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What's up in mental rotation?

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We report two experiments on the influence of head tilt on mental rotation. In Experiment I, subjects decided whether dot patterns were or were not repeated about a line. Their reaction times (RTs) were consistent with the interpretation that they mentally rotated the patterns so that the line was subjectively vertical before making their decisions. When the subjects tilted their heads, the RT functions shifted in the direction of the tilt, indicating that the subjective vertical lay closer to the retinal than to the gravitational vertical. In Experiment II, subjects decided whether singly presented alphanumeric characters in various orientations were standard or backward (mirror-reversed). Again, analysis of their RTs suggested mental rotation to the standard upright, but the function was unaffected by head tilt; in this case, the subjects operated in subjective gravitational rather than retinal coordinates. The choice of retinal or gravitational coordinates may depend on whether the stimuli are interpreted egocentrically or as part of the external world.

Recent studies have suggested that human observers can "mentally rotate" the internal representations of various shapes in order to make certain decisions about them. For example, Shepard and Metzler (1971) showed subjects pairs of perspective line drawings of three-dimensional shapes, and had the subjects decide as quickly as possible whether or not each pair represented the same shape. The reaction time (RT) for correct "same" decisions was a linear function of the angular difference between the portrayed orientations. From this result, Shepard and Metzler inferred that the subjects mentally rotated one of the shapes in order to match it to the other. In other experiments, subjects have been required to match some visually presented shape, not to another visually presented shape, but to some internally generated image. For instance, Cooper and Shepard (1973) found that the RT for discriminating normal from backward (mirror-reversed) versions of singly presented alphanumeric characters was an increasing function of the angular departure of the character from the standard, upright orientation. The subjects evidently rotated each character mentally to the upright in order to match it with some internal representation of the standard version of the character. In similar fashion, observers apparently perform mental rotations to determine whether a schematic drawing of a human hand is of a left or a right hand (Cooper & Shepard, 1975), or whether a random, angular form is a standard or reflected version (Cooper, 1975).

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In experiments where shapes are mentally rotated to some standard or "upright" position, it is pertinent to ask whether the "upright" is defined according to the subject's retinal (or egocentric) coordinates, or whether it is tied to gravitation axes. In one study, by Corballis and Roldan (1975), the mental upright appeared to be bound more closely to retinal than to gravitational coordinates. The subjects made rapid judgments as to whether dot patterns were or were not symmetrical about a line. The RT functions suggested that the subjects mentally rotated the patterns so that the line was vertical before making their decisions. When the subjects tilted their heads, the functions shifted in the direction of the head tilt—although there was some evidence for a slight gravitational influence. But for the most part, it appeared that the mental vertical corresponded fairly closely to the *retinal* vertical.

Yet most of our perceptual experience appears to be based on gravitational coordinates. The perceptual world remains stable when we tilt our heads. Rock (1973) has suggested, however, that our initial perceptual impressions are formed in retinal coordinates, but are rapidly corrected for head tilt (or for rotations of stimuli). Citing the work of Shepard and Metzler (1971), Rock further suggested that this correction process may itself be one of mental rotation. If this is so, then we can perhaps understand why subjects might prefer to perform speeded tasks involving mental rotations within retinal rather than gravitational coordinates: it would be both unnecessary and a waste of time to correct for head tilt *as well as* for rotation of the stimulus.

There is some evidence, though, that subjects can correct for head tilt in advance of a stimulus. Attneave and Olson (1967) found that RTs to horizontal and vertical lines were shorter than those to oblique lines, even when the subjects tilted their

Table 1
Percentage of Errors for Each Type and Orientation of Pattern Under Each Head Tilt Condition in Experiment I

Head Position	Pattern Type	Orientation* (Deg) 0	45	90	135
Left (135 Deg)	Repeated	6.9	9.7	6.3	4.9
	Symmetrical	8.3	3.5	8.3	8.3
Upright (0 Deg)	Repeated	9.7	9.0	8.3	4.9
	Symmetrical	7.6	2.8	4.9	6.3
Right (45 Deg)	Repeated	6.9	5.6	7.6	7.6
	Symmetrical	9.7	6.9	9.0	2.1

*Clockwise from gravitational vertical

heads 45° so that with respect to *retinal* coordinates the former were then oblique and the latter horizontal and vertical. The responses were names which the subjects had previously learned to associate with the lines. Attneave and Olson also found that there was excellent transfer to the head-tilt condition if the names continued to be assigned according to gravitational directions, but negative transfer if they were assigned according to retinal directions. The subjects operated within a gravitational framework, even though the task was a speeded one. Attneave and Reid (1968) have shown, however, that subjects can adopt a retinal frame of reference for this task if explicitly instructed to do so.

