Perceived numerosity*

LESTER E. KRUEGER†
The City College of the City University of New York,
New York, New York 10031

In Experiments 1 and 2, dots appeared less numerous when bunched together on a sheet than when spread out over a larger area, and apparent numerosity proved to be a power function, with exponents of 0.72 (Experiment 1) and 0.78 (Experiment 2), of objective numerosity. In Experiment 3, where Xs were shown instead of dots, the exponent was 0.77. In Experiment 4, where Ss made magnitude productions rather than magnitude estimations of the Xs, the exponent was 0.94. The overall results indicate an exponent of about 0.85 for numerosity, as well as a striking tendency for Ss to underestimate the number of dots or Xs presented.

The effect that one stimulus dimension can have on the perception of another is well illustrated by the size-weight illusion, in which increasing the size or volume of an object, while keeping its actual weight constant, decreases its apparent weight (e.g., Nyssen & Bourdon, 1956). Another instance is Fazil's finding (cited by Brunswik, 1956) that larger and more valuable coins appear more numerous than smaller and less valuable coins. Ansbacher (1937), similarly, found that more valuable stamps appear more numerous, and Bevan, Helson, and Maier (1963) found that increasing the size of a jar increased Ss' estimates of the number of beans in the jar.

The present study tested whether spread-out dots would appear more numerous than dots bunched together in a smaller area. The prediction that such an effect would occur came from Krueger's (1970) finding that the farther apart two lines are placed, the greater their judged combined length. Bevan and Turner (1964) also provided indirect support for the prediction. When their Ss were cautioned to separate dots from a surrounding frame (SG and LG conditions), the dots appeared more numerous as the frame was made smaller and set closer to the dots. The smaller frame may have made the dots seem more numerous by making the area covered by the dots seem larger by contrast. The Bevan, Helson, and Maier finding mentioned above (a larger jar made beans seem more numerous) also supports the prediction, although their study involved perception of beans lying in a jar, not dots on a sheet, and the larger jar had a larger width, so that the bean collection varied in shape as well as size.

On the other hand, Mokre (1928) had Ss compare two dot displays that were presented briefly, one after the other, and he found a tendency for numerosity to be inversely related to spatial separation. Mokre's results are inconclusive, however, because he found the reverse tendency (greater numerousness with greater dispersion) for two of his seven Ss. Further, Mokre spaced the dots equidistantly, thus imparting a consistent texture to each display which may have enabled his Ss to judge the displays simply on the basis of relative density. In the present study, dots were distributed more randomly over the area assigned.

In Experiment 1, described below, dots bunched together appeared less numerous than dots spread further apart, but the effect only held between the smallest dot area and all the other larger dot areas. To ensure that the particular type of effect in Experiment 1 was not simply a chance occurrence, a replication (Experiment 2) was conducted several months later, with a new E and new Ss. The largest dot area in Experiment 1 (2,025 cm²), which covered nearly the entire sheet and thus may have altered the figure-ground separation of dot region and surrounding white area, was not used in Experiment 2. Even so, the findings of Experiment 2 closely replicated those of Experiment 1, with an effect evident only for the smallest display area.

Experiments 1 and 2 also provide new estimates (0.72 and 0.78, respectively) of the power-function exponent for numerosity. Stevens (1957), relying on Taves's (1941) fractionation data, set the exponent for numerosity at 1.34. That is, $R = kS^{1.34}$, where R is apparent numerosity, S is objective numerosity, and k is a constant. Abbey (1962) used magnitude estimation and reported an exponent of 1.2 for numerosity.

