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Eigenvectors of the inertia tensor and perceiving the orientation of a hand-held object by dynamic touch

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Subjects wielded an object, hidden from view, and reported the orientation in which the object was positioned in the hand. The object consisted of a stem with two branches forming a V attached perpendicularly to the stem's distal end. The branches were differentially weighted so that the same spatial orientation of the object was associated with different orientations of its principal (symmetry) axes or eigenvectors. Perceived orientation was found to be dependent on the eigenvectors of the object's inertia tensor, computed about the point of rotation in the wrist, rather than on its spatial orientation. The results underscore the significance of the inertia tensor to understanding the perception of spatial properties by dynamic touch.

Many manual activities entail the grasping and wielding of objects. In performing these acts, the hand is usually in contact with only a part of the object, as when a hammer is grasped by its handle, swung to strike a nail, and returned to the table. Two kinds of touching are involved—cutaneous touching, in which the skin and deeper tissue are stimulated by the grasping of the object, and effortful or dynamic touching, in which muscles and tendons are stimulated by the wielding of the object (Gibson, 1966). Gibson (1966, p. 127) hypothesized that "the stimulus information from wielding can only be an invariant of the changing flux of stimulation in the muscles and tendons, an exterospecific invariant in this play of forces." Rotational motions about a fixed point of the kind characteristic of wielding about a joint—as when the hand rotates around the wrist or the forearm around the elbow in the manipulation of a hammer—follow from $N = \mathbf{I} \cdot d\omega/dt + \omega \times (\mathbf{I} \cdot \omega)$ (Goldstein, 1980). In wielding a given object, the torque N, angular velocity ω, and angular acceleration $d\omega/dt$, will vary from moment to moment. In contrast, **I** remains invariant. By Gibson's (1966) hypothesis, one might expect that **I** plays a prominent role in perception by dynamic touch.

I is the inertia tensor, represented mathematically by a symmetric 3 × 3 matrix (Goldstein, 1980). The diagonal terms (I_{xx}, I_{yy}, I_{zz}) quantify the object's rotational inertia with respect to the three axes. The terms above (I_{xy}, I_{xz}, I_{yz}) and below (I_{yx}, I_{zx}, I_{zy}) the diagonal quantify the object's rotational inertia in directions perpendicular to the axial rotations and reflect the asymmetries of the object's mass distribution about its point of rotation. Because $I_{ij} = I_{ji}$, **I** reduces to six independent numbers. Experiments have shown that, consonant with Gibson's (1966) hypothesis and with the expectation above, perceiving by dynamic touch is tied to **I**. Moments of inertia constitute the relevant independent quantity in perceiving by wielding the lengths of rods or rod segments that cannot be seen rather than other potentially relevant quantities such as mass, center of mass, center of oscillation, average torque, and average kinetic energy (Burton & Turvey, 1990; Pagano & Turvey, in press; Solomon & Turvey, 1988; Solomon, Turvey, & Burton, 1989a, 1989b; Turvey, Solomon, & Burton, 1989). Moments of inertia also seem to underlie the perception of the shapes of wielded objects hidden from view (Burton, Turvey, & Solomon, 1990). Moments and products of inertia together have been found to affect the perceived orientation, with respect to the hand, of occluded objects consisting of a stem and a branch perpendicular to the stem (Turvey, Burton, Pagano, Solomon, & Runeson, 1992). Specifically, subjects in the latter study grasped firmly the stem of an L-shaped object and through wielding perceived the direction of its branch relative to the hand. The research reported here is directed at this perceiving of orientation by wielding.

In line with previous research on dynamic touch, a reasonable expectation is that branch direction of an L-shaped object is a single-valued function of some property of **I**. The particular magnitudes of the moments and products of inertia cannot be candidate properties because they vary as a function of the length and weight of an object's parts and, therefore, will be different for different-sized objects that have coincident orientations. What is needed is a property of **I** that varies only according to the orienta-

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tion of the object. In pursuit of the relevant property, let us consider **I** more fully, particularly its behavior under coordinate transformations.

The calculations of **I**'s diagonal and off-diagonal terms are done with respect to a rectangular coordinate system with origin at O, that is, $Oxyz$. Patently, there are indefinitely many sets of three perpendicular axes that can be anchored at O, and the choice of axes can be arbitrary. For each choice of $Oxyz$, the components of **I** will differ, but the specificity of **I** to the dimensions of the object does not change. This is a basic property of tensors; **I** measured about one set of axes can be transformed to **I** measured about a different set of axes (Lovett, 1989). In general, a tensor is a hypernumber—a matrix of independent numbers that expresses a physical state of affairs and that behaves as a single quantity. Different rotations of a given coordinate system about O, as well as different translations of O, result in different tensorial components (the numbers in the matrix change). But the manner in which the tensor transforms is such that it continues to be specific to the environmental property it quantifies; it is invariant under coordinate transformation (Moon & Spencer, 1965, 1986).