We report two experiments here. The first is a variation of one reported by Corballis and Roldan (1975). Instead of investigating RT to detect whether or not dot patterns are symmetrical about a line, however, we studied how long it takes observers to decide whether or not patterns are *repeated* about a line. We measured RT as a function of the angular orientation of the line and the tilt of the subjects' heads. Our objectives were to determine whether mental rotation is necessary for this task, and if so, whether the subjects would mentally rotate the line to the gravitational or to the retinal vertical. The second experiment is concerned with the influence of head tilt on RT in the Cooper-Shepard task. In judging whether alphanumeric characters are forward or backward versions, would subjects mentally rotate them to the gravitational or to the retinal upright?

EXPERIMENT I

Method

Subjects. The subjects were six women and six men, all right-handed and possessing normal or corrected vision, and ranging in age from 18 to 24 years.

Stimuli. There were 96 stimulus patterns constructed from six basic half patterns, each consisting of a random cluster of six dots. Each half pattern was either reflected or repeated about a line. The dots and the line appeared white against a black background, and the line appeared in eight different orientations in 45° steps from 0° (vertical) to 315°. Since there was no absolute distinction between top and bottom of a pattern, however, there were effectively only four orientations, represented by the clockwise rotations of 0°, 45°, 90°, and 135° from the vertical. The same 12 symmetrical and 12 repeated patterns were photographed at each orientation to make up 96 slides, which were rear-projected onto a translucent screen. The patterns subtended about 10° along and 10° perpendicular to the axis—with slight variations—at the subjects' eyes. These patterns are the same as those used by Corballis and Roldan (1975), and examples are illustrated in Figure 1 of their report.

Procedure. The subject sat at a table about .85 m from the screen on which the patterns were displayed. In preparation for each presentation, he/she was instructed to fixate the screen so that the pattern would be centrally projected. He/she was to rest the index finger of each hand on a response button, and was told to press one button if the pattern was repeated about the line, the other if it was not. Subjects were not told that the nonrepeated patterns were symmetrical. Half of the subjects responded to repetition with the right hand, half with the left hand. Each subject was told to respond as rapidly as possible, but to try to avoid errors. Each trial began with a 500-msec warning tone, followed 1 sec later by presentation of the pattern for 2 sec. RT was measured from the onset of the pattern.

Each subject was first given several free practice trials with his/her head upright until it was clear that the task was understood. He/she was then given three sequences of trials, one with the head tilted 45° to the left, one with the head upright, one with the head tilted 45° to the right. The three conditions of head tilt were counterbalanced over subjects, according to a Latin square. Each sequence consisted of 24 practice trials followed by the 96 test trials, randomly ordered. The subject's head was held firmly in a padded wooden frame, and the level of head tilt was measured in terms of an imaginary line between the pupils of the subject's eyes. This line was tilted at 45° for the two head-tilt conditions, and was horizontal for the upright condition.

Results and Discussion

Errors. Percentages of errors under the different conditions of the experiment are shown in Table 1. Rank tests failed to reveal any significant trends in the data.

Reaction times. Mean RTs, for correct responses only, were computed for each judgment at each orientation and head-tilt condition for each subject, and were submitted to analysis of variance.

The first point of interest is that there was essentially no difference in overall RT between *yes* (repeated) and *no* judgments, $F(1,6) = .01$; the means were 1,032 and 1,030 msec, respectively. In this respect, the results are clearly different from those of Corballis and Roldan's (1975) study, where *yes* judgments referred to judgments of symmetry and were consistently and significantly faster than *no* judgments. In Experiment 3 of that study, in all other respects identical to the present experiment, the mean RTs for *yes* and *no* judgments were 819 and 904 msec, respectively. Thus it is clear that the change of instructions to the subjects did influence their performance. Moreover, if the two sets of results are taken together, it is clear that there is an overall advantage for symmetrical over repeated patterns (cf. Corballis & Roldan, 1974).

