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Sum of responding as a function of sum of reinforcement on two-key concurrent schedules

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Four pigeons pecked for food reinforcers delivered by several two-key concurrent schedules. The sum of the rates of reinforcement provided by the component schedules varied from 25 to 300 reinforcers/h. The ratio of the rates of reinforcement remained constant at 1:4. The sum of the rates of responding generated by the component schedules increased with increases in the sum of the rates of reinforcement which the components provided. The increase in response rate was predicted by equations proposed by Catania (1963) and by Herrnstein (1970). But the data conformed more closely to Herrnstein's equation.

Several theories predict that the sum of the rates of responding generated by the components of a concurrent schedule will increase with increases in the sum of the rates of reinforcement that the components provide. For example, Catania (1963) proposed that Equation 1 describes the rates of responding generated by the components of a concurrent schedule:

$$P_1 = \frac{cR_1}{(R_1 + R_2)^{5/6}}. \quad (1)$$

P_1 is the rate of responding generated by a rate of reinforcement equal to R_1. R_2 is the rate of reinforcement generated by responding on the other component schedule. C is a constant of proportionality. The formula for the rate of responding generated by component 2 is identical, but subscripts 1 and 2 are interchanged.

Catania's formula for the sum of the rates of responding generated by the component schedules appears in Equation 2:

$$P_1 + P_2 = c(R_1 + R_2)^{1/6}. \quad (2)$$

Taking the log of both sides of Equation 2 yields Equation 3. Equation 3 predicts that the log of the sum of the rates of responding will be a linear function of the log of the sum of the rates of reinforcement. The function will have a slope equal to 1/6, and a y intercept equal to log c:

$$\log(P_1 + P_2) = 1/6 \log(R_1 + R_2) + \log c. \quad (3)$$

Herrnstein (1970) proposed that Equation 4 describes the rate of responding generated by each component of a concurrent or multiple schedule:

$$P_1 = \frac{kR_1}{R_1 + mR_2 + R_0}. \quad (4)$$

Again, P_1 is the rate of responding generated by a rate of reinforcement equal to R_1. R_2 is the rate of reinforcement generated by responding on the other component schedule (P_2). And k, m, and R_0 are parameters estimated from the data with certain restrictions.

K is the subject's asymptotic rate of responding, measured in responses per time unit. Its size varies only with the subject and the topographical form of the response. Its size does not change with changes in reinforcement (Herrnstein, 1974). M is a parameter which describes how closely the component schedules are related. The value of m is 1.0 for subjects responding on concurrent schedules. R_0 is the rate of reinforcement which the subject obtains spontaneously, without responding. It is measured in reinforcers per time unit (Herrnstein, 1974). Its value may change as an orderly function of several variables. For example, Herrnstein and Loveland (1974) argued that the value of R_0 should increase relative to R_1 and R_2, and perhaps absolutely, as the subject's need for the programmed reinforcers decreases. They assumed that subjects turn increasingly to other sources of reinforcement as they become satiated for the programmed one.

Herrnstein's formula for the sum of the rates of responding generated by the components of a concurrent schedule appears in Equation 5:

$$P_1 + P_2 = \frac{k(R_1 + R_2)}{R_1 + R_2 + R_0}. \quad (5)$$

The equation predicts that the sum of the rates of

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responding will increase with increases in the sum of the rates of reinforcement unless $R_0 = 0$. If $R_0 = 0$, then $P_1 + P_2$ will equal k, which does not depend on $R_1 + R_2$.

Increases in the sum of the rates of responding with increases in the sum of the rates of reinforcement have been confirmed for pigeons (Catania, 1963; Findley, 1958) and people (Schmitt, 1974) responding on Findley concurrent schedules. But they have not been confirmed for pigeons responding on two-key (Fantino, Squires, Delbrück, & Peterson, 1972; McSweeney, 1975) or three-key (Davison & Hunter, 1976) concurrent schedules. Findley schedules present both component schedules on the same response manipulandum. The subjects change schedule by responding on a second manipulandum. Two-key and three-key concurrent schedules present the components on different response manipulanda. The subjects change schedule by moving from one manipulandum to the other.

