Perceptual separability and spatial models[1]

RAY HYMAN[2] AND ARNOLD WELL
UNIVERSITY OF OREGON

Highly analyzable two-dimensional color stimuli were generated using stimulus cards such that one part of each card varied only on the first dimension and the other part varied only on the second. Subjects were required to make judgments of similarity between pairs of these analyzable stimuli, between pairs of relatively unanalyzable color stimuli, and between pairs of geometric stimuli. The results support previous findings that the Euclidean combining rule is appropriate for judgments of single color patches but indicate that the city block combining rule may be appropriate for simple stimuli that vary on perceptually distinct dimensions.

In an earlier study (Hyman & Well, 1967), the problem of scaling color and geometric stimuli varying in two dimensions was investigated. Psychological distances between stimuli were obtained using the method of multiple ratios (Indow & Uchizono, 1960) and the adequacy of various spatial models was assessed using modifications of methods which had been employed by Attneave (1950) and Torgerson (1952). The results obtained supported both the conclusion of Attneave that the city block spatial model seems to fit the data for certain geometric stimuli and the conclusion of Torgerson and others that the Euclidean spatial model fits the data for color stimuli.

As extensive efforts were made to eliminate possible antifacts due to experimental techniques, methods of data analysis, and characteristics of the stimulus sets such as the number of stimuli in the sets and the configuration (distribution relative to one another) of these stimuli, it seems likely that the difference in spatial models is due to some intrinsic characteristic of the stimulus materials themselves.

Torgerson (1958), Attneave (1962), and Shepard (1964) have all suggested that the type of spatial model which is most suitable for a given set of stimuli may depend on the analyzability of the stimuli (i.e., the phenomenological obviousness or perceptual distinctness or separability of the component dimensions). If the separate dimensions of a stimulus set are obvious, it seems reasonable that a S judging dissimilarities between pairs of stimuli might not judge the over-all differences directly but rather add up the differences along the dimensions, thus accounting for the appropriateness of the city block model for data obtained with certain geometric stimuli. The dimensions along which our color stimuli varied were not obvious and it is tempting to try and relate the lack of strongly preferred directions in the color space to the rotationally invariant character of Euclidean geometry.

In the present study, highly analyzable color stimuli were generated using stimulus cards each made up of two parts (one part varying along one dimension and the other part varying along the other dimension) in an attempt to determine whether enhancing the analyzability of the stimuli results in Ss performing in a more additive manner.

EXPERIMENT 1

Subjects

Eight Ss were hired through the University of Oregon student employment service and were paid at the rate of $1.50 an hour. The Ss were required to have normal color vision as determined by the Ishihara test for color blindness. Six Ss attended ten sessions of approximately 1 h 45 min in duration, while two others attended nine sessions.

Stimulus Objects

Eight sets of stimuli were used, four sets of color stimuli and four sets of geometric stimuli, each with eight members. One color and one geometric stimulus set were "compact" and the other six sets "distributed." The color compact stimulus cards (set CI in Table 1) were 2-3/4 in. x 5 in. white cards on each of which was pasted a 3/4 in. square Munsell 5R glossy color patch. These could vary from one another in value or chroma or both. The geometric compact stimulus cards (set GI in Table 2) were white cards of the same size, on each of which had been drawn a Shepard circle-with-radius (Shepard, 1964). These circles-with-radius could vary from one another in size of circle or in inclination of the drawn-in radius or both. The distributed stimulus sets contained cards of the same size on which were pasted two color patches or drawn two circles-with-radius. Stimulus sets CII and GII were such that the left-hand part of the stimulus card varied along one dimension while the right-hand part of the card varied along the other. Stimulus sets CIII, CIV, GIII, and GIV were such that for each set, both parts of the stimulus cards varied along the same dimension,

Table 1. Color Stimulus Sets (Munsell 5R Glossy Color Patches)

CI	CII		CIII		CIV	
(Value/Chroma)	Left	Right	Left	Right	Left	Right
3/4	3/6	5/4	5/4	5/2	3/6	4/6
3/8	3/6	5/8	5/8	5/2	3/6	6/6
4/2	4/6	5/2	5/2	5/4	4/6	3/6
4/10	4/6	5/10	5/10	5/4	4/6	7/6
6/2	6/6	5/2	5/2	5/8	6/6	3/6
6/10	6/6	5/10	5/10	5/8	6/6	7/6
7/4	7/6	5/4	5/4	5/10	7/6	4/6
7/8	7/6	5/8	5/8	5/10	7/6	6/6