By means of a transformation that rotates the axes $Oxyz$ about O, **I** can be expressed in an invariant form independent of $Oxyz$. This invariant form is with respect to the principal axes or eigenvectors of **I**—the only nonarbitrary coordinate system at O. A vector $\boldsymbol{a}$ is called an eigenvector of **I**, with eigenvalue λ, if $\mathbf{I} \cdot \boldsymbol{a} = \lambda \boldsymbol{a}$, or, equivalently, $(\mathbf{I}-\lambda\mathbf{1}) \cdot \boldsymbol{a} = 0$, where $\mathbf{1}$ is the Kronecker delta or unit matrix. The condition for the existence of a non-trivial solution is that the determinant of the coefficients vanishes, that is, $\det(\mathbf{I}-\lambda\mathbf{1}) = 0$. The roots of the cubic equation in λ expressed by the determinant are the eigenvalues, scalar quantities that are invariant over the indefinitely many coordinate systems anchored at O. For a symmetric tensor such as **I**, the three eigenvalues are distinct and the three eigenvectors (given by substituting the λ values into $[\mathbf{I}-\lambda\mathbf{1}] \cdot \boldsymbol{a} = 0$) are orthogonal. If the eigenvectors are chosen as the axes, **I** must take the diagonal form, with the eigenvalues on the diagonal and all other entries equal to zero (that is, no products of inertia) (Goldstein, 1980). The eigenvectors are the directions about which the mass of the object is distributed evenly; they are the symmetry or body axes. These principal directions are fixed in the object and rotate with it. The experiments of Turvey et al. (1992) demonstrated that the perception of a hand-held object's orientation is related to **I** and suggested that it is related, more specifically, to **I**'s eigenvectors. In the present article, we report two experiments directed at evaluating this eigenvectors hypothesis.

Both experiments were designed so that objects with coincident spatial orientations were characterized by differentially oriented eigenvectors. The expected outcome was that the subject's perception of orientation would go as the object's eigenvectors rather than the object's spatial orientation.

In both experiments, the wielded object consisted of a stem with two branches forming a V perpendicular to the stem. Two coordinate systems can be identified for this object. One coordinate system provides a reference frame for the spatial orientation of the branches relative to the stem and is depicted in Figure 1A. This reference frame consists of angles within the plane of the branches about axes coincident with the central axes of the stem. The second coordinate system (anchored at the object's point of rotation in the wrist) provides a reference frame for calculating **I** and, in turn, the eigenvectors of the object and is depicted in Figure 1B. Each eigenvector is defined by three x,y,z coordinates of these axes. By differentially weighting the branches, it is possible to construct V objects such that, when the objects are oriented similarly within the former reference frame, their eigenvectors are oriented differently within the latter reference frame. (In the experiments of Turvey et al., 1992, the orientations in the two reference frames covaried perfectly.) If per-

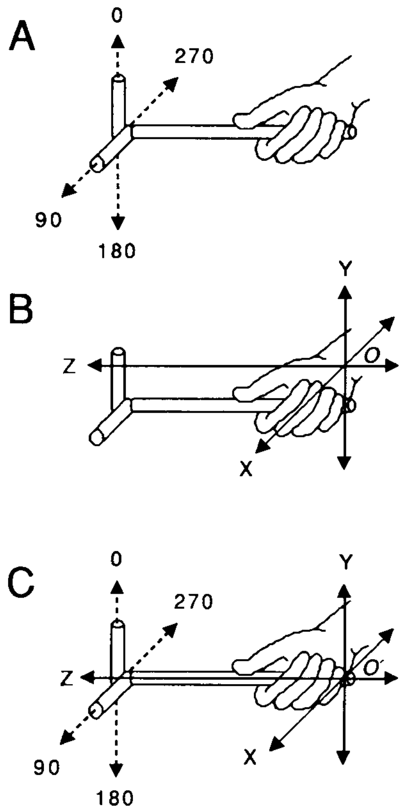

Figure 1. (A) A depiction of the coordinate system which provides a reference frame for the spatial orientation of the branches relative to the stem. This coordinate system consists of a plane perpendicular to the stem of the object. Branch orientations within this plane are defined relative to the subject's thumb, which lies along the longitudinal axis of the stem at 0°. Perceived orientation is defined within this reference frame. (B) A depiction of the coordinate system which provides a reference frame for the eigenvectors of the object. It consists of a set of axes with their origins at the point of rotation in the wrist. The *z*-axis is parallel to the stem, and the *y*-axis is perpendicular to the stem. Each eigenvector is defined by three coordinates of these axes. (C) The coordinate system of the eigenvectors can be transformed so as to allow for their expression in the spatial frame of reference of the branches.

ception of object orientation through dynamic touch is dependent only on the actual geometric orientation of the object, differentially weighted objects should be perceived to be at the same orientation within the spatial reference frame. If, however, the perception of object orientation is tied to the object's eigenvectors, differently weighted objects with coincident orientations within the spatial reference frame will be perceived to be at different orientations within that reference frame. As we will show below, the eigenvectors hypothesis can be put into an especially simple and useful form by taking further advantage of the transformation properties of tensors.