EXPERIMENTS 1 AND 2

Method

Test stimuli. Small black circles (Chartpak RDC 2 dry-transfer symbols, .16 cm in diam) were affixed on 47.5 x 47.5 cm white sheets. To position the dots properly, master matrices were created, on each of which was centered a 24 x 24 set of dot locations (576 points). On one master matrix, the center-to-center distance between adjacent dot locations in the 24 rows and 24 columns was held to .31 cm, thus providing a 7.5 x 7.5 cm matrix (56 cm²) in which dots might appear. On a second master matrix, the interpoint distance was .63 cm, thus providing a 15 x 15 cm display area (225 cm²). On a third, the distance was 1.25 cm (900 cm²), and on a fourth it was 1.88 cm (2,025 cm²). The four matrices served as templates and allowed dots on a given stimulus sheet to be distributed over one of four spatial extents: 56, 225, 900, or 2,025 cm². In positioning the dots, the matrices were centered on the stimulus sheets. Key reference points of the master matrix were lightly penciled onto the stimulus sheet and later were completely erased when all dots had been affixed.

The same random pattern for a given number of dots was used with all four master matrices, since during testing the stimulus sheets could be rotated by 0, 90, 180, or 270 deg. Thus, for instance, four identical patterns were created of the 75-dot set, one in conjunction with each

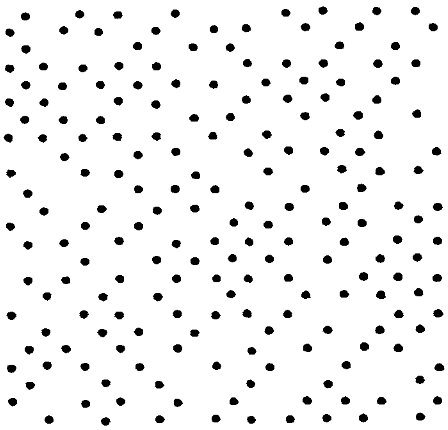

Fig. 1. Experiments 1 and 2: the 200-dot display with a display area of 56 cm². The figure shown here has been photographically reduced. The 7.5 x 7.5 cm dot region was centered on a 47.5 x 47.5 cm white sheet during testing.

*This study was supported by NIH Grant HD04869-01. The author is grateful to Allan Frosch for collecting and tabulating the data in Experiment 1, to Paul Kadis and Ora Ezrachi for collecting and tabulating the data in Experiments 2, 3, and 4, and to S. S. Stevens for commenting on an earlier version of the report.

†Requests for reprints should be sent to Lester E. Krueger, Department of Psychology, City College of the City University of New York, New York, N.Y. 10031.

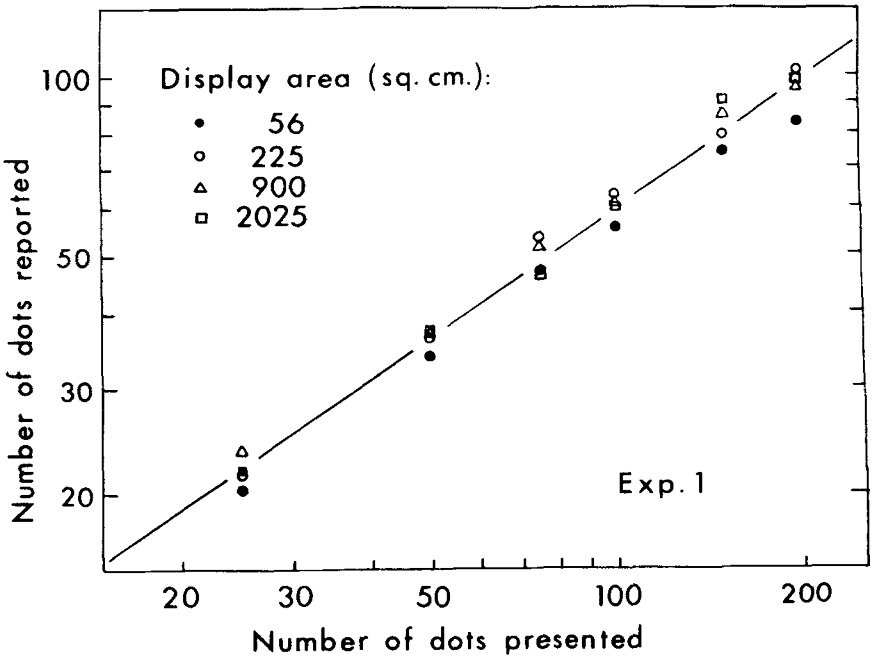

Fig. 2. Experiment 1: number of dots reported by number of dots presented. (Each point represents a geometric mean across 30 Ss.)

master matrix, but Ss saw four different patterns because the four stimulus sheets were rotated by different amounts.