Table 2. Geometric Stimulus Sets (Circles with one radius drawn in)

GI	GII Left	GII Right	GIII Left	GIII Right	GIV Left	GIV Right
2.88/80.0[a]	2.88/45.0	2.59/80.0	2.59/59.5	2.59/80.0	2.88/45.0	3.25/45.0
3.25/59.5	3.25/45.0	2.59/59.5	2.59/80.0	2.59/59.5	3.25/45.0	2.88/45.0
3.25/30.5	3.25/45.0	2.59/30.5	2.59/80.0	2.59/30.5	3.25/45.0	2.30/45.0
2.88/10.0	2.88/45.0	2.59/10.0	2.59/59.5	2.59/10.0	2.88/45.0	1.90/45.0
2.30/10.0	2.30/45.0	2.59/10.0	2.59/30.5	2.59/10.0	2.30/45.0	1.90/45.0
1.90/30.5	1.90/45.0	2.59/30.5	2.59/10.0	2.59/30.5	1.90/45.0	2.30/45.0
1.90/59.5	1.90/45.0	2.59/59.5	2.59/10.0	2.59/59.5	1.90/45.0	2.88/45.0
2.30/80.0	2.30/45.0	2.59/80.0	2.59/30.5	2.59/80.0	2.30/45.0	3.25/45.0

a diameter of circle (cm.) / inclination of radius (degrees)

the dimensions of variation being chroma, value, inclination of radius from the horizontal, and size of circle, respectively.

For all stimulus sets except CI, the dimensions along which the members of the set varied were perceptually distinct. The configuration of each stimulus set formed an octagon in stimulus space.

Procedure

Our procedure was based on Indow's method of multiple ratios (Indow & Uchizono, 1960) and has been described in detail in an earlier paper (Hyman & Well, 1967). Briefly, the stimulus objects were presented, one set per session, on a white sheet of heavy cardboard onto which were fixed eight horizontal trays. For each trial, one stimulus card was designated as the "standard" and was placed at the extreme left end of the fourth tray. The other seven stimuli were placed, one per tray, in a vertical column at the right side of the board. The S was instructed to move these seven comparison stimuli in such a manner that the more similar to the standard a stimulus was judged to be, the further to the left end of the board it was to be placed. During an experimental session, each stimulus object assumed the position of the standard. The orders in which the different stimulus objects became the standard as well as the vertical positions of the comparison stimuli were determined by reference to random number tables. A different randomization was used in each session.

After each trial, the horizontal distance between the center of each comparison card and the center of the standard card was measured and recorded to the nearest tenth of a centimeter. The next standard was then placed in position, the other cards randomized, and the next trial commenced.

For compact stimulus sets the instructions emphasized that each card could differ from one another in two different ways and that two cards were to be considered dissimilar to the extent that they differed in either or both ways. For distributed stimulus sets Ss were told that two cards were to be considered dissimilar to the extent that they differed in either or both of their corresponding parts.

Six Ss attended 10 sessions and two Ss attended nine complete sessions. The first two and the last two sessions were devoted to compact stimulus sets and the intervening six sessions were devoted to the six distributed sets which were presented in a different order for each S.

Results

Each S's settings for a session were converted into psychological interpoint distances by the procedure of Indow and Uchizono (1960). The internal consistency of S's judgments was measured by correlating the similarity judgments for stimulus pairs (i, j) when i was the standard with the judgments when j was the standard. The correlations for each stimulus condition are reported in Table 3.

The matrix of interpoint distances was transformed into a matrix of scalar products and the eigenvalues and eigenvectors of this matrix were obtained (this

Table 3. Summary of Results for Experiment 1

Stimulus Set	Mean correlation between (i, j) and (j, i)	Mean percentage of variance accounted for by first two vectors	Median Mann-Whitney z score	Median direct analysis classification
CI	.82	95.4%	0.9	E
CII	.91	88.9%	4.0***	CB
CIII	.84	85.4%	4.0***	CB
CIV	.85	86.4%	3.8***	CB+
GI	.84	86.1%	3.9***	CB
GII	.88	82.6%	4.0***	CB
GIII	.86	84.4%	3.9***	CB+
GIV	.84	87.1%	3.6***	CB