It is possible to derive the angular position of the eigenvectors in the spatial reference frame of the branches by translating the axes $Oxyz$ about O depicted in Figure 1B to a new point of reference O' located at the proximal end of the stem. As depicted in Figure 1C, this new point of reference allows the z-axis of the eigenvector's frame of reference to be coincident with the axis about which the orientations of the branches are defined. The transformation for moving the point of reference for **I** is called the *parallel axis theorem*. It states that the moment of inertia (I) about any axes equals the moment of inertia about parallel axes through the object's center of mass (I_{cm}) plus the product of the object's mass (m) and the square of the distance (a) between the two axes (Goldstein, 1980). That is, $I = I_{cm} + ma^2$. There is a similar parallel axis theorem for the products of inertia. The importance of the parallel axis theorem is that it provides a way to translate **I** from one point of reference to another. The major consequence of applying the theorem in the present case is that the x,y,z coordinates of the eigenvectors about O' can be translated into angles within the plane of the branches. The eigenvector extending from O' forward through the object (the z eigenvector) can be projected onto a plane orthogonal to the stem, or more specifically, the plane in which the branches lie. Accordingly, the eigenvectors hypothesis can now be expressed as a prediction that perceived orientation will be a single-valued function of the z eigenvector within the spatial reference frame of the branches.[1]

EXPERIMENT 1

In the first experiment, the more clockwise of the two branches of a V object was weighted in one third of the trials, the more counterclockwise branch was weighted in one third of the trials, and neither branch was weighted in one third of the trials. Subjects were asked to perceive the orientation of both branches. The predicted outcome of Experiment 1 was as follows: If perceived orientation is specific to the inertial eigenvectors, perceived orientations should be drawn away from the branch's spatial orientation to the orientation of the object's rotational-inertia symmetry axes. That is, in the conditions in which the more clockwise branch is weighted, perceived orientations should be more clockwise, and in the conditions in which the more counterclockwise branch is weighted, perceived orientations should be more counterclockwise. These expectations can be expressed succinctly in terms of the orientation of the z eigenvector in the spatial reference frame of the branches, as is shown below.

Method

Subjects. Six graduate students associated with the University of Connecticut participated on a volunteer basis in Experiment 1. Two subjects were men and 4 were women; all 6 subjects were right-handed.

Materials. The objects used in the experiment were constructed out of oak dowels 1.2 cm in diameter with a density of .711 g/cm³. The stems were 30 cm long with two 11.5-cm branches attached perpendicularly to the stem at their ends. When no weights were attached to either branch, the objects weighed 46.9 g. (A smaller object of the same proportions was used for instruction.) A 9.2-g thin strip of lead could be affixed lengthwise on a branch, which approximately doubled the branch's weight.

Apparatus. Figure 2 shows the experimental arrangement. The subject sat with the right forearm on a horizontal surface attached to the seat and occluded by a screen. A visible report dial mounted vertically in front of the subject consisted of two independently movable pointers. Only the 12 o'clock position was marked for the subject. From the experimenter's side, the position of the pointers could be matched onto angular markings.

Procedure. The angular separation between the two branches—the angle of the two branches forming a V—was always 90°, and the object was placed in the hand so that the branches were at 0° and 90°, 90° and 180°, 180° and 270°, and 270° and 360° within the spatial frame of reference, meaning that the bisector of the V was at 45°, 135°, 225°, and 315°, respectively.

In the first experiment, the conditions of (1) neither branch weighted, (2) the more counterclockwise branch (Branch 1) weighted, and (3) the more clockwise branch (Branch 2) weighted occurred equally often. Table 1 presents the **I**s for the 12 configurations (3 weight and 4 angular conditions) computed about the point of rotation O in the wrist (the mean distance between O and the stem was 6.8 cm). The tensor matrices in Table 1 were diagonalized to get the eigenvectors. The eigenvectors about O are given in Table 2. Using the parallel axis theorem (Goldstein, 1980), the **I**s were transformed by translating O at the point of rotation in the wrist 6.8 cm to O'. These new **I**s were diagonalized to get the eigenvectors about O'. The orientation of the z eigenvector (about O') in the spatial reference frame of the branches is given in Table 3. The equivalence of the eigenvectors over transformation from O to O' was confirmed by a multiple regression of the 12 new eigenvector orientations (see Table 3) on the original 3 × 3 coordinates (see Table 2), which yielded an r^2 of 1.0 ($p < .0001$).