In Experiment 2, the largest display area (2,025 cm^2) was eliminated, so only three areas were used, and a new system of rotation of stimulus sheets was employed. The 56 cm^2 sheets were rotated 90 deg counterclockwise relative to the 225 cm^2 ones, and 900 cm^2 sheets were rotated 90 deg clockwise. In addition, for eight Ss each, the entire set of stimulus sheets was rotated by 0, 90, 180, or 270 deg, so that across the total set of 32 Ss, each stimulus sheet for each area appeared equally often in all four orientations.

Six different numbers of dots were used: 25, 50, 75, 100, 150, and 200. The dots were distributed randomly among the 576 points in the 24 x 24 matrix, except that one-sixth of the dots had to fall within each successive set of four rows (to prevent excessive concentration and clustering of dots in any one portion of the matrix), and adjacent dot locations on a row or column could not both be filled (to ensure that individual dots would be discriminable). The most dense dot array, in which 200 of the 576 positions in a 56 cm^2 region were filled, is shown in Fig. 1; as can be seen, all dots are clearly discriminable.

In Experiment 1, each S saw 24 stimulus sheets, representing all possible combinations of four sizes of display area (56, 225, 900, 2,025 cm^2) and six numbers of dots (25, 50, 75, 100, 150, 200). In Experiment 2, 18 rather than 24 sheets were shown, because the 2,025 cm^2 display area was not used. The stimulus sheets were presented to each S in a different random order, except that no two sheets presented in succession contained the same number of dots.

Procedure. The Ss were asked to state how many dots *appeared* to be present on each sheet. The Ss were asked to make perceptual judgments and not to count or mentally compute the number of dots present. The Ss went at their own pace, but were encouraged to look at each sheet for only a few seconds. Timings made on several Ss by a second hidden E indicate that most responses were made 3 to 5 sec after the stimulus sheet was presented. The E immediately removed the sheet when the S gave his report. The S stood and looked down at a 0.75-m-high table on which the stimulus sheet lay centered on a 1 x 1.2 m (Experiment 1) or 0.6 x 1 m (Experiment 2) white sheet.

Subjects. Thirty City College undergraduates served as Ss in Experiment 1, and 32 served in Experiment 2.

Results

The average number of dots reported (geometric mean) for each dot display is shown on log-log coordinates in Figs. 2 (Experiment 1) and 3 (Experiment 2). Size of display area had a significant effect on number of dots reported by Ss, both in Experiment 1, $F = 3.30$, $df = 3,87$, $p < .025$, and Experiment 2, $F = 4.92$, $df = 2,62$, $p < .025$. Although, as predicted, the larger the display area the larger the number of dots reported, the effect was confined for the most part to the comparison between the 56 cm^2 array and the two or three larger arrays, as Figs. 2 and 3