*** $p < .001$

process is equivalent to the factor analytic approach suggested by Torgerson (1958)). Euclidean distances were reconstituted from the first two eigenvectors (since two-dimensional stimuli were used) and these distances were compared with the original interpoint distances. As the reconstituted distances were obtained on the assumption of a Euclidean space, a noise-free S acting in a Euclidean manner would be expected to yield judgments such that the original psychological distances and the reconstituted distances would be identical. As there is always noise in the system, these distances are not identical—but the important thing is that an actual S acting in a Euclidean manner would yield a random pattern of relatively small deviations, while a "city block" S would yield a rather specific pattern of deviations depending on the configuration of the stimulus set. For an octagonal configuration of stimuli, a noise-free city block S would give results such that the distances reconstituted from the first two eigenvectors were always larger than the original interpoint distances for unidimensional comparisons and always smaller than the original interpoint distances for bidimensional comparisons.

A useful, distribution-free index of the extent to which Ss performed in a city block manner, then, would be the Mann-Whitney z score based upon the ranked deviations between the original and the reconstituted distances. A noise-free Euclidean S should yield a value of 0 on this measure and because of the nature of our stimulus configurations, a noise-free city block S should yield a value of 4.0. It appears quite clear from Table 3 that by this criterion Ss behaved in a city block fashion for all stimulus sets except CI, the only set for which the dimensions were not perceptually distinct. For stimulus set CI, behavior did not differ significantly from that predicted by the Euclidean model.

A criterion of goodness of fit to the Euclidean model suggested by Torgerson (1958) is that of the proportion of variance among the scalar products accounted for by the first two Euclidean vectors. It is difficult to decide what percentage is high enough to denote a "good fit" and we have previously shown that this criterion is not particularly sensitive, especially when Ss weight one dimension more than the other. In any event, by this criterion the goodness of fit to the Euclidean model is significantly better for stimulus set CI than for any of the other seven stimulus sets ($p < .005$ by Friedman rank test).

A third criterion is based on the fact that when the interpoint distances between stimuli with an octagonal configuration in stimulus space have been obtained, twelve of these distances can be looked upon as forming the sides of eight right-angled triangles. Each of these right triangles can be classified as to whether the hypotenuse is related to the other two sides in such a manner as to suggest the appropriateness of the Euclidean model (E), the city block model (CB), the dominance model (D), a violation of the Minkowski metric at the dominance end (DV), or a violation of the triangle inequality (TV). The median classification of the eight triangles on the ordinal scale DV, D, E, CB, TV was taken as being representative of that session. A more detailed account of this procedure is given in an earlier paper (Hyman & Well, 1967). Table 3 shows the median classification for stimulus set CI to be E while the classifications for the other sets are CB or CB+.

Conclusions to Experiment 1

By all three criteria that were considered, the behavior of Ss when dealing with stimulus set CI was considerably different from that when dealing with the other seven stimulus sets. The similarity judgments of color compact stimuli were consistent with a Euclidean spatial model, while those for the other stimuli were consistent with the city block spatial model. By enhancing the distinctness of the dimensions of color stimuli, it seems possible to shift from the Euclidean to the city block model.

There are, however, certain difficulties with Experiment 1. Sitting through so many experimental sessions, a number of Ss realized, at least for the geometric stimuli, that there were only four values on each dimension. In the case of one S this resulted in a strategy being employed in which similarity judgments were given on the basis of the total number of steps by which stimuli differed on their two dimensions and not necessarily on how similar they appeared to the S.[3] While only one S gave evidence of acting in so overtly analytical a manner, it was decided to run a second experiment in which such effects would be minimized.

EXPERIMENT 2

Subjects

Twenty-four new Ss were obtained and were paid at the same rate as in Experiment 1. They were required to have normal color vision.

Stimulus Objects

Stimulus sets CI, CII, GI, and GII were used.

