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Manual discrimination and identification of length by the finger-span method

N. I. DURLACH, L. A. DELHORNE, A. WONG, W. Y. KO,
W. M. RABINOWITZ, and J. HOLLERBACH
Massachusetts Institute of Technology, Cambridge, Massachusetts

Experiments were conducted on length resolution for objects held between the thumb and forefinger. The just noticeable difference in length measured in discrimination experiments is roughly 1 mm for reference lengths of 10 to 20 mm. It increases monotonically with reference length but violates Weber's law. Also, it decreases when the subject is permitted to maintain a constant finger span between trials; however, it tends to increase when the nondominant hand is used. As would be expected from studies of other stimulus dimensions in other sense modalities, resolution is considerably poorer in identification experiments than in discrimination experiments. For stimulus sets that cover a broad range (90 mm), the total information transfer is roughly 2 bits; for those that cover a relatively small range (18 mm), it is roughly 1 bit. The data are analyzed and interpreted using analysis techniques and models that have been used previously in studies of audition (e.g., Durlach & Braida, 1969).

This paper concerns the ability of humans to discriminate and identify the extent of an object by holding it between the thumb and forefinger of a single hand (the *finger-span* method of length estimation). We are interested in the ability to resolve length in this fashion for a number of reasons.

First, we want to characterize, understand, and model the human's ability to recognize objects manually. We want to achieve this goal both as an end in itself and as background for the design of improved robots. Although some previous work has been conducted in this area (e.g., Lederman & Klatzky, 1987), the results are still very limited.

To understand object recognition of any kind, it is necessary to (1) determine the dimensions (first in physical space, then in perceptual space) that are relevant to the given recognition task, (2) measure the ability to resolve differences along these dimensions, and (3) construct a model of how the sensed values along the different dimensions are combined and used to select a specific recognition response (a model that must take into account a priori information, payoffs, decision making, short-term memory, etc.). The experiments reported in this paper are addressed to requirement 2. Although the specific dimensions that are important in a given recognition task will obviously depend on the collection of objects in the stimulus set, many practical recognition tasks involve object extent as a significant parameter.[1]

A second motivation for studying length resolution stems from our previous research on tactual communication of speech by the deaf-blind (e.g., Norton et al., 1977; Reed, Rabinowitz, Durlach, & Braida, 1985). To understand the remarkable performance achieved by these individuals, we need an improved understanding of the hand's ability to discriminate and identify patterns of stimulation that involve changes not only in vibration and airflow, but also in shape and compliance.[2]

Third, we are interested in man-machine interfaces for teleoperator and virtual-environment systems. Knowledge of the operator's resolution limits is essential to the design of efficient interfaces; the resolution of the interface should be appropriately matched to that of the operator.

Finally, there is the continuing uncertainty (and controversy) over the physiological mechanisms that underlie the sense of finger position. Although information on finger position is available from cutaneous and muscle receptors and also, to some extent, from joint receptors, the relative contribution of each of these mechanisms to various systems under various conditions is not yet well understood (e.g., see the review by Clark & Horch, 1986). Since measurement of object length by the finger-span method involves the sensing of differential finger position, a comprehensive and accurate characterization of finger-span length resolution could provide useful data for the evaluation of hypotheses concerning the mechanisms that underlie the sense of finger position.

In general, there are several methods of estimating length manually: (1) by the finger-span method considered in this paper (or one in which the two fingers are on different hands); (2) by the *temporal-sweep* method, in which a fixed area of skin (e.g., a finger pad) is swept across

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the object in question (or the object is swept across the skin); and (3) by the *cutaneous-extent* method, in which the length is impressed upon the skin.

Corsini and Pick (1969) studied the effect of texture on tactually perceived length using an unspecified combination of the temporal-sweep and cutaneous-extent methods and found a Weber fraction (for the no-texture case) of 7% for a reference length of 178 mm. Jones and Vierck (1973) examined length discrimination for reference lengths of 13 and 127 mm using the cutaneous-extent method (along the forearm) and found a roughly constant difference limen of approximately 8 mm. When the stimuli were placed across the arm, rather than along it, the difference limen was reduced from 8 to 5 mm. When comparisons were made across arms, the difference limen increased roughly from 8 to 11 mm.

Experiments using the finger-span method (tips of the thumb and forefinger) have been performed by Dietze (1961), Evans and Howarth (1966), Gaydos (1958), and Stevens and Stone (1959). Using relatively crude psychological methods (probable error in a matching task with an uncontrolled number of corrective adjustments), Gaydos (1958) obtained results over the range 35–100 mm which satisfied Weber's law and showed a Weber fraction of approximately 0.03; however, the difference limen did not continue to decrease below 35 mm. Dietze (1961) found a Weber fraction of approximately 0.02 at 30 and 50 mm; however, the fraction increased to roughly 0.06 at 10 mm. Stevens and Stone (1959) determined a difference limen $(\Delta L)_0$ that satisfied the equation $(\Delta L)_0 = 0.0286L + 0.8$, where L (the reference length) lies in the range $10 \leq L \leq 90$ mm. The nonzero value of the intercept in this linear equation violates Weber's law. Evans and Howarth, in a study of the effect of *grip tension* on length discrimination, found a Weber fraction of roughly 0.03 at a reference length of 76 mm (grip tension was found to affect mean response, but not variability).

