*Perception & Psychophysics*
*1987, 41 (6), 482-488*

# Influence of the diatonic tonal hierarchy at microtonal intervals

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**Previous studies have shown that listeners' perceptions of tones within a musical context are influenced by a hierarchy of tonal functions. This study investigated the influence of this hierarchy on the perception of microtones finer than the chromatic tones. Following the probe tone method introduced by Krumhansl and Shepard (1979), listeners rated each of the 48 eighth tones that occur within an octave interval according to how well that tone fit in with a major scale played just before the tone. Listeners discriminated in their preferences for tones separated by quarter-tone—but not eighth-tone—intervals, and their rating patterns conformed to a hierarchy of tonal functions extended to the quarter tones. A Fourier analysis of the data showed that listeners' rating patterns over the 48 eighth tones were necessarily and sufficiently estimated by a Fourier curve generated by the subset of 24 quarter tones, and that ratings for all tone frequencies not probed could be justifiably interpolated from this curve.**

In recent years, a number of investigators have reported empirical evidence of a diatonic hierarchy of tonal functions as an active cognitive mechanism in the perception of musical tones. These tonal functions are defined by a tone's stability relative to the tonic tone within a musical context, with less stable tones having a tendency to move toward more stable tones for resolution. Krumhansl and Shepard (1979) developed the *probe-tone method* to quantify listeners' perceptions of single tones within a musical context and found that the context defined such a tonal hierarchy for listeners. In that study, listeners heard an incomplete major diatonic scale followed by a single probe tone and judged how well that tone completed the scale they had just heard. Response patterns of listeners' preferences for tones conformed to the hierarchy of tonal functions, with the diatonic tones (e.g., C, D, E, F, A, B, C in the key of C major) preferred over the nondiatonic tones, and the three most stable tones, the tonic (C), dominant (G), and mediant (E) preferred, in that order, over all other tones.

Krumhansl and Kessler (1982) found a similar hierarchy of tonal stabilities using the probe-tone method for musical contexts other than a scale. In musical contexts consisting of either single harmonic chords or 3-chord cadences, listeners' response patterns conformed to a stable hierarchy of tonal functions within major/minor key modes. Evidence of a hierarchy of tonal stabilities has also been found in the musical perceptions of children (Krumhansl & Keil, 1982) and in the musical perceptions of people from non-Western cultures whose music is different from Western music (Castellano, Bharucha, & Krumhansl, 1984; Kessler, Hansen, & Shepard, 1984.)

These studies suggest that cognitive mechanisms exist for organizing the 12 chromatic tones within musical contexts according to relations consistent with music theory. However, studies on tonal discriminability show that listeners' perceptions of the tonal frequency continuum extend to a granularity finer than that of the chromatic tones. For example, in tasks in which listeners judge which of two intervals is larger, the discriminability threshold between intervals has been as low as 16 cents for musically trained listeners and as high as 75 cents for untrained listeners (Burns & Ward, 1978; Dobbins & Cuddy, 1982; Houtsma, 1968), all well below the 100-cent size of the semitone interval that separates two chromatic tones. Evidently, musical intervals that deviate in size within these discriminable microtonal ranges are categorically interpreted by listeners as being one of the familiar intervals derived from the chromatic tones (Burns & Ward, 1978; Seigel & Siegel, 1977a, 1977b; Zatorre & Halpern, 1979). How do listeners perceive tones that are finer than the chromatic tones in relation to the well-established hierarchy of tonal functions?

In this study, I addressed this question and investigated the influence of a diatonic hierarchy of tonal functions at microtonal intervals. To extract listeners' musical perceptions of nonchromatic tones, I extended the probe-tone method introduced by Krumhansl and Shepard (1979) and systematically considered probe tones that were denser than the chromatic tones. The objective was to determine how finely a set of probe tones must divide the octave to ensure that a profile of preference ratings captured all the significant psychological structure possible. To achieve this objective, I followed an approach used in sig-

This research was supported by the author's National Institute of Mental Health Traineeship (MH-15157-06) at Stanford University. I am indebted to Roger Shepard for many discussions of this work and for invaluable comments on drafts of this article. I also thank Eddie Kessler for assistance in generating the tone sequences, and Carolyn Foss, Tad Hogg, and Jeff Shrager for discussions of various portions of this work. Portions of this article were presented at the annual meeting of the Society for Mathematical Psychology, Chicago, 1984. Reprint requests and correspondence may be sent to: Daniel S. Jordan, Intelligent Systems Laboratory, Xerox Palo Alto Research Center, 3333 Coyote Hill Road, Palo Alto, CA 94304.

nal processing and performed a Fourier analysis on the preference rating data. Two constraints were observed to select a probe-tone sampling rate for this analysis: First, the set of probe tones had to contain the 12 chromatic tones, since these tones are the primitive units of Western music; second, the probe tones had to occur at equal (log-frequency) intervals to enable the use of a Fourier analysis algorithm. Together, these constraints imply that the selected probe-tone sampling rate must be an *integral multiple* of the semitone sampling rate (which, for the octave interval, is 12). A semitone sampling rate has been used in Fourier analyses of profiles in other studies (Cuddy & Badertscher, 1987; Krumhansl, 1982). In these analyses, I used a sampling rate of 48, which corresponds to the eighth tones.