The results of these experiments do not prove that the sum of the rates of responding increases with increases in the sum of the rates of reinforcement for Findley, but not for two-key concurrent schedules, however. First, the Findley studies varied the ratio of the rates of reinforcement when they varied the sum. The two-key studies held the ratios constant. Changes in the ratios may have contributed to the differences between the studies. Second, the two-key studies are difficult to interpret. Fantino et al. (1972) studied only three total rates of reinforcement over the range from 9 to 900 reinforcers/h. They did find an increase in the sum of the rates of responding with increases in the sum of the rates of reinforcement up to an intermediate value. But the rate of responding decreased when the rate of reinforcement increased to 900 reinforcers/h. This decrease may have been produced by other variables, such as transient states of satiation (cf. Collier & Myers, 1961), which may confound the effects of reinforcement. If so, then the Findley studies also might have found a decrease in response rates, if they had studied a rate of reinforcement as high as 900 reinforcers/h. The McSweeney (1975) study varied the subjects' body weight as well as the total rate of reinforcement. The lengthy manipulation of body weight that occurred between successive changes in the total rate of reinforcement may have obscured orderly changes in the rate of responding which would have otherwise appeared.

The prediction that the sum of the rates of responding increases with increases in the sum of the rates of reinforcement should be tested in a study that employs a two-key concurrent procedure. Such a study could test predictions of Catania's and Herrnstein's theories, investigate differences between the rates of responding generated by Findley and two-key concurrent schedules, and provide evidence that the sum of the rates of responding increases with increases in the sum of the rate of reinforcement even when the ratio of the rates is held constant.

METHOD

Subjects

Four experimentally experienced pigeons served as subjects. Subjects 3174 and 1530 were White Carneaux pigeons. Subjects 6443 and 60 were homing pigeons.

Apparatus

The apparatus was a standard three-key, Grason-Stadler experimental enclosure for pigeons, Model E6446C, enclosed in a Grason-Stadler Model E3125A-300 sound-attenuating chamber. The two outside keys were used as response manipulanda. The key located to the subject's left, as it faced the wall containing the food magazine, was illuminated with white light. The key located to the subject's right was illuminated with red light. Responses on these keys were recorded and each response produced a brief feedback click. The middle key, located directly above the food magazine, remained dark throughout the experiment. It did not produce a feedback click when pecked, and pecks on it were not recorded. A houselight, shining through a Plexiglas panel near the ceiling at the right side of the box, illuminated the chamber throughout the experimental session. Electromechanical equipment, located in another room, scheduled the experimental events.

Procedure

Subjects, maintained at approximately 80% of their free-feeding body weights, responded on a series of concurrent variable interval variable interval (conc VI VI) schedules of reinforcement. The ratio of the rates of reinforcement supplied by the component schedules was held constant at 1:4. But the sum of the programmed rates of reinforcement varied from 25 to 300 reinforcers/h. The schedules used and the number of sessions for which each was presented appear in Table 1 in order of presentation. In each case, the component schedule listed first was presented on the key illuminated with white light. The component schedule listed second was presented on the key illuminated with red light. The key which provided the higher rate of reinforcement changed with most changes of schedule.

Pecks emitted during a 3.0-sec changeover delay (COD) were not reinforced at any time. CODs were initiated by all switches from one key to the other. Pecks were also ineffective, and the key lights were extinguished during reinforcement. Reinforcers consisted of 3-sec presentations of the magazine which contained mixed grain. A 20-interval series, generated by a procedure out-

Table 1
Number of Sessions for Which Each Schedule Was Conducted

	VI Schedule†		Number of Sessions: Subject			
Sum*	White Key	Red Key	6443	3174	1530	60
300	1 min	15 sec	19	19	19	19
150	30 sec	2 min	15	15	15	15
75	4 min	1 min	14	14	14	13
37.5	2 min	8 min	12	13	13	14
37.5	8 min	2 min	27	27	27	26
75	1 min	4 min	19	24	18	18
150	2 min	30 sec	20	19	27	21
300	15 sec	1 min	11	9	9	11
50	6 min	1.5 min	16	13	15	24
25	12 min	3 min	17	20	16	15
25	3 min	12 min	11	11	10	10
50	6 min	1.5 min	16	16	17	17

**Of rates of reinforcement* *†80% free-feeding weight* lined by Catania and Reynolds (1968, Appendix 2), programmed the interreinforcer intervals.