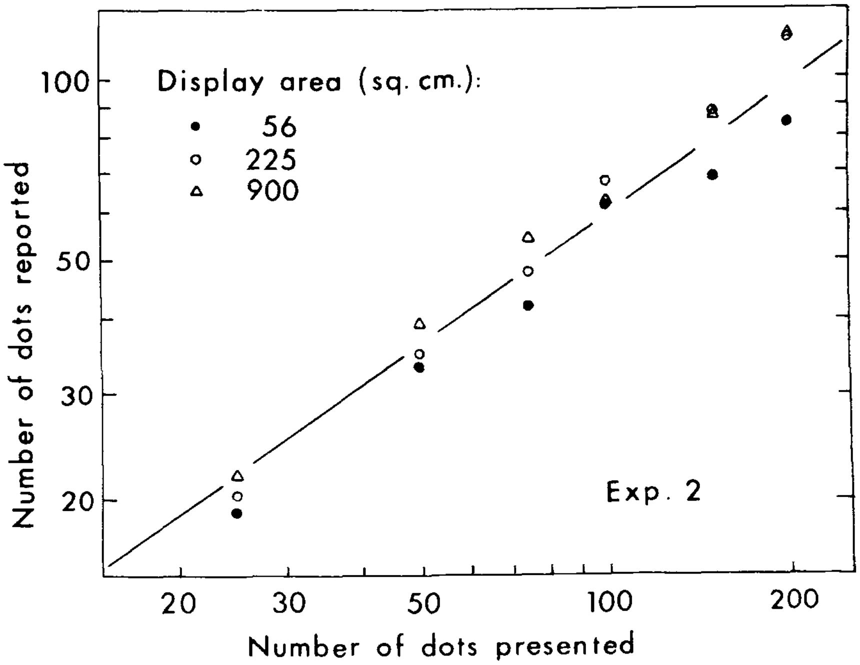

Fig. 3. Experiment 2: number of dots reported by number of dots presented. (Each point represents a geometric mean across 32 Ss.)

Table 1
Power Function Exponents for Each S in Each Condition in Experiments 1, 2, 3, and 4

	Experiment 1				Experiment 2				Experiment 4	
	Display Area (cm^2)							Experiment	Starting No.	
Exponent	56	225	900	2025	56	225	900	3	Low	High
.20- .29	—	—	1	—	1	—	—	—	—	—
.30- .39	2	2	—	—	—	—	—	2	—	—
.40- .49	5	—	5	4	3	1	5	2	—	—
.50- .59	4	7	4	4	3	3	2	2	1	2
.60- .69	7	5	8	4	9	4	5	4	3	5
.70- .79	2	7	3	3	7	5	4	8	9	10
.80- .89	4	3	4	8	4	6	7	7	8	4
.90- .99	2	3	3	5	3	7	5	4	5	4
1.00-1.09	2	1	—	—	—	2	3	2	3	2
1.10-1.19	1	—	1	1	1	2	—	—	1	1
1.20-1.29	1	—	1	1	—	1	—	—	2	1
1.30-1.39	—	2	—	—	1	—	—	1	—	2
1.40-1.49	—	—	—	—	—	1	—	—	—	—
1.50-1.59	—	—	—	—	—	—	—	—	—	1
1.60-1.69	—	—	—	—	—	—	—	—	—	—
1.70-1.79	—	—	—	—	—	—	1	—	—	—
N	30	30	30	30	32	32	32	32	32	32
Arithmetic Mean Exponent of Individual Ss	(.69)	(.74)	(.69)	(.75)	(.72)	(.84)	(.78)	(.76)	(.87)	(.86)
Exponent for Group Data	(.69)	(.74)	(.69)	(.75)	(.72)	(.84)	(.78)	(.77)	(.94)	(.93)

show. On Newman-Keuls tests (Winer, 1962), in both Experiments 1 and 2, the judged numerosity was significantly lower ($p < .05$) for the 56 cm^2 display area than for each of the other larger displays, which did not differ significantly among themselves.

In Experiment 1, for all array sizes, the power-function exponent for numerosity is about 0.7, as shown by the close fit to the line with 0.7 slope in Fig. 2. Least-squares computations produced exponent values of 0.69 for the 56 cm^2 display area, 0.74 for 225 cm^2, 0.69 for 900 cm^2 and 0.75 for 2,025 cm^2. For comparison, the 0.7-slope line of Fig. 2 is drawn on Fig. 3; the generally good fit shown in Fig. 3 indicates good agreement between Experiments 1 and 2. The power-function exponents were generally higher in Experiment 2: 0.72 for 56 cm^2, 0.84 for 225 cm^2, and 0.78 for 900 cm^2. The exponents did not vary significantly by size of display area, as indicated by the lack of interaction in either experiment, between size of display area and number of dots presented ($F < 1.0$).

Discussion

Both Experiments 1 and 2 showed, as predicted, that dots bunched together appear less numerous than dots spread out. Unexpectedly, the effect held only between the smallest display area (56 cm^2) and the other two (Experiment 2) or three (Experiment 1) larger areas. It is not clear why the effect was not shown among the larger areas. Perhaps a ceiling effect was involved, with the maximum effect of area being achieved with a medium-size, 225 cm^2 display area.