The object was placed in the subject's hand, with the thumb on top, aligned with the stem, and parallel to the ground. In the course

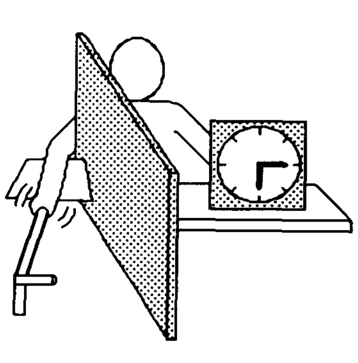

Figure 2. The apparatus used in the two experiments.

Table 1
Moments (e.g., I_{xx}) and Products (e.g., I_{xy}) of Inertia (g·cm²) About the Rotation Point O for the Objects of Experiment 1 as a Function of the Object's Spatial Orientation and Weighted Branch

Inertia Term	Bisector Orientation 45°	135°	225°	315°
	Counterclockwise Branch Weighted			
I_{xx}	4,249	4,417	4,521	4,281
I_{yy}	4,103	4,118	4,103	4,118
I_{zz}	241	446	514	310
I_{xy}	−34	−68	34	68
I_{xz}	153	317	−153	−317
I_{yz}	−649	−1,118	−1,283	−813
	Clockwise Branch Weighted			
I_{xx}	4,208	4,449	4,345	4,177
I_{yy}	4,505	4,008	4,045	4,009
I_{zz}	310	514	446	241
I_{xy}	−68	−34	68	34
I_{xz}	305	159	−305	−159
I_{yz}	−799	−1,262	−1,116	−653
	No Added Weight			
I_{xx}	3,257	3,393	3,393	3,257
I_{yy}	3,102	3,102	3,102	3,102
I_{zz}	228	364	364	228
I_{xy}	−34	−34	34	34
I_{xz}	153	159	−153	−159
I_{yz}	−597	−908	−914	−603

Note—**I** is a symmetric tensor, $I_{ij} = I_{ji}$.

of wielding about the wrist with the forearm remaining on the armrest, the thumb's relation to the ground plane would change. The subject was instructed to return the hand to the thumb-parallel-to-the-ground configuration before each adjustment of the pointers. The task in Experiment 1 was to position the pointers of the report dial to correspond to the felt orientation of the V at the end of the stem. The hand could be moved relative to the wrist, but the stem was not allowed to move within the hand. There was no time limit within a trial for wielding the object nor for adjusting the pointers of the report dial; the subject could wield the object and adjust the pointers as often as he/she needed to reach a confident judgment. Each subject was shown the smaller sample object before starting; the subjects did not see the actual objects. No practice or feedback of any sort was provided in the experiment. The subjects were told that the two branches could be positioned at any orientation in the plane perpendicular to the stem, and at any angular separation in relation to each other. The subjects were not informed that the branches could be of different weights, and they were asked to attend to both branches. Four trials at each configuration were given, for a total of 48 trials per subject. A different randomization of configurations across trials was used for each subject.

Results

For each trial in Experiment 1, the pointer angles were recorded and the bisector was determined. For example, if one branch was felt at 0° and the other at 90°, the bisector would be 45°. For each subject and each configuration, a resultant bisector was computed through circular statistics (Batschelet, 1965, 1978; see also Turvey et al., 1992, for a discussion and application of these statistics) from the four bisectors reported in the four trials at each configuration. Circular statistics were chosen because an arithmetic mean is not suited to measurements of angle. For example, an arithmetic mean of a 350° and a 30° judgment would be 190°, while the mean of 10° given by circular statistics is clearly more representative of the two judgments. The main component of circular statistics consists of finding an average judgment by taking the sine and cosine of each judgment, summing these judgments over repetitions, and transforming this sine and cosine combination back into the angle they pertain to.

Figure 3A shows the resultant perceived bisectors (perceived V orientations) for each configuration, averaged across subjects. As can be seen, the perceived V orientations with the clockwise branch weighted differed from the perceived orientations with the counterclockwise branch weighted, and this difference was in the clockwise direction. Our analysis of the data depicted in Figure 3A will proceed in two steps. First, we will investigate how the different conditions manipulated explicitly in the experiment—namely, orientation and differential weighting—affected perceived orientation. Second, we will evaluate the quantity manipulated implicitly and dictated theoretically—namely, the eigenvectors of **I**.