In most other respects, however, our results closely resemble those of Corballis and Roldan (1975).

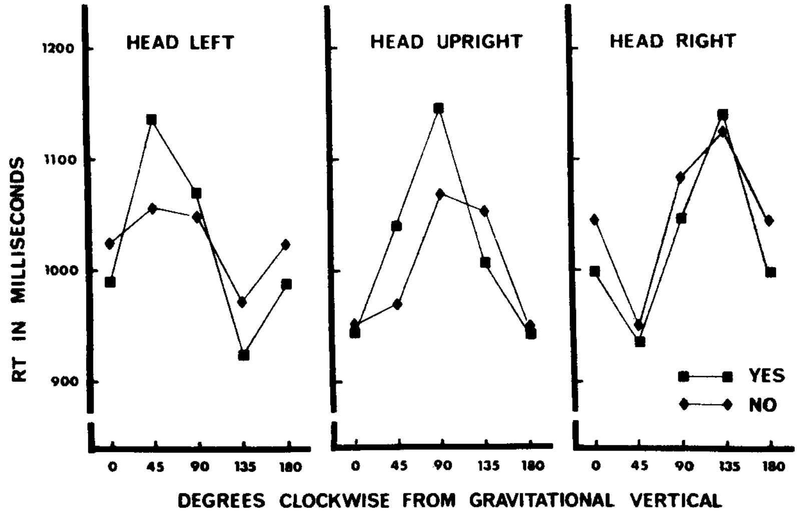

Figure 1. Mean RTs as a function of angular orientation of patterns, shown for each condition of head tilt, in Experiment I.

Figure 1 shows how mean RTs vary as a function of judgment and orientation under each condition of head tilt. RT is clearly a function of orientation, and the orientation function is more nearly invariant on retinal than on gravitational coordinates. When orientation is measured on gravitational coordinates, the interaction between orientation and head tilt is highly significant, $F(6,36) = 16.79$, $p < .001$. But if we suppose each retina to have been rotated 45° to the left or right under the head-tilt conditions (which neglects counterrolling of the eyes—see below) and measure orientation on these presumed retinal coordinates, the interaction is much reduced, $F(6,36) = 3.52$, $p < .01$—and is not significant if we adopt the conservative degrees of freedom recommended for repeated-measurements effects (Greenhouse & Geisser, 1959). The estimate of the population variance attributed to the interaction between orientation and head tilt is 5,206 $msec^2$ if orientation is measured on gravitational coordinates, but only 364 $msec^2$ if it is measured on retinal coordinates (cf. Vaughan & Corballis, 1969).

When measured on presumed retinal coordinates, the effect of orientation was highly significant, $F(3,18) = 24.68$, $p < .001$. If we suppose that the subjects mentally rotated the patterns so that the axes were retinally vertical, the orientation function would be ideally described by the contrast (−1, 0, 1, 0), where the coefficients are ordered clockwise in 45° steps from the retinal vertical. This contrast was highly significant, $F(1,18) = 73.66$, $p < .001$, leaving a residual that was insignificant, $F(2,18) = .19$. The results therefore support the notion that the subjects performed the task by mentally rotating the patterns to the upright on retinal coordinates.

We can, however, assess more accurately the relative roles of gravitational and retinal coordinates by estimating where the orientation which would produce the shortest RT might be expected to lie, on the assumption that RT increases linearly with angular departure from this orientation (see Corballis & Roldan, 1975, p. 227). This orientation for optimal detection of repetition can be taken to represent the subjective vertical. We computed it separately for each type of pattern under each condition of head tilt:

(1) *Head tilted left.* With the head tilted 45° to the left, the orientation was 32.9° to the left of the gravitation vertical for *yes* judgments and 34.7° to the left for *no* judgments. Again, we see that the subjective vertical lies closer to the retinal vertical than to the gravitational vertical, although there may have been some gravitational influence. Part of the discrepancy between the subjective vertical and the actual degree of head tilt, but perhaps not all of it, can be attributed to counterrolling of the eyes, which is thought to be about 10% of the angle of head tilt—in this case about 4°-5° (Miller, 1962).