Another unexpected finding was the low power-function exponent for numerosity: an average 0.72 in Experiment 1 and 0.78 in Experiment 2. Previous studies indicated the exponent to be much higher. Taves's fractionation data, for example, indicate an exponent of 1.34, as Stevens (1957) has reported, though Taves's magnitude estimation data (see his Fig. 10) indicate an exponent of about 1.0 or less.

Abbey (1962) used magnitude estimation and found an exponent of 1.2 for numerosity. Abbey's method differed from that of the present study in that the display area was smaller (30 cm in diam) and the duration of exposure was limited (3 sec). Also, he told his Ss to make subsequent judgments proportional to their judgment on the first stimulus, whereas in the present study, Ss judged each stimulus on its own as to how many dots it appeared to contain. Abbey's plotted data are negatively accelerated, so that the exponent would be less than 1.2 if computed only for the more numerous displays (i.e., 85, 130, 200 points). Another peculiarity, perhaps due to the way his Ss kept their judgments proportional to the first judgment, was that his Ss seemed consistently to underestimate the number of dots presented. Normally, underestimation is associated with an exponent of less than 1.0, with apparent magnitude increasing at a slower rate than objective magnitude.

Bevan, Helson, and Maier also found a consistent underestimation of the number of beans in a jar, and their data indicate an exponent of less than 1.0, although their task may have involved more the judgment of volume

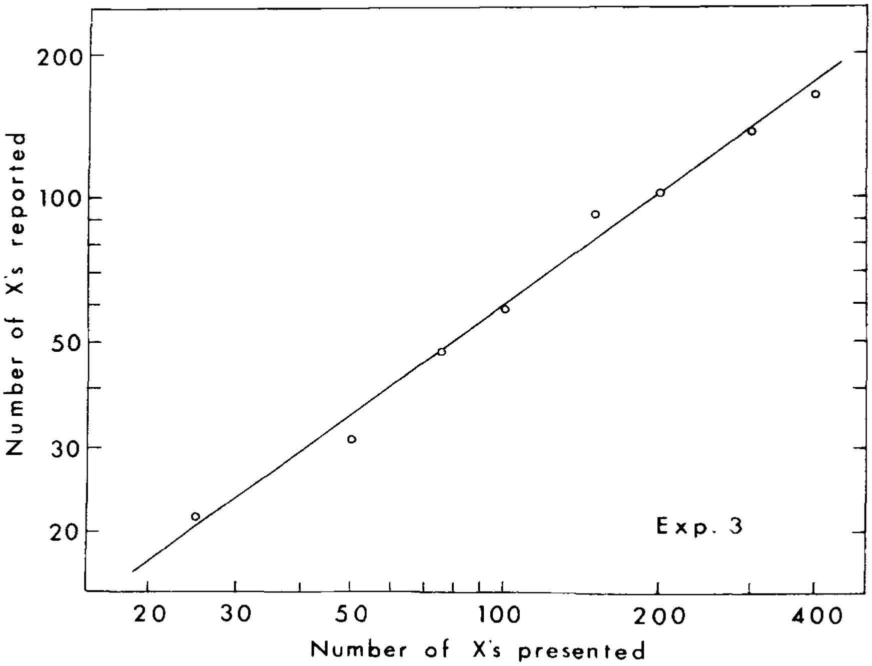

Fig. 4. Experiment 3: number of Xs reported by number of Xs presented. (Each point represents a geometric mean across 32 Ss.)