Finally, note that our investigation includes experiments on both discrimination and identification. Whereas discrimination data reflect more directly the basic underlying sensitivity and are more easily related to peripheral physiology, the identification task more closely approximates the tasks encountered in natural situations. Also, as discussed previously by Pollack (1962) and others, and as modeled quantitatively in our previous work on discrimination and identification of sound intensity (e.g., Berliner & Durlach, 1973; Braida & Durlach, 1972; Braida et al., 1984; Durlach & Braida, 1969; Lim, Rabinowitz, Braida, & Durlach, 1977), identification performance is limited not only by imperfect basic sensitivity, but also by imperfect short-term memory.

GENERAL METHOD

Subjects

Eighteen college students were paid on an hourly basis to serve as subjects in these experiments. In many cases, a given subject participated in only one experiment.

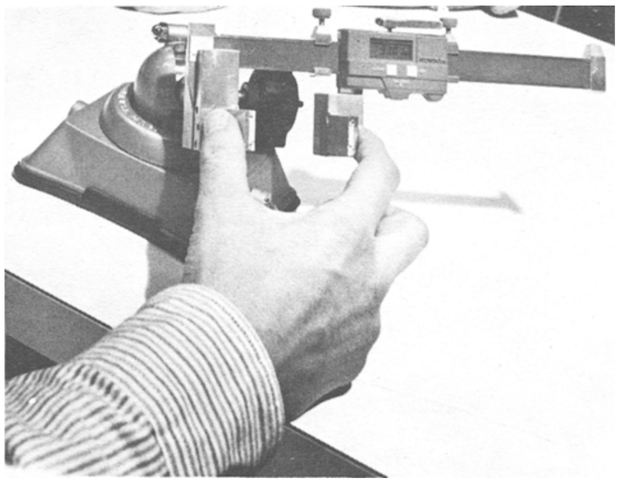

Figure 1. The apparatus used for length-resolution experiments. (See text for details.)

Apparatus

The apparatus used in all experiments is shown in Figure 1. A vernier caliper with a digital readout was modified to accept two rectangular pads defining the length to be estimated and against which the thumb and forefinger were placed in order to make the estimate. In the discrimination experiments, the length was varied between the two relevant values (L and $L+\Delta L$) by moving the variable portion of the caliper back and forth between two stops. In the identification experiments, the appropriate length was achieved by inserting a step wedge between the pads. In general, by using this device to vary the separation between fixed pads (as opposed, for example, to using different objects of different fixed lengths), we eliminated the possibility of the subject's attending to stimulus properties other than length. It should be noted, however, that the device, as configured, did not permit us to present stimuli with $L < 10$ mm.

Procedure

All experiments used a single-interval forced-choice paradigm with trial-by-trial correct-answer feedback. In other words, on each trial the subject was presented with one of N lengths (chosen randomly with equal a priori probabilities), was forced to choose one of N responses, and was then told the correct response. In the discrimination experiments, one of two lengths (L and $L+\Delta L$) was presented to the subject, and the subject was required to respond "short" or "long." In the identification experiments, one of many lengths ($N = 10$ or 19, depending on the experiment) was presented, and the response set consisted of the first N integers.

The apparatus, hidden from view, was adjusted between trials by the experimenter; the subject's hand was removed from the apparatus while it was being adjusted. The standard procedure, referred to below as Method B, required that the subject touch his or her thumb and forefinger together during the intertrial interval (eliminating the use of fixed finger span as a mechanical intertrial memory aid). In the alternative procedure, referred to below as Method H, the subject held his or her finger span fixed during the intertrial interval. Method H was only used in Experiment 1B, an experiment in which the two methods were compared.

EXPERIMENT 1: DISCRIMINATION

On each trial of each discrimination experiment, the subject was presented with a short (L) or long ($L+\Delta L$)

stimulus and was asked to judge which of the two had been presented. A run consisted of 60 trials; the first 10 were treated as practice trials and the last 50 were used to measure performance. The sensitivity index d' and response bias β were estimated from the appropriate 2×2 matrices formed after each run (e.g., Berliner & Durlach, 1973). Values of d' and β estimated for sets of runs taken under fixed conditions were computed by averaging these quantities over the runs. As discussed in the Appendix, using the full 60 trials in each run (rather than only the last 50) to estimate performance, or combining runs by pooling matrixes (rather than averaging d' and β) has little effect on the results. Consistent with the latter finding is the result, also discussed in the Appendix, that β is sufficiently small to ignore. Finally, as shown in Experiment 1A, the dependence of d' on the length increment ΔL for a fixed reference length L can be described as a straight line through the origin. Thus, performance can be summarized for a fixed L by the slope $\delta' = d'/\Delta L$ of this line (i.e., the sensitivity per millimeter). The just noticeable difference (jnd) $(\Delta L)_0$, defined by the performance criterion $d' = 1$, is given simply by $(\Delta L)_0 = 1/\delta'$.