An analysis of variance (ANOVA) revealed significant effects for weight condition [$F(2,5) = 8.36, p < .01$] and V orientation [$F(3,10) = 56.23, p < .001$]. The interaction was insignificant [$F(6,30) = 1.25, p > .05$]. To determine whether the simple regressions depicted in Figure 3A differed, a multiple regression was conducted with

Table 2
Eigenvector Coordinates (Expressed in the Unit Vectors of $Oxyz$) for the Objects Used in Experiment 1 as a Function of the Object's Spatial Orientation and Weighted Branch

Eigenvector Coordinate	Bisector Orientation 45°	135°	225°	315°
	Counterclockwise Branch Weighted			
X^*x†	.837	.735	.756	.749
X^*y	−.560	−.685	−.663	−.676
X^*z	−.121	−.241	.236	.191
Y^*x	.576	.717	−.704	−.691
Y^*y	.836	.716	.728	.738
Y^*z	.116	.149	.211	.099
Z^*x	.034	.063	−.029	−.068
Z^*y	−.155	−.253	−.285	−.190
Z^*z	.951	.908	.891	.937
	Clockwise Branch Weighted			
X^*x	752	.801	.730	.867
X^*y	−.674	−.604	.693	.510
X^*z	−.190	−.222	.245	.118
Y^*x	.690	.650	−.724	−.528
Y^*y	.740	.777	.709	.868
Y^*z	.101	.229	.151	.126
Z^*x	.067	.030	−.061	−.036
Z^*y	−.190	−.288	−.256	−.160
Z^*z	.935	.892	.905	.951
	No Added Weight			
X^*x	.830	.778	.772	.819
X^*y	−.570	−.635	.644	.585
X^*z	−.148	−.226	.228	.157
Y^*x	.586	.675	−.684	−.607
Y^*y	.830	.756	.749	.813
Y^*z	.133	.197	.197	.134
Z^*x	.043	.040	−.039	−.045
Z^*y	−.187	−.270	−.271	−.188
Z^*z	.939	.900	.898	.939

†Read as x coordinate of X eigenvector.

Table 3
Spatial Orientation of the Bisector and the z Eigenvector (in Degrees) About O' as a Function of the Actual Orientation (in Degrees) of the Two Branches in Each Condition of Experiment 1

Actual Orientation		Bisector	z Eigenvector
Branch 1	Branch 2	Orientation	Orientation
	No Added Weight		
0	90	45	43.9
90	180	135	133.9
180	270	225	223.9
270	360	315	313.9
	Branch 1 Weighted		
0	90	45	25.5
90	180	135	115.5
180	270	225	205.5
270	360	315	295.5
	Branch 2 Weighted		
0	90	45	62.8
90	180	135	152.8
180	270	225	242.8
270	360	315	332.8

the continuous independent variable of spatial orientation and the categorical variable of weight condition (Pedhazur, 1982). A significant effect for weight [$F(2,68) = 190.62, p < .001$] revealed that the intercepts of the regression lines differed as a function of weight condition. In contrast, the nonsignificant interaction [$F(2,66) = 1.68, p > .05$] revealed that the slopes of the regression lines did not differ as a function of weight condition. Since weight condition was significant, it is not legitimate to characterize the data by using a single regression; rather, separate regression lines are needed, indicating that perceived V orientation was not a single-valued function of V's spatial orientation. The preceding outcome was expected on the eigenvectors hypothesis.

Figure 3B shows the perceived bisectors for each configuration, replotted as a function of the orientation of the z eigenvector relative to the stem. We are now in a position to address directly the question as to which better predicts perceived orientation: the actual bisector orientation, or the orientation of the eigenvectors? Regressing the 72 (6 subjects × 12 condition means) values corresponding to the perceived bisector orientations onto actual bisector orientations yielded an r^2 of .79 ($p < .0001$). Regressing perceived bisector orientations on the eigenvector orientations identified in Table 3, in contrast, yielded an r^2 of .82 ($p < .0001$). Multiple regression with only the 48 (6 subjects × 8 condition means) values from the weighted conditions (those for which the spatial and eigenvectors hypotheses predict differently, see Table 3) confirmed that eigenvector orientation was the relevant variable from which to predict perceived orientation: The r^2 was .85 ($p < .0001$), with the partial F corresponding to eigenvector orientation equal to 13.22 ($p < .001$) and the partial F corresponding to actual orientation equal to .32 ($p > .05$). Table 4 compares the simple regressions predicting perceived bisector orientation from actual bisector orientation, and eigenvector orientation for the 6 subjects in Experiment 1. Also shown in Table 4 is the r^2 and partial Fs for the multiple regression of both variables on perceived bisector orientation. In sum, perceived bisector orientation appears to be a single-valued function of I's eigenvectors about O', and, therefore, by the transformation laws of tensors, about O.