(2) *Head tilted right.* With the head tilted 45° to the right, the computed orientations for *yes* and *no* judgments were 36.6° and 36.9° to the right, respectively. These lie even closer to the presumed retinal vertical, but again there may have been some gravitational influence beyond that attributable to counterrolling of the eyes.

(3) *Head upright.* In this case, the orientations for *yes* and *no* judgments were 6.1° to the left and 18.6° to the right, respectively. We are unable to explain why these values deviate so far from the gravitational (and presumed retinal) vertical; moreover, the discrepancy between them is particularly surprising in view of the close agreement between the corresponding values computed under the head-tilt conditions. We attribute no special importance to this anomaly.

We also estimated the rates of mental rotation to the estimated subjective vertical, and these are shown in Table 2. They were computed separately for each judgment, since the interaction between judgments

Table 2
Estimated Rates of Mental Rotation in Degrees Per Millisecond for Each Type of Pattern Under Each Head Tilt Condition in Experiment I

Pattern Type	Head Tilted: Left (135 Deg)	Upright (0 Deg)	Right (45 Deg)
Repeated	313	382	358
Symmetrical	809	440	415

and orientation was significant, $F(3,18) = 3.85$, $p < .05$—although, again, this is insignificant according to reduced degrees of freedom. Figure 1 shows that the orientation functions were flatter for *no* judgments (symmetrical patterns) than for *yes* judgments (repeated patterns), and Table 2 confirms that the estimated rotation rates were higher for *no* than for *yes* judgments. (We have no ready explanation for why one estimate—that of 809°/sec—is deviant from the others.)

Although other interpretations are certainly possible, our results are reasonably consistent with the notion that the subjects mentally rotated each pattern to the subjective vertical before judging whether or not it was repeated about the line. If anything, the case for mental rotation is more compelling here than in our previous study, where the subjects were required to detect symmetry rather than repetition (Corballis & Roldan, 1975). In studies of the mental rotation of letters and digits, the average rate of mental rotation is typically around 400°/sec (e.g., Cooper & Shepard, 1973). One might expect the mental rotation of the dot patterns we used in our study to be, if anything, slower than this, since they are somewhat more complex and unfamiliar than letters and digits—although it is still a moot point whether complexity influences the rate of mental rotation (Cooper, 1975). Table 2 shows that the estimated rotation rates for judgments of repetition lie between 300°/sec and 400°/sec, which is about what one might expect. The estimated rates for *no* judgments (symmetrical patterns) are faster, and include one estimate of 809°/sec, which is clearly beyond what one would expect on the basis of the earlier evidence. In our earlier study, too, the estimated rotation rates in the detection of symmetry ranged from 499°/sec to 1,095°/sec, with part of the variation probably attributable to practice (Corballis & Roldan, 1975). We suspect that subjects may sometimes be able to detect symmetry without having to rotate the pattern to the subjective vertical, especially if the pattern is familiar. Even in the present study, the subjects may well have based their *no* judgments on the detection of symmetry rather than on the nondetection of repetition, on at least some trials. This would help explain why *no* judgments took no longer, on average, than *yes* judgments.

But whether or not the RT functions reflect a process of mental rotation, it is clear that they show greater invariance with respect to retinal coordinates than with respect to gravitational coordinates. This belies the everyday observation that the world remains perceptually invariant when we tilt our heads. The next question is whether mental rotation is always tied more closely to retinal than to gravitational coordinates in speeded tasks. We therefore examined the influence of head tilt on the mental rotation of letters and digits in the paradigm described by Cooper and Shepard (1973).

EXPERIMENT II

Method

Subjects. The subjects were seven women and five men, drawn from undergraduate classes, and possessing normal or corrected vision.

Stimuli and apparatus. The stimuli were the uppercase letters G, J, and R, and the Arabic numerals 2, 5, and 7 (Letraset No. 193, mounted on double-glass slides). Each character was presented in normal and backward (mirror-reversed) form, in six different angular orientations ranging from 0° to 300° in 60° steps from the upright. There were thus a total of 72 slides.

