Perception & Psychophysics
1988, 44 (5), 500

Notes and Comment

A theoretical note on R. Ulrich, "Threshold models of temporal-order judgments evaluated by a ternary response task"

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Recently, Ulrich (1987) published an important unification and extension of existing temporal-order judgment models. The aim of this note is to present a simple theorem for Ulrich's general model, in order to help one grasp Ulrich's theoretical notions intuitively. The notation is in strict accordance with Ulrich's.

Under the assumptions of Ulrich's (1987) Theorem 1 and the additional assumption that $E[D_R]$ and $E[D_L]$ are both finite, we can prove the following:

THEOREM. *The area A between the two functions Y_L and Y_R is equal to the sum of the expected thresholds: $A = E[C_{XY}] + E[C_{YX}]$.*

PROOF. First, note that: $Y_L(d) - Y_R(d) = P\{-C_{YX} \leq A_Y - A_X \leq C_{XY}\} = P\{si\} \geq 0$ (see also Ulrich, 1988). Hence, we may write

$$A:=\int_{-\infty}^{+\infty}|Y_L(d)-Y_R(d)|\,dd$$

$$=\int_{-\infty}^{+\infty}[Y_L(d)-Y_R(d)]\,dd$$

$$=\int_{-\infty}^{0}[Y_L(d)-Y_R(d)]\,dd+\int_{0}^{+\infty}[Y_L(d)-Y_R(d)]\,dd$$

$$=-\int_{-\infty}^{0}Y_R(d)\,dd+\int_{0}^{+\infty}[1-Y_R(d)]\,dd$$

$$+\int_{-\infty}^{0}Y_L(d)\,dd-\int_{0}^{+\infty}[1-Y_L(d)]\,dd$$

$$=-\int_{-\infty}^{0}P(D_R\leq d)\,dd+\int_{0}^{+\infty}[1-P(D_R\leq d)]\,dd$$

$$+\int_{-\infty}^{0}P(D_L\leq d)\,dd-\int_{0}^{+\infty}[1-P(D_L\leq d)]\,dd.$$

For any random variable X with finite expectation, we have that

$$E[X]=-\int_{-\infty}^{0}P(X\leq u)\,du+\int_{0}^{+\infty}[1-P(X\leq u)]\,du$$

(Kendall & Stuart, 1969, p. 93). Applying this to the last expression, we get $A = E[D_R] - E[D_L]$. Finally, from the definitions of $D_R \equiv U + C_{XY}$ and $D_L \equiv U - C_{YX}$, we also have $A = E[U + C_{XY}] - E[U - C_{YX}] = E[C_{XY}] + E[C_{YX}]$. Q.E.D.

Hence, the area between Y_L and Y_R is equal to the sum of the means of the two thresholds and is, more importantly, independent of the arrival latency distributions. Thus, it may be taken as a nonparametric index of the response bias toward the intermediate category, which is not confounded by the perceptual sensitivity per se. It may also be seen that, apart from temporal-order judgments, the index A can generally prove useful for the investigation of the problems associated with psychophysical tasks that allow for three response categories (see Woodworth & Schlosberg, 1954, chap. 8, for an elementary exposition).

I wish to thank Rolf Ulrich for the frank discussion of his work. Address correspondence to Wolfgang Schwarz, Philipps University, Department of Psychology, Gutenbergstr. 18, D-3550 Marburg, Federal Republic of Germany.

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(Manuscript received May 10, 1988;
accepted for publication May 16, 1988.)