It is possible that listeners will perceive these eighth tones as musically distinct elements despite the fact that they are rarely, if ever, heard in Western music. Alternatively, these tones, if distinctly perceived, might simply be interpreted as mistunings of their familiar chromatic-tone neighbors, and thus be "categorically assigned" the tonal function associated with the presumed mistuned chromatic tone. If so, we would expect the preference rating patterns of these microtones to not appreciably deviate from those obtained with the less dense chromatic-tone probes.

Performing a Fourier analysis on preference ratings also offers the ability to interpolate, on the Fourier curve, the ratings of tones not probed. Interpolation would be particularly useful for studying profiles obtained in musical contexts generated by tones not found in the chromatic scale, such as in scales from non-Western cultures or in artificially constructed scales (such as the distorted scales reported by Jordan & Shepard, 1987). Profiles from such musical contexts may be compared to profiles from a diatonic context, for example, by interpolating in the diatonic context ratings for tones not probed in that context, but probed in the nondiatonic context.

In the following experiment, listeners were presented with a major scale followed by one of the eighth-tone probes, and were asked to rate how well they thought the probe fit in with the scale they had just heard.

## METHOD

### Subjects

Twelve Stanford University undergraduates participated in the experiment for credit toward an introductory psychology course requirement. On the basis of their responses to a questionnaire concerning their musical background (handed out at the beginning of the course), listeners were selected over a wide range of musical experience. Each listener participated in a single 1-h experimental session.

### Apparatus

Tones were generated on a 64K Apple II Plus microcomputer operating a MountainComputer Music System synthesizer. The synthesizer samples at 32,000 points per second and filters out all frequencies above 14000 Hz. Sine waves with 30-msec rise and decay times were used for all tones. Amplification of the tones was equalized to compensate for the effect of the filters. Analog output was recorded on a Sanyo Plus D 55 stereo tape recorder, and during the experimental session was played on that same tape recorder.

### Stimulus Materials

In each trial, listeners were presented with either an ascending or descending sequence of tones consisting of the eight tones (including the octave) of a diatonic major scale, followed by a probe tone. In all sequences, the tone duration was 0.5 sec, with an intertone interval of 0.05 sec. The probe-tone duration also was 0.5 sec, and was presented 1.5 sec after the final tone of the sequence. Listeners were then given 7 sec in which to record their responses before the next trial began.

Different frequency ranges were used for variety. (Previous studies have shown that the structure of perceived musical relations is invariant under transposition; e.g., Attneave & Olson, 1971; Dowling, 1978; Dowling & Fujitani, 1971.) The ascending scale range was C–C′ (based on a 440-Hz A), and the descending scale range was D♯–D♯′. Probe tones consisted of tones obtained from a sampling rate of 49 (inclusive) points per octave—that is, the eighth tones—for each of the two frequency ranges. The equation for these eighth-tone frequencies is

$$f(i) = f_0 \, R^{i/48},$$

where $f_0 = f(0)$ is the frequency of the starting tone of the scale, $f(i)$ is the frequency of the tone that is $i$ eighth tones away from $f_0$ (where $i = 1, 2, 3, \ldots$ for ascending scales and $i = -1, -2, -3, \ldots$ for descending scales), and $R = 2$ is the ratio of the frequencies of an octave. The 98 trials were presented in four blocks of length 24 or 25, and the ascending and descending scales were randomly ordered across the four blocks.

### Procedure

Similar to Krumhansl and Shepard (1979), listeners were instructed to rate on a 7-point scale how well the probe tone fit in, musically, with the immediately preceding musical scale. On the rating scale, 1 was marked *does not fit at all* and 7 was marked *fits very well*. Listeners were encouraged to use the full range of the rating scale, and it was emphasized to them that the rating was a subjective rating. Listeners were told that the scales would be both ascending and descending and would vary in the frequency range they covered, and that the probe tone would not necessarily be one of the tones they heard in the musical scale.

## RESULTS

### Individual differences

Listeners were chosen to approximate a uniform distribution of musical background, which ranged from no musical background to 9 years of music instruction and performance. To evaluate the response consistency of listeners with such diverse musical backgrounds, intersubject correlations for the response profile were computed; these ranged from .387 to .914 and averaged .604. Results of applying the additive tree-fitting method, ADDTREE (Sattath & Tversky, 1977), to the matrix of correlations suggested two major groups of listeners plus two single listeners, a grouping that explained 92% of the variance in the correlational data (Formula 1 stress = .062). These two groups, referred to as Groups 1 and 2, differed significantly in musical background. The average numbers of years for performing (voice or instrument) were 5.3 and 2.3 [$t(8) = 2.51, p < .05$] and for musical training were 7.2 and 2.0 [$t(8) = 3.38, p < .01$] for Groups 1 and 2, respectively. The ungrouped listeners both reported having no musical background and are not considered further in this analysis.

