILL, F. A. *Introduction to the theory of statistics*. New York: McGraw-Hill, 1963.
PITZ, G. F. Sample size, likelihood, and confidence in a decision. *Psychon. Sci.*, 1967, 8 (6), 257-258.

Fig. 1. Average confidence as a function of the proportion of the predominant color in the sample, and of sample size, *N*.

Fig. 2. Proportion of optimal responses made as a function of the proportion of the predominant color, and of *N*.