Sessions terminated when 40 reinforcers had been collected for all schedules except the conc VI 3-min VI 12-min. Terminating the sessions after 15 reinforcers for this schedule prevented the sessions from becoming extremely long. Sessions were conducted daily, 6 to 7 times per week. Subjects responded on each schedule until their responding had stabilized. Responding was considered to be stable when the sum of the rates of responding, generated by the component schedules over the last five sessions, all fell within the range of the rates generated during the preceding sessions.

RESULTS

The rates of responding emitted, and the rates of reinforcement obtained, by each subject on each component of each concurrent schedule appear in Table 2. Each rate represents the mean of the rates generated over the last five sessions for which each schedule was presented. The rates were obtained by dividing the number of responses emitted on, or the number of reinforcers obtained from, each component schedule by the total session time. The time for which the magazine was available was excluded from total session time. The rates obtained when the schedules were conducted in order of increasing and decreasing rates of reinforcement have been averaged because hysteresis did not occur. Rates of responding are reported in responses per minute; rates of reinforcement are reported in reinforcers per hour. The more favorable schedule is the component of each concurrent schedule which provided the higher rate of reinforcement. The less favorable schedule is the component which provided the lower rate of reinforcement.

Figure 1 presents the log of the sum of the rates of responding plotted as a function of the log of the sum of the obtained rates of reinforcement. The fifth set of axes in each figure represents the mean of the data generated by all subjects. Each of the other set of axes represents the data generated by an individual subject. The best fitting straight line, estimated by a least squares technique, has been drawn for each subject, and its equation has been reported. The slopes of the best fitting lines in Figure 1 are smaller than the .167 slope reported by Catania (1963) for subjects responding on Findley concurrent schedules, while maintained at 80% of their free-feeding weights.

Herrnstein's theory predicts that the function which relates the inverse of the sum of the rates of responding to the inverse of the sum of the rates of reinforcement should be a straight line (cf. Cohen, 1973). The formula for this line appears in Equation 6. The y intercept is $1/k$. The slope is R_0/k.

$$\frac{1}{(P_1 + P_2)} = \frac{R_0}{k}\left(\frac{1}{R_1 + R_2}\right) + \frac{1}{k}. \qquad (6)$$

Figure 2 presents the inverse of the sum of the rates of responding plotted as a function of the inverse of

Table 2

Rates of Responding Generated by and Rates of Reinforcement Obtained from the Components of Several Concurrent Schedules for Subjects Maintained at 80% of Their Free-Feeding Weight