EXPERIMENT 2

The results of Experiment 1 suggest that perceived orientation is specific to I's eigenvectors. Perceived orientations were drawn away from the branch's spatial orientation to the orientation of the object's rotational-inertia symmetry axes. That is, in the conditions of Experiment 1, in which the more clockwise branch was

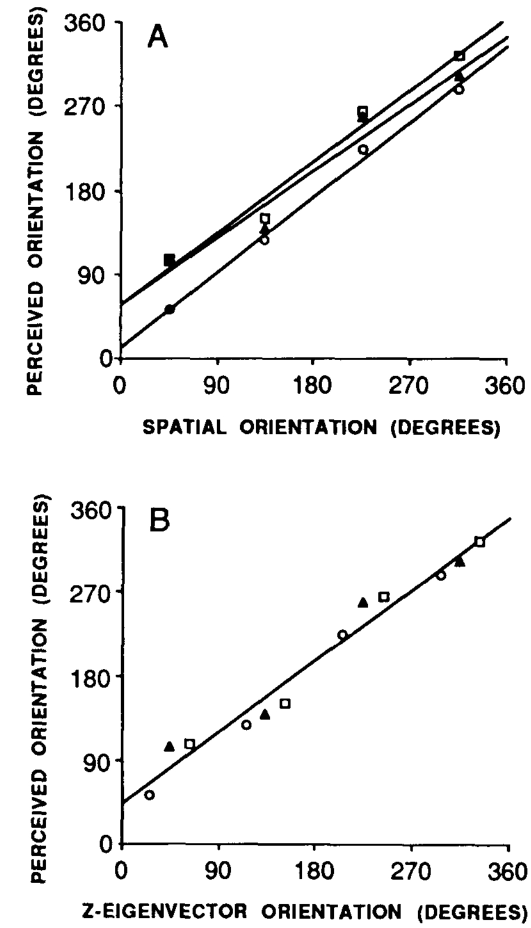

Figure 3. (A) Relation of perceived orientation to spatial orientation in Experiment 1. Open circles indicate conditions with the counterclockwise branch weighted; $y = 14.48+.90x$, $r^2 = .99$. Open squares indicate conditions with the clockwise branch weighted; $y = 65.32+.81x$, $r^2 = .97$. Closed triangles indicate conditions without added weight; $y = 66.70+.74x$, $r^2 = .94$. Each data point represents the 24 (6 subjects × 4 repetitions) judgments. (B) Perceived bisector orientation as a function of actual orientation of the z eigenvector about O' for the 6 subjects in Experiment 1; $y = -42.80+.86x$, $r^2 = .97$.

Table 4
Simple and Multiple Regression Predicting Perceived Bisector Orientation From Actual Bisector and From *z* Eigenvector Orientation About *O′* for the Subjects in Experiment 1

	Simple Regression		Multiple Regression		
				Partial *F*	
Subject	Actual	Eigenvector	r^2	Actual	Eigenvector
1	.84	.89	.97	1.21	11.94*
2	.91	.92	.94	.50	.90
3	.93	.89	.94	5.00	.32
4	.74	.83	.95	5.36	16.55†
5	.80	.87	.97	1.59	5.40
6	.92	.94	.97	.03	5.51
Overall	.79	.82	.85	.32	13.22‡

Note—All r^2 values are significant at $p < .01$. For individual simple regressions, $n = 12$; for the overall simple, $n = 72$. For individual multiple regressions, $n = 8$; for the overall, $n = 48$. $^*p < .05$. $^†p < .01$ $^‡p < .001$.

weighted, perceived orientations were more clockwise, and in the conditions in which the more counterclockwise branch was weighted, perceived orientations were more counterclockwise. Experiment 2 was designed to generalize these findings to the case in which subjects were informed that one of the branches was weighted. The subjects were asked to perceive the orientation of the weighted branch only. It was predicted that, as in the first experiment, the subjects' perception of orientation would be drawn away from the branch's spatial orientation and to the orientation of the *z* eigenvector about *O′*.

In Experiment 2, under conditions in which the unweighted branch was 90° clockwise with respect to the weighted branch, perceived orientation of the weighted branch should be more clockwise. Likewise, under conditions in which the unweighted branch was 90° counterclockwise with respect to the weighted branch, the perceived orientation of the weighted branch should be more counterclockwise. Note that for the conditions of the second experiment, the eigenvectors hypothesis predicted that perceived orientation would be drawn in the direction of the *unweighted* branch. For the conditions of Experiment 1, the eigenvectors hypothesis had predicted that perception would be drawn in the direction of the *weighted* branch. Though different, both predictions followed from the understanding that perceived orientation is a function of **I**'s eigenvectors, rather than the actual spatial orientation of either branch.

Method

Subjects. Seven graduate students associated with the University of Connecticut participated on a volunteer basis in Experiment 2. None of the subjects had participated in Experiment 1. Four subjects were men and 3 were women; all were right-handed.

Materials. The weighted objects from Experiment 1 were used.

Apparatus. The apparatus from Experiment 1 was used, with the modification that the report dial consisted of one pointer instead of two.

Procedure. The procedure for Experiment 2 was essentially the same as that used in the first experiment. The angle of the two branches forming a V was always 90°, and the object was placed in the hand so that the branches were at 0° and 90°, 90° and 180°, 180° and 270°, and 270° and 360° within the spatial frame of reference. On half of the trials, the unweighted branch was oriented 90° clockwise with respect to the weighted branch (e.g., the weighted branch was at 90° and the unweighted branch was at 180°), and it was 90° counterclockwise with respect to the weighted branch on the other half of the trials (e.g., the weighted branch was at 90° and the unweighted branch was at 0°). The orientation of the *z* eigenvector in the spatial reference frame of the branches is given in Table 5.