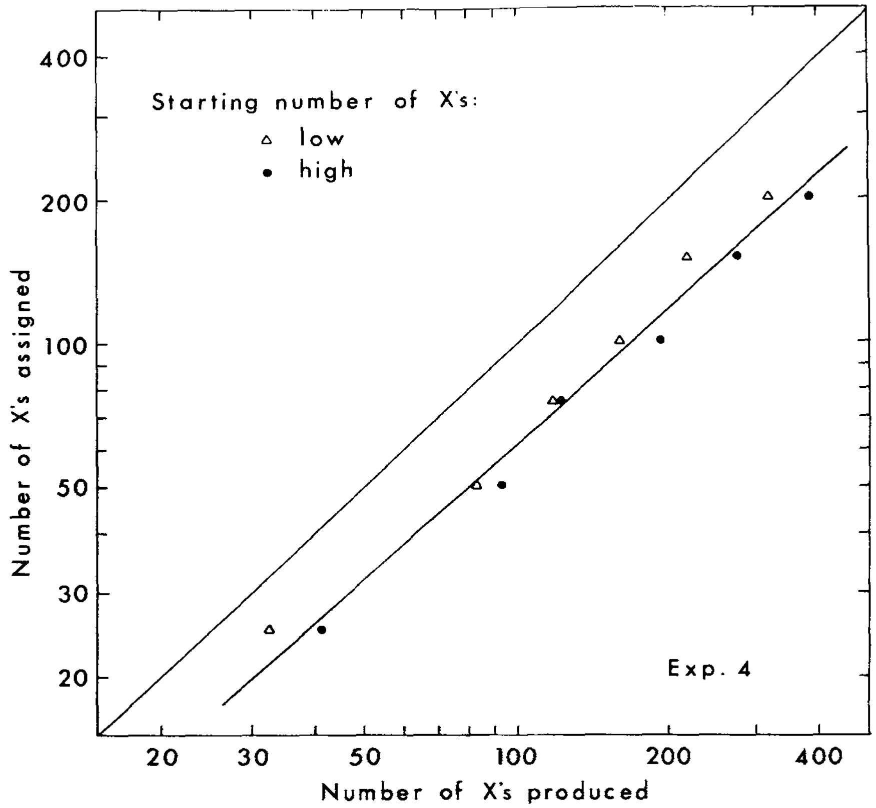

Fig. 5. Experiment 4: number of Xs produced by number of Xs assigned to be produced. (Each point represents a geometric mean across 32 Ss.)

than of numerosity. Bevan and Turner, who investigated numerosity of dots, also found a consistent tendency to underestimate, and the data for their control (C) condition indicate an exponent of about 0.9.

The considerable variability across studies in the size of the exponent may reflect its sensitivity to changes in the format of the stimuli or in the method of judgment. Differences among the studies are mentioned above, but it is not clear which differences might actually have affected the exponent. Large variability among individual Ss also may be a factor. Table 1 tabulates exponents obtained for each S in each condition in the present experiments. The increased variability of the data for individual Ss, as compared with the geometric means for the groups, affected the least-squares computations (regression of log apparent numerosity on log objective numerosity) so as to reduce the average size of exponent slightly, as can be noted in Table 1, but not so much as to explain the considerable consistency with which Ss had exponents below 1.0. The data for individual Ss, then, support the case that the exponent for numerosity is less than 1.0.

Another reason so low an exponent for numerosity was found in the present study may be that the instructions in Experiments 1 and 2 stressed that Ss were to report how many dots *appeared* to be present, not how many dots actually *were* present. That Ss' attitudes may have considerable influence is shown by Teghtsoonian's (1965) finding of an exponent of 0.8 for area of circles when Ss judged how large the area of a circle looked, and an exponent of 1.0 when Ss judged how large the area of a circle actually was. Another explanation for the low exponent may lie in the tendency for the method of magnitude estimation, used in Experiments 1 and 2, to underestimate the size of the exponent due to regression effects (Stevens, 1966).

To test if some feature unique to the particular type of stimulus used in Experiments 1 and 2 produced the low power-function exponent, in Experiment 3 Xs on computed printout sheets were used instead of dots, and sheets containing 300 and 400 items were included so that the number of items per sheet ranged from 25 to 400. In Experiment 4, Xs on computer printout sheets also were used, but the method was changed from magnitude estimation to magnitude production, with S leafing through a stack of sheets to find the one that appeared to contain a particular number of Xs. Since magnitude production tends to overestimate the size of the exponent (Stevens, 1966), if the true exponent is 1.0 or higher, then the estimate obtained in Experiment 4 ought to be above 1.0.