Subject		VI 3 min VI 12 min	VI 2 min VI 8 min	VI 1.5 min VI 6 min	VI 1 min VI 4 min	VI 30 sec VI 2 min	VI 15 sec VI 1 min
				More Favorable Component Schedule			
6443	Responses	28.2	33.5	23.9	39.0	30.0	38.1
	Reinforcers	18.7	29.3	34.4	55.7	119.1	231.4
3174	Responses	22.8	23.5	19.7	28.1	26.8	34.2
	Reinforcers	18.5	26.3	32.5	52.9	109.0	234.4
1530	Responses	26.0	25.6	41.1	32.7	34.1	33.0
	Reinforcers	16.6	27.0	36.0	52.8	110.2	234.4
60	Responses	30.0	42.7	43.1	44.5	40.9	47.8
	Reinforcers	16.8	27.5	38.7	53.9	109.2	237.7
Mean	Responses	26.8	31.3	31.9	36.1	33.0	38.3
	Reinforcers	17.6	27.5	35.4	53.8	111.9	233.9
				Less Favorable Component Schedule			
6443	Responses	2.8	7.5	8.1	6.6	7.2	4.3
	Reinforcers	4.7	8.0	8.4	11.8	25.0	30.6
3174	Responses	6.7	4.7	7.9	6.9	6.9	4.7
	Reinforcers	4.4	4.3	8.6	11.5	21.0	27.6
1530	Responses	10.2	8.5	5.8	7.5	12.2	11.5
	Reinforcers	4.7	6.3	4.4	8.9	21.5	40.3
60	Responses	5.5	12.0	5.4	8.5	10.3	4.8
	Reinforcers	3.3	6.2	6.5	9.0	25.0	29.8
Mean	Responses	6.3	8.2	6.8	7.4	9.2	6.1
	Reinforcers	4.3	6.2	7.0	11.1	23.1	32.1

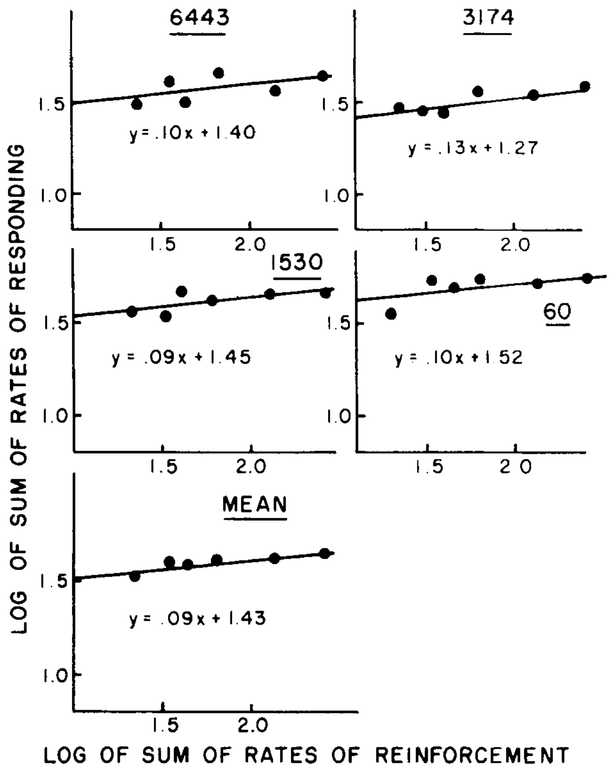

Figure 1. Log of the sum of the rates of responding as a function of the log of the sum of the rates of reinforcement.

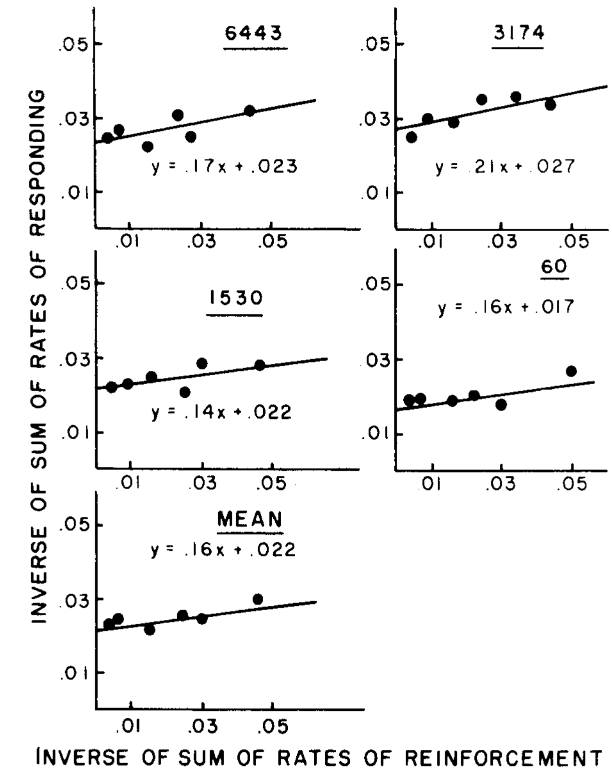

Figure 2. Inverse of the sum of the rates of responding as a function of the inverse of the sum of the rates of reinforcement.