## Ratings Following the Major Scale

Figure 1 shows the rating profiles for Groups 1 and 2. (To facilitate comparison, all scales in these figures are shown to range from C to C′, although the actual scale frequency ranges differed). Considering only the chromatic tones, these profiles are consistent with those obtained in earlier studies (e.g., Krumhansl & Kessler, 1982; Krumhansl & Shepard, 1979), with the more musical listeners more strongly exhibiting the hierarchy of tonal functions expected from music theory. However, listeners here also discriminated in their preferences of microtones finer than the chromatic tones.

For the set of chromatic tones, Group 1 listeners strongly preferred the eight diatonic tones (C, D, ... C′) over the five nondiatonic tones [$t(11) = 6.40, p < .0005$, and $t(11) = 4.89, p < .0005$, for the ascending and descending scales, respectively], but Group 2 listeners (for whom these differences were insignificant) did not. For the set of quarter tones, both groups showed a strong preference for the 13 chromatic quarter tones over the 12 nonchromatic quarter tones [for ascending scales, $t(23) = 5.69, p < .0001$, and $t(23) = 2.58, p < .01$; for descending scales, $t(23) = 5.15, p < .0001$, and $t(23) = 3.93, p < .001$, for Groups 1 and 2, respectively.] For the set of eighth tones, neither group showed a significant difference in preference for the 24 quarter-tonal versus the 24 non-quarter-tonal eighth tones.

## Fourier Analysis

To analyze the adequacy of the eighth-tone sampling rate, a Fourier analysis was performed on each of the pro-

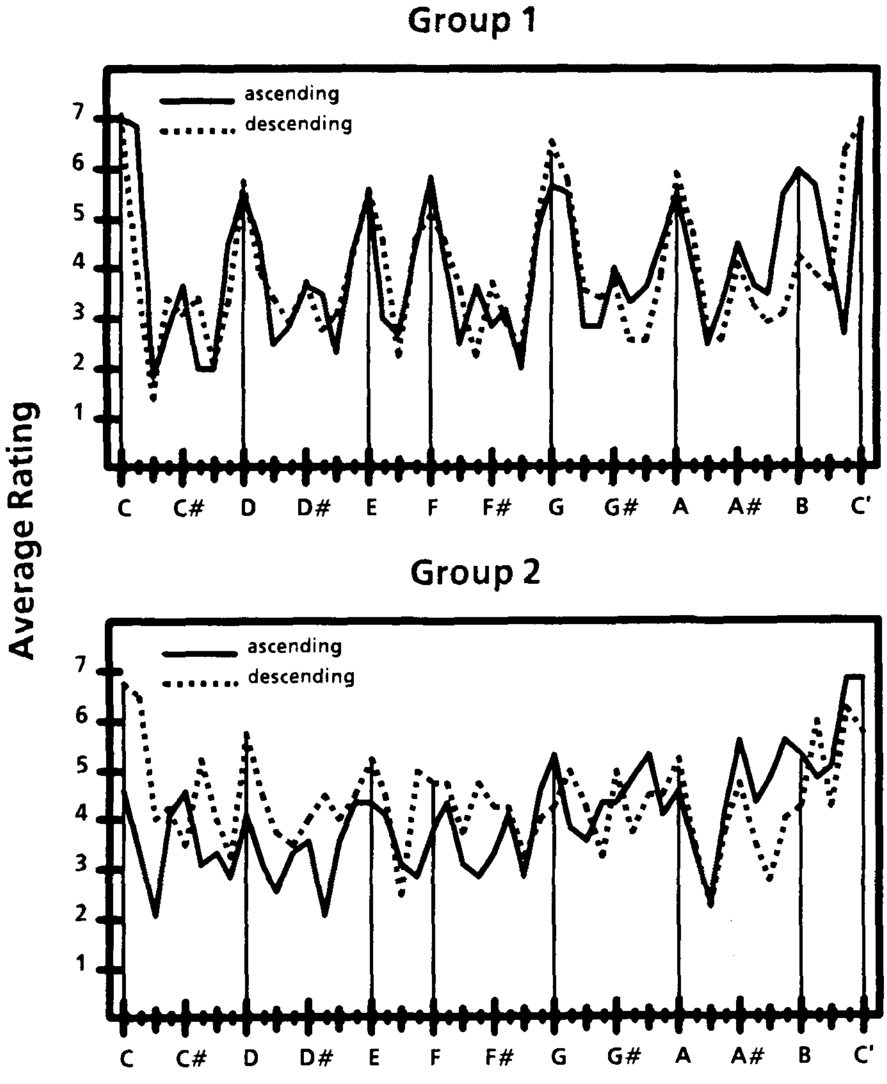


Figure 1. Rating profiles with eighth-tone probes obtained from the more musical Group 1 listeners (top) and the less musical Group 2 listeners (bottom) for ascending (solid-line) and descending (dashed-line) scale contexts. Vertical lines connect the diatonic tones (on the $x$-axis) to their ratings.