Procedure

To minimize the development of strategies based on extreme familiarity with stimulus materials, Ss attended only four experimental sessions. In addition, all but four Ss were run using a modified version of the Indow procedure used in Experiment 1. Ss were still required to indicate similarity judgments by displacing comparison stimuli from standard stimuli, but they never saw more than three cards at one time: (a) the standard, (b) a fixed reference card which had a middle value on both dimensions and which was placed at a constant distance from the standard to provide a modulus,

Table 4. Summary of Results for Experiment 2

Stimulus Set	Indow Method (4 Ss) Median Mann-Whitney z score	Median direct analysis classification	Modified Indow Method (12 Ss) Median Mann-Whitney z score	Median direct analysis classification
CI	0.0	E+	0.4	E–
CII	3.4***	CB	2.7**	CB–
GI	3.8***	CB	1.6	CB–
GII	3.4***	CB	2.6**	E+

** $p < .01$
*** $p < .001$

and (c) the comparison card which S was to move to indicate similarity. One stimulus set was used in each session and the order in which Ss encountered the stimulus sets was counterbalanced.

Results

The data were analyzed exactly as in Experiment 1 and the results for 16 Ss are summarized in Table 4. Twenty-four Ss participated in the experiment but the data for eight of them were collected by an undergraduate assistant who performed in a somewhat erratic manner with the result that the data he collected were appreciably "noisier" (had lower internal consistency) than that collected by the other experimenters. Although the results for these eight Ss point to the same conclusions as those of the 16 Ss whose data were collected by the other experimenters, these results have not been included in Table 4.

The results for the four Ss run using the Indow procedure (as in Experiment 1) were very similar to those obtained in Experiment 1 in that they suggest Ss act as though they were using the Euclidean combining rule for stimulus set CI and the city block rule for the other three stimulus sets. For the 12 Ss run using the modified Indow procedure, the results are not so clear-cut but still suggest that Ss act in a much more additive manner for stimulus sets CII, GI, and GII than for stimulus set CI. It should be pointed out that although the median Mann-Whitney z score for stimulus set GI is only 1.60 (not significant at the .05 level), pooling the z scores for the 12 Ss by the procedure suggested by Mosteller and Bush (1954) gives a z score of 4.9 which is significant ($p < .00001$).

Conclusions to Experiment 2

The results of Experiment 2 support the basic conclusion of Experiment 1, namely, that Ss perform in a much more additive manner when dealing with highly analyzable stimuli than they do for relatively unanalyzable stimuli like single color patches. The use of the modified Indow method in Experiment 2, while causing greater variation and more inconsistencies between Ss than had been the case in Experiment 1, did prevent Ss from learning enough about the stimulus sets to use strategies based on knowing the number of steps on each dimension.

DISCUSSION

Our findings are consistent with those obtained in an earlier study (Hyman & Well, 1967) that Ss act in a seemingly Euclidean fashion when dealing with "compact" color stimuli and that they act in a much more additive fashion when dealing with certain geometric stimuli. Moreover, our results indicate that the major factor upon which this differential behavior is based may well be, as was suggested by Torgerson (1958), Attneave (1962), and Shepard (1964), the perceptual distinctness or separability of the dimensions of the stimulus sets.

An interesting finding was that so long as the dimensions were perceptually separable, they were combined in the same fashion. Differences in value seemed to combine with differences in chroma in the same fashion as differences in angle combined with differences in size. In fact, in Experiment 1, differences along noncommensurable dimensions (angle-size, value-chroma) seemed to be combined in the same fashion as differences along commensurable dimensions (e.g., value of left color patch-value of right color patch). To check this more closely, a third experiment was run using the method of Experiment 2. Four Ss using stimulus sets CII, GII, CIII, and GIII and four Ss using stimulus sets CII, GII, CIV, and GIV showed nothing to indicate that commensurable and noncommensurable dimensions were being dealt with differently. As was the case in Experiment 2, however, more noise appeared in the data and there was a shift from the city block combining rule in the direction of the Euclidean combining rule.

Enhancing the perceptual separability of dimensions causes a shift in the appropriate combining rule towards the city block model. Introduction of additional noise into the system tends to act in the opposite fashion. It seems that we are dealing with a continuum of combining rules and it may prove more profitable to conceive of this continuum simply in terms of efficiency of informational usage than in terms of families of spatial models with all their assumptions and implications. Insights into what actually is going on can likely be obtained by investigating more fully those sets of circumstances causing shifts of various kinds in the combining rules used by Ss.

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Notes

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2. At Instituto di Psicologia, Bologna, Italy for the academic year 1967-1968.

3. This S arbitrarily assigned the values 1, 2, 3, 4 to the four circle sizes and to the four radius inclinations. His subsequent judgments were made on the basis of these arbitrary numbers rather than directly on the apparent similarities.

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