The basic apparatus was the same as that in Experiment I. The stimuli were rear-projected, and subtended about 1°35′ at the subject's eyes.

Procedure. The basic procedure and conditions of testing were the same as those of Experiment I. Each trial began with a 500-msec warning tone, followed 1 sec later by presentation of a stimulus character for 1 sec.

Prior to the experimental trials, each subject was carefully instructed to press one of the two buttons if a character was displayed in the normal form and the other if it was backward (i.e., mirror-reversed). The nature of the task was illustrated by means of characters drawn on cards, and the subjects were also given practice trials with extra slides until they were sure they understood. They were told to respond as quickly as possible, without sacrificing accuracy. They used the index finger of each hand to press the buttons, and the assignment of hands to the responses "normal" and "backward" was counterbalanced over subjects.

The 72 slides were arranged in random order in the projector tray. During the experimental trials, each subject received the 72 slides three times, once each with his/her head upright, tilted left and tilted right. In this experiment, we attempted to compensate for counterrolling of the eyes (Miller, 1962) by tilting the head 66° to the left or right, so that the eyes would be tilted about 60°. The three conditions of head tilt were counterbalanced according to a Latin square.

Results and Discussion

All subjects achieved an error rate of less than 5%. When an error occurred, the RT for the cell was estimated from the remaining "correct" RTs in the block for the particular subject, character, and head tilt in question, according to the formula

$$RT' = \overline{RT}_f + \overline{RT}_o - \overline{RT},$$

where $\overline{RT}_f$ is the mean for the particular form (normal or backward) of the character, $\overline{RT}_o$ the mean for the particular angular orientation, and $\overline{RT}$

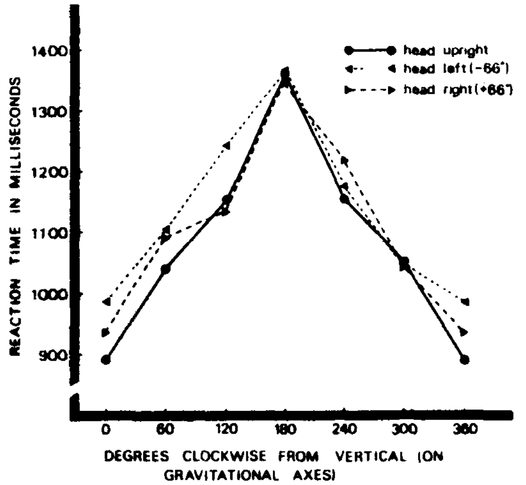

Figure 2. Mean RTs as a function of clockwise angular departure of the test characters from the gravitational upright, shown separately for each condition of head tilt, in Experiment II.

the overall mean for the block (Winer, 1971, p. 488).

The actual and estimated "correct" RTs were then subjected to analysis of variance. Judgments of "normal" proved to be significantly faster than judgments of "backward," $F(1,10) = 19.14$, $p < .01$; the mean RTs were 1,056 and 1,203 msec, respectively. There was a significant difference among the characters, $F(5,50) = 10.46$, $p < .01$; judgments about the letter R were the fastest (1,037 msec) and those about the numeral 7, the slowest (1,222 msec). And there was a significant effect due to angular orientation, $F(5,50) = 28.10$, $p < .01$.

There were no significant effects related to head tilt. Although RTs were slightly faster when the head was upright than when tilted, the main effect of head tilt was not significant, $F(2,20) < 1$. More importantly, the interaction between head tilt and angular orientation was also insignificant, $F(10,100) = 1.11$. Figure 2 shows the orientation functions for each head tilt condition, and it is clear that the functions are very largely unaffected by head tilt.

Following Cooper and Shepard (1973), we may suppose that the orientation functions reveal a process of mental rotation; the subjects mentally rotated the characters to the *gravitational* upright before judging them to be normal or backward. Assuming the rotation to be linear, the least-squares estimate of the rotation rate from our data is 441°/sec, which is close to the average rate estimated by Cooper and Shepard. However, Cooper and Shepard also observed a systematic, concave nonlinearity in the ascending and descending arms of their functions, suggesting more rapid or less frequent rotation of characters closer to the upright than of characters further away. Such a nonlinearity was not readily apparent in our data; the contrast (−3, −1, 1, 3, 1, −1) applied to the successive orientation from 0° to 300°, representing linear mental rotations, was highly significant, $F(1,50) = 138.8$, $p < .01$, leaving a residual that was not significant, $F(4,50) = 2.27$, $p > .05$. We do not know why the systematic nonlinearity did not appear in our data. It may be related to practice with the task. Cooper and Shepard tested their subjects in sessions lasting several hours, while individual sessions in our experiment were typically less than ½ h.