The task in Experiment 2 was to position the single pointer to correspond to the felt orientation of the weighted branch only. The subjects were told that the two branches could be positioned at any orientation in the plane perpendicular to the stem, and at any angular separation in relation to each other. The subjects were told that one branch would weigh approximately twice as much as the other branch, and they were asked to attend only to the heavier branch. Four trials at each configuration were given, for a total of 32 trials per subject.

Results

For each trial in Experiment 2, the angle of the single pointer was recorded. The results of the data averaged over trials and subjects are depicted in Figure 4A. An ANOVA showed that the difference between the two weight conditions was significant [$F(1,6) = 92.3$, $p < .001$], whereas the interaction between them was not [$F(3,18) < 1$]. To confirm that there were two different mappings between spatial and perceived orientation in Figure 4A, a multiple regression was conducted with the continuous independent variable of spatial orientation and the categorical variable of weight condition (Pedhazur, 1982). The multiple regression yielded an r^2 of 0.90 ($p < .0001$) with main effects of weight and spatial orientation [$F(1,53) = 33.33$ and 419.30, respectively, both *p*s $< .001$], identifying the presence of two mappings in Figure 4A.

Figure 4B shows the resultant perceived orientation for each configuration, replotted as a function of the orientation of the *z* eigenvector relative to the stem. As is evident from inspection of Figure 4B and the corresponding regression equation, perceived orientation was almost perfectly predicted by the the *z* eigenvector about *O′*. Table 6 compares the simple regression predicting perceived orientation from actual orientation, and eigenvector orientation, for the 7 subjects in Experiment 2. Also shown

Table 5
Spatial Orientation of the *z* Eigenvector (in Degrees) about *O′* as a Function of the Actual Orientation (in Degrees) of the Two Branches in each Condition of Experiment 2

Actual Branch Orientation		*z* Eigenvector
Weighted	Unweighted	Orientation
	Unweighted Branch Clockwise	
0	90	25.5
90	180	115.5
180	270	205.5
270	360	295.5
	Unweighted Branch Counterclockwise	
0	−90	−27.2
90	0	62.8
180	90	152.8
270	180	242.8

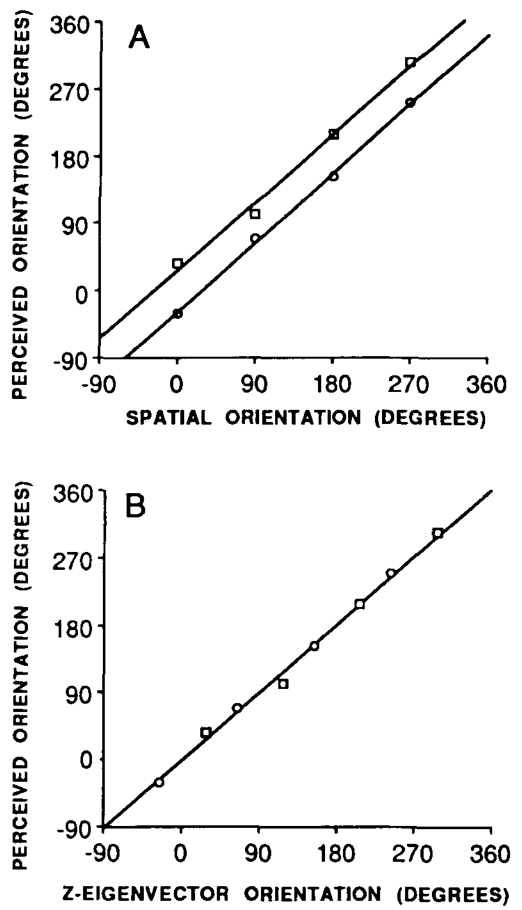

Figure 4. (A) Relation of perceived orientation to spatial orientation in Experiment 2. Open squares indicate conditions with the unweighted branch clockwise with respect to the weighted branch; $y = 27.59+1.02x$, $r^2 = .99$. Open circles indicate conditions with the unweighted branch counterclockwise with respect to the weighted branch; $y = -27.39+1.02x$, $r^2 = .99$. Each data point represents the 28 (7 subjects × 4 repetitions) judgments. (B) Perceived orientation of the weighted branch as a function of actual orientation of the *z* eigenvector about *O′* for the 7 subjects in Experiment 2; $y = -1.06+1.01x$, $r^2 = .99$.

in Table 6 are the r^2 values and partial *F*s for the multiple regression of perceived orientation on both variables. In sum, as concluded in Experiment 1, perceived orientation appears to be a single-valued function of **I**'s eigenvectors about *O′* and, therefore, by the transformation laws of tensors, about *O*.