files. A Fourier analysis fits a continuous curve composed only of cosine (or, equivalently, sine) curves to a given set of data points. Each cosine term is called a *Fourier harmonic* (not to be confused with acoustic harmonics) and has an associated phase and amplitude. The equation for a Fourier curve, which exactly fits the data at the given points and gives estimates of all points in between, is:

$$f(\theta) = \sum_{k=0}^{ndata/2} A_k \cos(k\theta - \Phi_k),$$

where $f$ is the estimated Fourier curve over the variable $\theta$, *n*data is the number of data points, $A_k$ is the amplitude, and $\Phi_k$ is the phase of the *k*th harmonic in the curve. In these analyses, there are 48 data points, which determine 24 Fourier harmonics for each of the profiles.

In interpreting Fourier analysis results, our primary interest here is in a measure of the relative prominence of each of the Fourier harmonics to the total curve, called the PCV (for percent contribution to the total variance of the curve) of a harmonic:

$$\text{PCV}_k = \left(A_k^2 / \sum_k A_k^2\right) \times 100.$$

This equation shows that only the amplitude and not the phase is of interest. A graph of PCV versus Fourier harmonics is called a power spectrum.

**Calculating the Fourier parameters.** To calculate the Fourier parameters, the data were slightly altered in two ways. First, because we assume periodicity of the ratings on the octave interval (to meet the Fourier assumption of periodicity of the data), the ratings for the starting and ending tones of the scale were replaced with one rating equal to their average. Second, the 0th Fourier harmonic, which is an additive constant whose only effect is to translate the curve vertically, was factored out of the PCVs by subtracting from the data the mean rating for each profile (resulting in an amplitude of zero for this harmonic.)

The fast Fourier transform (see, e.g., Bracewell, 1973; Brigham, 1974) was applied to the data. Figure 2 shows the power spectra of the 24 Fourier harmonics for the profiles, and the appendix gives the amplitude, phase, and PCV of these harmonics. The most striking consistency in these results is that the combined PCV of the 12 upper harmonics (i.e., those above the 12th harmonic) is quite small: 15.7% and 8.0% for the ascending and descending scales, respectively, for Group 1; and 14.7% and 25.9% for the two respective scales for Group 2. Thus, in most of the analyses, one-half of the terms in the Fourier series, in particular the *last half*, together account for less than one-sixth of the total variance of the curve. This result implies that removal of these upper 12 harmonics would result in a *minimal* loss of information in the Fourier curve. Removal of these 12 harmonics reduces the number of harmonics to 12, or one-half that of the present number, and corresponds to reducing the sampling rate of probes from eighth tones to quarter tones.

However, the prominence of the 12th harmonic prohibits the removal of any more than these upper 12 harmonics from the analysis. This harmonic accounts for 37.5% and 35.2% of the total variance of the curve for the ascending and descending scales, respectively, for

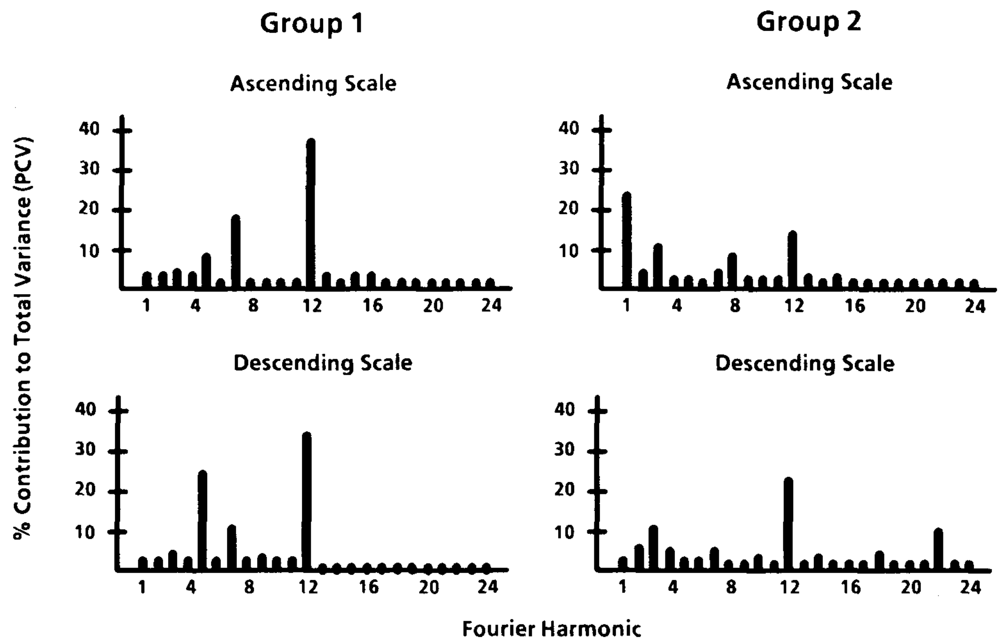


**Figure 2. Power spectra of the Fourier curves generated by the rating profiles for Group 1 (left) and Group 2 (right) listeners for ascending (top) and descending (bottom) scale contexts.**

Group 1 listeners, and 15.5% and 23.2% of this variance for the respective scales for Group 2 listeners.