EXPERIMENTS 3 and 4

Method

Test stimuli. An IBM 360-50 computer generated the printout sheets containing Xs. The printout sheet (381.0 cm wide x 279.4 cm high) contained 66 lines, with a light gray line printed beneath every second line. The matrix of possible X positions consisted of 25 rows (odd-numbered lines extending from 9 to 57 on the 66-line sheet) with 35 X positions, spaced two positions apart, on each row. The overall 25 row by 35 column matrix extended 26.0 cm left to right and 20.6 cm top to bottom and was separated from the left edge of the sheet by 4.5 cm and from the right edge by 6.8 cm, from the top edge by 3.2 cm and from the bottom edge by 3.8 cm. Using every other line and placing two blank spaces between each possible X position ensured that each X printed would be discriminable. The printed capital X was .25 cm high and .16 cm wide, with a .60-cm vertical separation between adjacent X positions in each of the 35 columns and a .60-cm horizontal separation between adjacent X positions in each of the 25 rows.

The 25 by 35 matrix, containing a total of 875 positions for Xs, was divided into five 5 by 35 submatrices, each containing 175 positions. A random-number generating subroutine produced random permutations of 175, one permutation per submatrix, so that every position in the submatrix, reading across each of the five rows in succession, was assigned a value, which was equally likely to be any number from 1 to 175.

Each of the five submatrices always contained one-fifth of the Xs on a stimulus sheet. When the sheet contained 200 Xs, each submatrix contained 40 Xs. To distribute randomly the 40 Xs within the submatrix, the computer simply printed an X in each position whose assigned value from the random permutation listing fell in the 1 to 40 range and left blank those positions falling in the 41 to 175 range.

For Experiment 3, eight stimulus sheets were prepared, one for each of the following number of Xs: 25, 50, 75, 100, 150, 200, 300, 400. A new set of random permutations was used by the computer in preparing each sheet.

For Experiment 4, instead of tearing apart the sheets, the continuous printout of fanfold sheets was kept intact. In the stack, 10 blank sheets in the front were followed by a sheet containing 15 Xs, then an intervening blank sheet, then one with 20 Xs, and so on, so that each succeeding printed sheet contained five more Xs than the previous printed one, until finally a sheet with 870 Xs (only five blank positions) was printed, followed by 10 blank sheets. No notation on any sheet indicated how many Xs were present; a letter code was penciled lightly behind each sheet. The letter code ensured that not even the E, in recording S's choice, knew how many Xs were present. To save computer time, in Experiment 4, only one set of five random permutations, one for each submatrix, was prepared, but the order of the five submatrices in the printout was varied randomly so that a particular submatrix might occupy Lines 6-10 on one sheet, but Lines 21-25 on another. In leafing through the set of sheets, this investigator could detect no similarity between successive sheets; each sheet seemed to represent a new randomization of a set of Xs.

Procedure. In Experiment 3, Ss received the same basic instructions as in Experiments 1 and 2 and then viewed each of the eight stimulus sheets once. The S stood next to a 0.75-m-high 0.75 x 1.5 m desk, on whose light-green Formica top the sheet was laid out. No white background sheet was used. The E removed the sheet immediately after S made his judgment.

After participating in Experiment 3, each S then was seated and participated in Experiment 4, where his task was to leaf through a set of sheets to find the one that appeared to contain a particular number of Xs. The S was asked to bracket his judgments, that is, to leaf beyond the first sheet that appeared to match the target number of Xs and then come back so as to home in on the best possible match. As a further precaution, on 6 of the 12 trials the S began leafing from a low starting number of Xs (E opened the stack to the sheet containing 15 Xs) and on the other six the S began from a high starting number (E opened the stack towards the very end, to a sheet containing over 800 Xs). The stack was arranged so that S flipped a page away from himself to reveal the next page. Since a blank page intervened between successive printed pages, S only saw one printed page at a time.