DISCUSSION

The dominant performatory aspect of everyday manipulations tends to obscure the hand's role as an organ of perception. Everyday tasks require that objects be grasped, lifted, pushed, pulled, carried, inserted, turned, and so on. It is evident that these tasks are executed under the guidance of both the visual and the haptic perceptual systems, but the perceptual contribution of the hand can go unnoticed because attention is directed at the performance capabilities and because awareness is dominated by what is seen rather than what is felt (Gibson, 1966). It is the case, however, that deformations of skin and subcutaneous tissues, articulations of joints, and distortions of muscles and tendons are inevitable accompaniments of manipulation. In consequence, it has been suggested that the haptic perceptual system may well play a more fundamental role in the control of manipulatory activity than vision (Gibson, 1966).

The challenge of understanding the hand-related haptic perceptual subsystem is to identify the mechanical quantities that both affect it and relate systematically to the properties of objects. Most everyday touching of objects is perfunctory in the sense that only a small part of the object is in contact with the hand—as with a cup grasped in the act of drinking, or a pencil held in the act of writing, or a box held in the act of stacking, or a hammer held in the act of striking. Touching in the service of commonplace manual activity, therefore, is primarily dynamic in that the relevant tactile impressions are tied to the pattern of muscular and tendon tensile states brought about in the course of the activity (Carello, Fitzpatrick, Domaniewicz, Chan, & Turvey, 1992).

As noted in the introduction, a substantial body of experimentation has identified that an object's resistance to rotational acceleration is the mechanical quantity underlying the spatial capabilities of dynamic touch. The resistance occurs in different directions (tangential and normal to the angular rotations about each of the three spatial axes), with different magnitudes in these different directions. The resistance to rotational acceleration, therefore, is represented mathematically as a hypernumber or *holor*—an entity built up of several independent elements (Moon & Spencer, 1965, 1986). When holors transform in a specific way, they are referred to as tensors (Moon & Spencer, 1965, 1986); the holor that captures resistance to rotation—namely, **I**—satisfies the tensor criterion. In the present research, we have sought to understand whether the diagonalized form of **I** for three-dimensional motion about a fixed point *O* provides a basis for per-

Table 6
Simple and Multiple Regression Predicting the Perceived Orientation From Actual Orientation and From *z* Eigenvector About *O′* for the Subjects in Experiment 2

	Simple Regression		Multiple Regression		
				Partial *F*	
Subject	Actual	Eigenvector	r^2	Actual	Eigenvector
1	.80	.88	.89	.18	3.95
2	.89	.99	.99	2.68	51.66‡
3	.84	.90	.90	.02	3.53
4	.95	.99	.99	1.64	21.22†
5	.78	.86	.86	.07	2.70
6	.94	.97	.97	.60	7.45*
7	.89	.96	.96	.03	9.21*
Overall	.83	.90	.90	.16	33.33‡

Note—All r^2 values are significant at $p < .01$. For individual simple and multiple regressions, $n = 8$; for overall regressions, $n = 56$.
*$p < .05$. †$p < .01$. ‡$p < .001$.

ceiving the orientation of an object in the hand. Specifically, we have addressed the role of the I's principal directions or eigenvectors in constraining the awareness one has of a grasped object's disposition relative to the hand. We used an uneven mass manipulation to break the coincidence between the directions of an object's parts in space and the directions of its rotational-inertia symmetry axes. The results of the two experiments concur with the hypothesis that perceived orientation goes with the directions of the symmetry axes. In future experiments, the number of mass asymmetry conditions can be expanded, and the point of rotation can be varied, in order to provide stronger tests of the dependence of perceived orientation on the inertial eigenvectors. Such research can be guided by the hypothesis that, regardless of an object's shape, composition, or orientation of its parts in space, perceived orientation by dynamic touch will be a function of the orientation of the object's eigenvectors.

In summary, it seems that a hand-held object's resistances to rotation, characterized by the diagonalized form of **I**, provide a means of quantifying the information available to dynamic or effortful touch (recognizing that, in its ultimate form, the information is probably defined over properties of tissue deformations expressed through the strain and rate of strain tensors; see Solomon & Turvey, 1988; Solomon et al., 1989a, 1989b; Turvey, Solomon, & Burton, 1989). A hypothesis to guide future research is that **I** provides the domains for two sets of functions, one consisting of the eigenvalues that map onto perceived object "magnitudes" (e.g., length, shape, and possibly weight; Burton & Turvey, 1990; Burton et al., 1990; Pagano & Turvey, in press; Solomon & Turvey, 1988; Solomon et al., 1989a, 1989b), and one consisting of the eigenvectors that map onto perceived object "directions" (e.g., orientation in the hand).

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NOTE

1. The transformation of **I** about O to **I** about O' eliminates the requirement for a "vectorial" response measure—one that expresses the subject's angular responses in $Oxyz$ coordinates—as used by Turvey et al. (1992).

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