### Comparison of Estimated Ratings From Lower Sampling Rates

Using the current set of data, estimated ratings of eighth tones were interpolated from Fourier curves generated by probe tones at the quarter-tone and semitone sampling rates and compared with the *actual* ratings of these eighth tones. The quarter-tone and semitone sampling rates were simulated by selecting, along the frequency continuum, every other data point and every fourth data point, respectively. Correlations between estimated and actual eighth tone ratings for each of these lower sampling rates and for the two listener groups are given in Table 1. The highest correlations occurred for eighth-tone ratings estimated from a quarter-tone Fourier curve; they ranged from .704 to .917. By comparison, the correlations for eighth-tone ratings estimated from a semitone Fourier curve ranged from only .347 to .581. Additionally, across sampling rates and scale contexts, correlations were higher for the more musical Group 1 listeners.

## DISCUSSION

The results of this experiment show that: (1) the influence of a diatonic hierarchy of tonal functions on listeners' perceptions of musical tones extends to frequencies finer than those of the 12 chromatic tones—specifically, to the 24 quarter tones, but not, apparently, to the much finer 48 eighth tones; (2) a Fourier curve generated by a rating profile obtained with quarter-tone probes estimates, with acceptable accuracy, the curve generated with eighth-tone probes—again verifying that listeners' discriminability level for tones in this rating task is closer to quarter-tone than to eighth-tone intervals; and (3) in fact, a quarter-tone sampling rate of probes is both *necessary* and *sufficient* for capturing, in a preference rating profile, the relevant psychological information, as indicated by our ability to remove, with minimal loss of information, the upper 12 and *only* the upper 12 Fourier harmonics from Fourier curves obtained at an eighth-tone sampling rate. This last point implies that ratings of tones not probed may be justifiably interpolated from the Fourier curve of a profile obtained with quarter tones (and not, in particular, from a curve obtained with only the chromatic tones).

**Table 1**
**Correlations Between Actual and Estimated Eighth-Tone Ratings**

| | Scale | |
|---|---|---|
| Listeners | Ascending | Descending |
| Estimated from Quarter-Tone Fourier Curve | | |
| Group 1 | .863 | .917 |
| Group 2 | .745 | .704 |
| Estimated from Semitone Fourier Curve | | |
| Group 1 | .530 | .581 |
| Group 2 | .432 | .347 |

The necessary and sufficient status of the quarter-tone sampling rate, together with the unnecessary status of the eighth-tone sampling rate for profiles, implies a discriminability interval of an eighth tone to a quarter tone, or 25-50 cents, for this rating task. This interval agrees with those reported in earlier discriminability studies (Burns & Ward, 1978: Dobbins & Cuddy, 1982; Houtsma, 1968).

The results of this experiment therefore extend earlier findings that a hierarchy of tonal functions strongly influences listeners' perceptions of tones within a musical scale context, and, furthermore, that this influence varies with musical background (Krumhansl & Kessler, 1982; Krumhansl & Shepard, 1979). Evidently, this hierarchy includes quarter tones as well. However, Krumhansl and Shepard (1979) found (in their second experiment) that, for a similar rating task, listeners did not discriminate between quarter tones. In that experiment, listeners rated how well a probe tone that occurred in either the octave above or the octave below a preceding major scale finished that scale. Possibly, this octave difference between probe and scale tones has an effect on listeners' discriminability for tone preferences. Octave changes of tones have been found to be disruptive in the recognition of melodies (Deutsch, 1972, 1978; Dowling & Hollombe, 1977) and in interference effects on pitch memory (Deutsch, 1973). Also, any preservation properties of octave equivalence in general have been questioned (Kallman, 1982; Thurlow & Erchul, 1977). However, the effect of octave differences on probe-tone discriminability for this rating task remains to be investigated.

It has been suggested that the division of the octave into 12 chromatic steps optimally satisfies cognitive processing constraints given our sensory-level perceptual abilities (Balzano, 1980; Dowling, 1978; Shepard, 1982). Following this premise, the discriminability of quarter tones as observed in this study might occur because the 12 nonchromatic quarter tones, each being midway between two chromatic tones, are maximally distant from, and therefore most easily distinguishable from, the chromatic tones.