Trials with a low starting number of Xs were interdigitated with the high-starting-number trials. For half the Ss, all odd-numbered trials were from low starting numbers and all even-numbered were from high starting numbers, and vice versa for the other Ss.

Each S had to leaf through the stack to find each of the following numbers, once from a low and once from a high starting position: 25, 50, 75, 100, 150, 200. Each S received a different order, which was random except that no two trials in succession assigned the same number of Xs for S to find.

Subjects. Thirty-two City College undergraduates served as Ss. Each S served in both experiments in the same session, participating first in Experiment 3 and then in Experiment 4.

Results

Magnitude estimations in Experiment 3 provided a good fit to a power function with an exponent of 0.77 (see the least-squares line fitted to the data points on the log-log coordinates of Fig. 4), which agrees quite well with the 0.72 found in Experiment 1 and the 0.78 in Experiment 2.

Magnitude productions in Experiment 4 resulted in a power-function exponent of 0.94 when the starting number of Xs was low and 0.93 when the starting number was high. An average of the two least-squares lines is drawn in Fig. 5. The matches were much lower when Ss started from a low number of Xs, but for both starting positions Ss consistently underestimated the number of Xs by a considerable extent, as shown by the fact that in Fig. 5 all data points fall to the right of the 45-deg diagonal line.

Discussion

If equal weight is given to the magnitude estimation exponents of Experiments 1, 2, 3 (0.72, 0.78, 0.77), as against the magnitude production exponents of Experiment 4 (0.93, 0.94), then the best estimate of the true power-function exponent for numerosity is about 0.85. Throughout, Ss showed a striking tendency to underestimate the number of dots or Xs present, estimating about 100 to be present when 200 were shown in Experiments 1, 2, and 3 and producing an average (geometric mean) 309 (low starting position) or 385 (high starting position) when asked to produce 200 in Experiment 4.

The various experiments showed considerable stability as to the size of the exponent; when data were plotted on log-log coordinates, size of display area (Figs. 2 and 3) and starting number of Xs (Fig. 5) affected the intercept, but not the slope of the function. In the power function, $R = kS^n$, then, the effect of size of display area and starting number of Xs was on the constant factor, k, rather than the exponent, n, which suggests that display and starting number of Xs affected Ss' estimates of the number of dots or Xs as an after-the-fact application of a constant correction factor—made after a more basic perceptual computation of the number of dots or Xs.

REFERENCES

ABBEY, D. S. Cross-modality matching of numerosity and pitch. Canadian Journal of Psychology, 1962, 16, 283-290.

ANSBACHER, H. Perception of number as affected by monetary values. Archives of Psychology, New York, 1937, No. 215.

BEVAN, W., HELSON, H., & MAIER, R. A. The influence of context upon the estimation of number. American Journal of Psychology, 1963, 76, 464-469.

BEVAN, W., & TURNER, E. D. Assimilation and contrast in the estimation of number. Journal of Experimental Psychology, 1964, 67, 458-462.

BRUNSWIK, E. *Perception and the representative design of psychological experiments.* Berkeley: University of California Press, 1956.

KRUEGER, L. E. Apparent combined length of two-line and four-line sets. Perception & Psychophysics, 1970, 8, 210-214.

MOKRE, H. Über den Einfluss von Grösse und Abstand der Elemente auf die Mengenauffassung. Zeitschrift für Psychologie, 1928, 105, 195-225.

NYSSEN, R., & BOURDON, J. A new contribution to the experimental study of the size-weight illusion. Acta Psychologica, 1956, 12, 157-173.

STEVENS, S. S. On the psychophysical law. Psychological Review, 1957, 64, 153-181.

STEVENS, S. S. Regression effect in psychophysical judgment. Perception & Psychophysics, 1966, 1, 439-446.

TAVES, E. H. Two mechanisms for the perception of visual numerousness. Archives of Psychology, New York, 1941, No. 265.

TEGHTSOONIAN, M. The judgment of size. American Journal of Psychology, 1965, 78, 392-402.

WINER, B. J. *Statistical principles in experimental design.* New York: McGraw-Hill, 1962.

(Accepted for publication June 10, 1971.)