GENERAL DISCUSSION

With respect to the influence of head tilt, the results of Experiment II clearly contrast with those of Experiment I and with the results reported by Corballis and Roldan (1975). Mental rotation of alphanumeric characters was evidently accomplished within gravitational coordinates, while the processing of dot patterns was tied more closely to retinal coordinates. Since the subjects in the two experiments, and indeed in the Corballis and Roldan study, were tested in the same room, with the same equipment, and under the same ambient conditions, the discrepancy in results must be attributed to the difference either in stimulus materials or in tasks.

Corballis and Roldan (1975) suggested that the detection of symmetry might be retinally tied because symmetry is a fundamentally egocentric concept, and depends on the symmetry of the nervous system itself (cf. Julesz, 1971; Mach, 1897). This argument is somewhat weakened by the fact that Experiment I of the present study required judgments of repetition, not of symmetry. Although Mach (1897) evidently thought that the perception of symmetry and of repetition were closely related, it is not so obvious how perception of repetition might be related to the structure of the nervous system itself. Even so, it is still conceivable that judgments of repetition, or of sameness, might be most efficiently accomplished by an interhemispheric comparison, so that the optimal projection condition is to present the pattern so that the two halves are to either side of the retinal vertical meridian.

But it is also possible that whether the judgment is retinally or gravitationally based depends on the nature of the stimuli rather than the task. The dot patterns in Experiment I and in Corballis and Roldan's (1975) study were artificial, without obvious counterparts or associations in the real world. This could be why the subjects apparently interpreted them egocentrically. By contrast, alphanumeric characters are highly familiar, and we are used to seeing them in real-world contexts—on signs, shop windows, television screens, etc.—which are rigid with respect to gravity. In Experiment II, therefore, the subjects may have interpreted the stimuli as being part of the real-world environment.

We must await further converging experiments to discover more exactly what are the factors which determine when mental rotation is carried out in retinal coordinates and when in gravitational coordinates. For the present, we can at least observe that the coordinate system can vary, depending on the circumstances. Moreover, it is of some interest that subjects *can* use gravitational coordinates, as in Experiment II. This suggests that the correction for head tilt differs in one respect from the correction for rotation of a stimulus, at least in the processing of letters and digits. In their study, Cooper and Shepard (1973) showed that subjects were obliged to mentally rotate each character to the upright even if they were told its orientation (but not its identity) in advance. Similarly, in Corballis and Roldan's (1975) study, advance knowledge of the orientation of a pattern evidently did not free the subjects from having to mentally rotate it to the upright in order to judge its symmetry. These findings suggest that observers cannot mentally rotate an abstract frame of reference in preparation for some stimulus of known orientation. Yet the data of Experiment II suggest that observers *can* adjust their frame of reference in advance of a stimulus to compensate for head tilt. Any RT component reflecting a post-stimulus correction for head tilt must have been small, at best. Although the RTs under the head-tilt conditions were, on average, 38 msec longer than those under the head-upright condition, the difference was not statistically significant and was considerably less than the estimated 136 msec required to rotate a *stimulus* through 60°. Moreover, if the subjects had had to make a poststimulus correction for head tilt, they would surely have preferred to operate within retinal rather than gravitational coordinates.

Most objects in the world about us are fixed with respect to gravitational axes, and it is we ourselves who move about. It is therefore not surprising that we should be better adapted to compensate for changes in our own viewing position relative to objects in the world than for changes in the position of objects relative to ourselves. But the point is a fine one, since the actual identification of disoriented letters appears to be more or less independent of their orientation (Cooper & Shepard, 1973). It is only when more subtle discriminations are involved, such as detecting symmetry or repetition, or determining whether forms are standard or backward, that mental rotation seems to be required.

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