### Interpretation of Fourier Harmonics

Certain individual Fourier harmonics emphasize specific diatonic features of listeners' preference rating patterns for tones. The 12th harmonic, by far the most prominent for most profiles, corresponds to a cosine curve with a period of 1 semitone. Given a phase shift such that the rating for the tonic (C) is a maximum, this curve produces other maxima only on the chromatic tones, and minima only on the nonchromatic quarter tones (see Figure 3). Thus, this harmonic maximally differentiates the 12 chromatic from the 12 nonchromatic quarter tones,

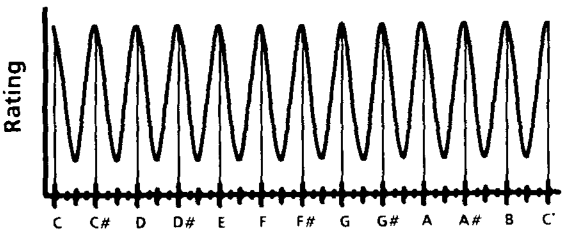


**Figure 3. Illustration of the 12th Fourier harmonic, which has a period of 1 semitone, cycling through tonal frequencies such that the chromatic tones receive a maximum rating and the nonchromatic quarter tones a minimum rating.**

verifying that listeners distinguish, rather sharply, between these tones and, in particular, tend to rate the chromatic tones high and the nonchromatic quarter tones low. However, this tendency as indicated by this harmonic is less pronounced for the less musical listeners.

Each of the fifth and seventh Fourier harmonics, also very prominent in the profiles, corresponds to a cosine curve that, given a phase shift such that the rating for the tonic (C) is a maximum, produces the following partial ordering of the chromatic tones based on ratings: ( {C}, {G, F}, {D, A♯}, {A, D♯}, {E, G♯}, {B, C♯}, {F♯}). Thus, each of these two harmonics captures the tendency of listeners to most strongly prefer the tonic tone C, followed by G (the dominant) and F (the subdominant)—that is, the most stable tones in the hierarchy of tonal functions. (Beyond these three tones, however, this partial ordering of chromatic tones departs from that given by listeners' ratings; e.g., listeners clearly prefer A over A♯).

The first Fourier harmonic, which was the most prominent for the less musical listeners in the ascending context, has a period of 1 octave and, with proper phase shift, corresponds to a trend effect due to pitch height. Apparently, for less musical listeners, pitch height is a principal factor in the musical interpretation of tones, consistent with findings from earlier studies (e.g., Attneave & Olson, 1971; Krumhansl & Shepard, 1979).

Thus, in this study, a Fourier analysis of the data highlighted the influence of specific diatonic structures on listeners' perception of tones, and conclusively verified that, at least for this rating task, probing listeners at quarter-tone intervals sufficiently samples cognitive representations of music to reveal their influence on the musical perception of *all* tonal frequencies within a diatonic context. These results suggest that this signal-processing approach may prove useful in analyzing psychological data involving cognitive phenomena other than—as well as including—those concerned with musical perception.

## REFERENCES


Attneave, F., & Olson, R. K. (1971). Pitch as a medium: A new approach to psychological scaling. *American Journal of Psychology*, **84**, 147-166.

Balzano, G. J. (1980). The group-theoretic description of twelvefold and microtonal pitch systems. *Computer Music Journal*, **4**, 66-84.

Bracewell, R. (1973). *The Fourier transform and its applications*. New York: McGraw-Hill.

Brigham, E. O. (1974). *The fast Fourier transform*. Englewood Cliffs, NJ: Prentice-Hall.

Burns, E. M., & Ward, W. I. (1978). Categorical perception—phenomenon or epiphenomenon: Evidence from experiments in the perception of melodic musical intervals. *Journal of the Acoustical Society of America*, **63**, 456-468.

Castellano, M. A., Bharucha, J. J., & Krumhansl, C. L. (1984). Tonal hierarchies in the music of North India. *Journal of Experimental Psychology: General*, **113**, 394-412.

Cuddy, L. L., & Badertscher, B. (1987). Recovery of a tonal hierarchy: Some comparisons across age and levels of musical experience. *Perception & Psychophysics*, **41**, 609-620.

Deutsch, D. (1972). Octave generalization and tune recognition. *Perception & Psychophysics*, **11**, 411-412.

Deutsch, D. (1973). Octave generalization of specific interference effects in the memory for tonal pitch. *Perception & Psychophysics*, **13**, 271-275.

Deutsch, D. (1978). Octave generalization and melody identification. *Perception & Psychophysics*, **23**, 91-92.

Dobbins, P. A., & Cuddy, L. L. (1982). Octave discrimination: An experimental confirmation of the "stretched" subjective octave. *Journal of the Acoustical Society of America*, **72**, 411-415.

Dowling, W. J. (1978). Scale and contour: Two components of a theory of memory for melodies. *Psychological Review*, **85**, 341-354.

Dowling, W. J., & Fujitani, D. S. (1971). Contour, interval, and pitch recognition in memory for melodies. *Journal of the Acoustical Society of America*, **49**, 524-531.

Dowling, W. J., & Hollombe, A. W. (1977). The perception of melodies distorted by splitting into several octaves: Effects of increasing proximity and melodic contour. *Perception & Psychophysics*, **21**, 60-64.

Houtsma, A. J. M. (1968). Discrimination of frequency ratios. *Journal of the Acoustical Society of America*, **44**, 383.