the sum of the rates of reinforcement. The fifth set of axes represents the mean of the data generated by all subjects. Each of the other sets of axes represents the data generated by individual subjects. The best fitting straight line, estimated by a least squares technique, has been drawn for each subject and for the mean of all subjects, and its equation has been reported. The k parameters were 43.5, 37.0, 45.5, 58.8, and 45.5 pecks/min for each subject and for the mean of all subjects. The R_0 parameters were 7.4, 7.8, 6.4, 9.4, and 7.3 reinforcers/h for each subject and for the mean of all subjects.

Table 3 contains the proportion of the variance accounted for by the best fitting lines plotted in Figures 1 and 2. Herrnstein's equation accounted for a larger proportion of the variance than Catania's equation for three of four subjects and for the mean of all subjects.

DISCUSSION

The data confirm a prediction of the theories proposed by Catania and Herrnstein. The sum of the rates of responding generated by the components of the two-key concurrent schedules did increase with increases in the sum of the rates of reinforcement that the components provided. The increase occurred when the ratio of the rates of reinforcement was held constant.

Herrnstein's equation described the data slightly better than Catania's. It accounted for a larger proportion of the variance in the data for three of four subjects and for the mean of all subjects. Herrnstein's equation might not be preferable to Catania's if that were its only advantage. Herrnstein's equation has two free parameters; Catania's has only one. Equations with more free parameters should fit the data better. But Herrnstein's equation has an additional advantage. Herrnstein has offered interpretations of his parameters which are in accord with the values of k and R_0 found in the present experiment. The obtained values of k, which ranged from 37 to 59 pecks/min, do not violate the interpretation of k as the subject's asymptotic rate of responding. The obtained values of R_0, which ranged from 6 to 10 reinforcers/h, do not violate the interpretation of R_0 as the reinforcers obtained from unprogrammed sources.

Additional evidence for the presence of R_0 should

Table 3
Proportion of the Variance Accounted for by Herrnstein's and Catania's Equations

	Subject				
Equation	6443	3174	1530	60	Mean
Herrnstein	.37	.63	.47	.57	.76
Catania	.31	.79	.42	.32	.66

be obtained. Catania's and Herrnstein's equations both contain multiplicative constants as parameters. But Herrnstein's equation contains the additional R_0 parameter in the denominator. Two studies have already provided evidence for R_0 (Herrnstein, 1974; McSweeney, 1975). Additional evidence could be obtained in an experiment which replicated the present experiment with subjects maintained at several different percentages of their free-feeding weights. If Herrnstein is correct, the slope of the line relating the inverse of the sum of the rates of responding to the inverse of the sum of the rates of reinforcement should increase with increases in weight. The slope of that line equals R_0/k. The size of R_0/k should increase if R_0 increases, but k remains constant, as subjects become less hungry (Herrnstein, 1974; Herrnstein & Loveland, 1974).

The slopes of the lines in Figure 1 were smaller than the .167 slope reported by Catania (1963). At least two factors may have produced this difference. First, the change in the ratio of the rates of reinforcement in the Catania study may have increased the size of the change in the rates of responding which was generated by changing the sum. The present experiment would have produced smaller changes in response rates because it held these ratios constant. Second, the differences in the slopes may represent a difference between the rates of responding generated by Findley and two-key concurrent procedures.

The factor responsible for the differences in results could be identified by conducting a study similar to the present one which used a Findley procedure. Finding slopes similar to those generated in the present study would suggest that variations in the reinforcer ratios contributed to Catania's results. Finding larger exponents for the Findley schedules in spite of constant ratios of reinforcement would suggest that Findley and two-key concurrent schedules exert subtly different control over responding.

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