Jordan, D. S., & Shepard, R. N. (1987). Tonal schemas: Evidence obtained by probing distorted musical scales. *Perception & Psychophysics*, **41**, 489-504.

Kallman, H. J. (1982). Octave equivalence as measured by similarity ratings. *Perception & Psychophysics*, **32**, 37-49.

Kessler, E. J., Hansen, C., & Shepard, R. N. (1984). Tonal schemata in the perception of music in Bali and in the West. *Music Perception*, **2**, 131-165.

Krumhansl, C. L. (1982, August). *A quantitative description of musical key structure*. Paper presented at the meeting of the Society for Mathematical Psychology, Princeton, NJ.

Krumhansl, C. L., & Keil, F. C. (1982). Acquisition of the hierarchyof tonal functions in music. *Memory & Cognition*, **10**, 243-251.

Krumhansl, C. L., & Kessler, E. J. (1982). Tracing the dynamic changes in perceived tonal organization in a spatial representation of musical keys. *Psychological Review*, **89**, 334-368.

Krumhansl, C. L., & Shepard, R. N. (1979). Quantification of the hierarchy of tonal functions within a diatonic context. *Journal of Experimental Psychology: Human Perception & Performance*, **5**, 579-594.

Sattath, S., & Tversky, A. (1977). Additive similarity in trees. *Psychometrika*, **42**, 319-345.

Shepard, R. N. (1982). Geometrical approximations to the structure of musical pitch. *Psychological Review*, **89**, 305-333.

Siegel, J. A., & Siegel, W. (1977a). Absolute identification of notes and intervals by musicians. *Perception & Psychophysics*, **21**, 143-152.

Siegel, J. A., & Siegel, W. (1977b). Categorical perception of tonal intervals: Musicians can't tell sharp from flat. *Perception & Psychophysics*, **21**, 399-407.

Thurlow, W. R., & Erchul, W. P. (1977). Judged similarity in pitch of octave multiples. *Perception & Psychophysics*, **22**, 177-182.

Zatorre, R. S., & Halpern, A. R. (1979). Identification, discrimination, and selective adaptation of simultaneous musical intervals. *Perception & Psychophysics*, **26**, 384-395.

## APPENDIX
### Fourier Analysis of Profiles Obtained with Eighth-Tone Probes

| Scale | Harmonic | $A_k$ | $\Phi_k$ | % Variance |
|---|---|---|---|---|
| Group 1 Listeners | | | | |
| Ascending | 0 | 0 | 0 | 0 |
| | 1 | 0.36172 | −0.87854 | 0.03770 |
| | 2 | 0.34845 | −0.97535 | 0.03499 |
| | 3 | 0.37988 | −0.75124 | 0.04158 |
| | 4 | 0.31794 | 1.34296 | 0.02913 |
| | 5 | 0.55243 | −0.79977 | 0.08794 |
| | 6 | 0.17086 | 2.64485 | 0.00841 |
| | 7 | 0.79263 | 0.86778 | 0.18104 |
| | 8 | 0.20061 | 2.15739 | 0.01160 |
| | 9 | 0.18518 | 1.72080 | 0.00988 |
| | 10 | 0.27194 | 0.86625 | 0.02131 |
| | 11 | 0.10928 | 1.46113 | 0.00344 |
| | 12 | 1.14066 | 0.12268 | 0.37493 |
| | 13 | 0.31722 | 1.16531 | 0.02900 |
| | 14 | 0.16630 | 1.66393 | 0.00797 |
| | 15 | 0.33796 | 0.98731 | 0.03291 |
| | 16 | 0.32577 | 1.50744 | 0.03058 |
| | 17 | 0.26303 | 1.43315 | 0.01994 |
| | 18 | 0.11551 | 0.96211 | 0.00384 |
| | 19 | 0.12583 | 1.00663 | 0.00456 |
| | 20 | 0.14835 | 1.98914 | 0.00634 |
| | 21 | 0.20385 | −1.53354 | 0.01197 |
| | 22 | 0.14742 | 2.38246 | 0.00626 |
| | 23 | 0.10862 | 0.27416 | 0.00340 |
| | 24 | 0.06583 | 0 | 0.00125 |
| Descending | 0 | 0 | 0 | 0 |
| | 1 | 0.13128 | 0.56492 | 0.00560 |
| | 2 | 0.20070 | −0.31399 | 0.01309 |
| | 3 | 0.41369 | −1.45812 | 0.05562 |
| | 4 | 0.06807 | −1.20699 | 0.00151 |
| | 5 | 0.90633 | −0.64747 | 0.26696 |
| | 6 | 0.26365 | 1.22678 | 0.02259 |
| | 7 | 0.59547 | 0.78342 | 0.11523 |
| | 8 | 0.28432 | −1.42002 | 0.02627 |
| | 9 | 0.38792 | −1.94964 | 0.04891 |
| | 10 | 0.15544 | −1.10777 | 0.00785 |
| | 11 | 0.09515 | −1.45463 | 0.00294 |
| | 12 | 1.04025 | 0.04568 | 0.35168 |
| | 13 | 0.11691 | −0.72692 | 0.00444 |
| | 14 | 0.08313 | −0.44751 | 0.00225 |
| | 15 | 0.11172 | −1.18250 | 0.00406 |
| | 16 | 0.02134 | 0.32707 | 0.00015 |
| | 17 | 0.22629 | −1.04235 | 0.01664 |
| | 18 | 0.20254 | 0.29281 | 0.01333 |
| | 19 | 0.09015 | 0.97537 | 0.00264 |
| | 20 | 0.20474 | 0.84848 | 0.01362 |
| | 21 | 0.21376 | 2.57854 | 0.01485 |
| | 22 | 0.16858 | 3.04309 | 0.00924 |
| | 23 | 0.03766 | 2.45576 | 0.00046 |
| | 24 | 0.01542 | 0 | 0.00008 |
| Group 2 Listeners | | | | |
| Ascending | 0 | 0 | 0 | 0 |
| | 1 | 0.70625 | −1.07255 | 0.23956 |
| | 2 | 0.33585 | −0.78593 | 0.05417 |

### APPENDIX (Continued)

| Scale | Harmonic | $A_k$ | $\Phi_k$ | % Variance |
|---|---|---|---|---|
| | 3 | 0.49767 | −0.70842 | 0.11895 |
| | 4 | 0.25528 | 0.91294 | 0.03130 |
| | 5 | 0.24983 | 1.01471 | 0.02998 |
| | 6 | 0.09931 | 0.34705 | 0.00474 |
| | 7 | 0.32899 | −0.24069 | 0.05198 |
| | 8 | 0.44535 | 1.19930 | 0.09526 |
| | 9 | 0.24491 | −0.92969 | 0.02881 |
| | 10 | 0.08697 | −0.62523 | 0.00363 |
| | 11 | 0.27057 | −1.16278 | 0.03516 |
| | 12 | 0.56808 | −0.33636 | 0.15500 |
| | 13 | 0.25004 | −1.23159 | 0.03003 |
| | 14 | 0.09250 | 0.33477 | 0.00411 |
| | 15 | 0.27572 | 0.40620 | 0.03651 |
| | 16 | 0.19673 | 0.69709 | 0.01859 |
| | 17 | 0.24920 | −0.04573 | 0.02983 |
| | 18 | 0.09968 | 2.56178 | 0.00477 |
| | 19 | 0.06691 | 0.11236 | 0.00215 |
| | 20 | 0.08516 | 2.97045 | 0.00348 |
| | 21 | 0.05120 | 0.58153 | 0.00126 |
| | 22 | 0.19379 | 0.25065 | 0.01804 |
| | 23 | 0.06931 | −0.56560 | 0.00231 |
| | 24 | 0.02854 | 0 | 0.00039 |
| Descending | 0 | 0 | 0 | 0 |
| | 1 | 0.25248 | 0.52555 | 0.03856 |
| | 2 | 0.35518 | 0.13154 | 0.07630 |
| | 3 | 0.43441 | 0.04248 | 0.11414 |
| | 4 | 0.33036 | −0.74298 | 0.06601 |
| | 5 | 0.20628 | −0.87489 | 0.02574 |
| | 6 | 0.22793 | −0.69848 | 0.03142 |
| | 7 | 0.32783 | 0.50416 | 0.06500 |
| | 8 | 0.13542 | 1.53233 | 0.01109 |
| | 9 | 0.14506 | 0.52622 | 0.01273 |
| | 10 | 0.25997 | 1.08575 | 0.04088 |
| | 11 | 0.18773 | 0.92343 | 0.02132 |
| | 12 | 0.61942 | 0.34302 | 0.23207 |
| | 13 | 0.10045 | 2.38498 | 0.00610 |
| | 14 | 0.26129 | 0.42689 | 0.04129 |
| | 15 | 0.09802 | 1.03199 | 0.00581 |
| | 16 | 0.08526 | 1.26041 | 0.00440 |
| | 17 | 0.19418 | −0.65046 | 0.02281 |
| | 18 | 0.26295 | −0.00282 | 0.04182 |
| | 19 | 0.04634 | 0.56156 | 0.00130 |
| | 20 | 0.12766 | 1.93422 | 0.00986 |
| | 21 | 0.06779 | 3.01182 | 0.00278 |
| | 22 | 0.42201 | 0.54676 | 0.10772 |
| | 23 | 0.12708 | 2.30176 | 0.00977 |
| | 24 | 0.13542 | 0 | 0.01109 |

Fourier equation:

$$f(\theta) = \sum_{k=0}^{ndata/2} A_k \cos(k\theta - \Phi_k),$$

Legend: $f$ is the estimated Fourier curve over the variable $\theta$, $n$data = 48 is the number of data points, $A_k$ is the amplitude of the $k$th harmonic in the curve, $\Phi_k$ is the phase (in radians) of the $k$th harmonic in the curve, and % Variance is the percent of variance explained by the $k$th harmonic.