Experiment 1A

The primary purpose of Experiment 1A was to determine the dependence of performance on the reference length L. We also wished to confirm the result, found for many other variables in a variety of sense modalities, that d' is proportional to the increment ΔL and, therefore, that resolution can be described simply by the slope parameter $\delta' = d'/\Delta L$ (for a discussion of this issue in the case of sound intensity, see Rabinowitz, Lim, Braida, & Durlach, 1976).

In each of eight test sessions, each of three values of L (10, 40, and 80 mm) was tested with each of two increments ΔL ($\Delta L = 1.0$, 2.0 mm for $L = 10$ mm; $\Delta L = 1.5$, 3.0 mm for $L = 40$ mm; and $\Delta L = 2.0$, 4.0 mm for $L = 80$ mm). The order in which the six conditions were tested was randomized individually for each session and each subject. The total number of trials obtained for each subject for each of the six pairs (L, ΔL) was 400. Using r and l to denote right and left hand, and y to denote ambidextrous, we denote the 4 subjects tested in this experiment by S_1 (r,r), S_2 (l,r), S_3 (r,y), and S_4 (l,y), where the first entry in the parentheses indicates the hand tested and the second describes the hand dominance. The role of handedness is considered further in Experiment 1C.

The results indicate that the psychometric function $d'(\Delta L)$ can be well represented by a straight line through the origin: $d' = \delta'\Delta L$. For each subject and each value of L, we estimated the value of d' for each of the two increments ΔL tested. We then computed the slope parameter $\delta' = d'/\Delta L$ for each ΔL, averaged these slope values to form the mean slope M, computed the root-mean square (RMS) deviation σ of the two slope values from the mean slope, and, finally, computed the quantity σ/M. The values of M, σ, and σ/M, averaged over the 4 subjects and over the three values of L, are 0.71, 0.08, and 0.12, respectively. (Unless stated otherwise, we shall assume throughout the rest of this paper that the relationship $d' = \delta'\Delta L$ is valid, and we shall report our resolution results in terms of the sensitivity-per-millimeter $\delta' = d'/\Delta L$.)

To determine how performance improved with practice, we examined the dependence of δ' on session number. Specifically, for each subject and each value of L, we computed the normalized quantity $\delta'/\bar{\delta}'$, where δ' is computed for each session n and $\bar{\delta}'$ is the average of δ' over n. As shown in Figure 2, practice generally has little or no effect. (The slight increase in performance with session number evident in the graph for the average across subjects is due primarily to the results for S_2, who used the nondominant hand.) Unless stated otherwise, in the remainder of this paper, a superscript bar (as in $\bar{\delta}'$) denotes the average over sessions or run number.

The dependence of resolution on reference length L is shown in Figure 3, in which we have plotted $1/\bar{\delta}'$ versus L. Recalling that $1/\delta'$ equals the jnd $(\Delta L)_0$, and noting that the difference between the two quantities $1/\bar{\delta}'$ and $\overline{1/\delta'}$ is relatively small in these tests (the latter was found to be, on the average, roughly 10% greater than the former), we see that the validity of Weber's law is measured by the extent to which the dependence in Figure 3 can be represented by a straight line through the origin. Clearly, the empirical dependence of $1/\bar{\delta}'$ on L does not satisfy Weber's law. Not only are the intercepts of the best-fitting straight lines for the various subjects much greater than zero (ranging from 0.786 to 0.995), but there appears to

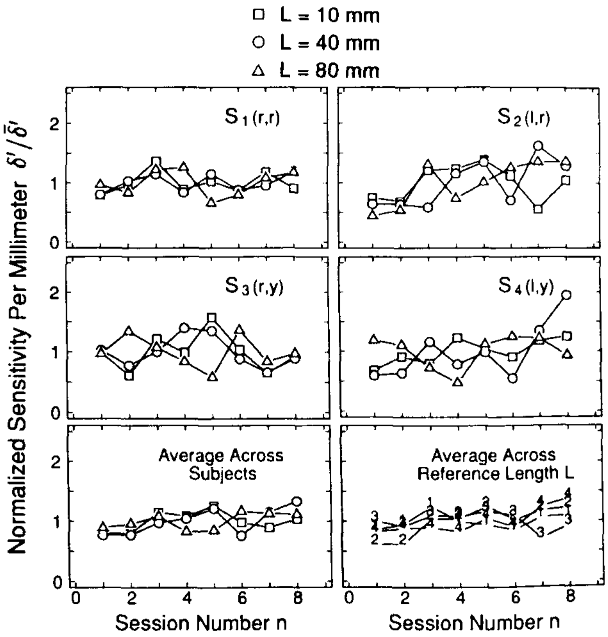

Figure 2. Dependence of discrimination performance on test session (Experiment 1A). Each of the top four panels shows the results for a single subject S; each of the curves within each panel corresponds to a different reference length L. The letters r and l denote right and left hand, respectively; y denotes ambidextrous. The first entry in parentheses indicates the hand tested, and the second entry indicates the subject's dominant hand. The bottom two panels show average results.

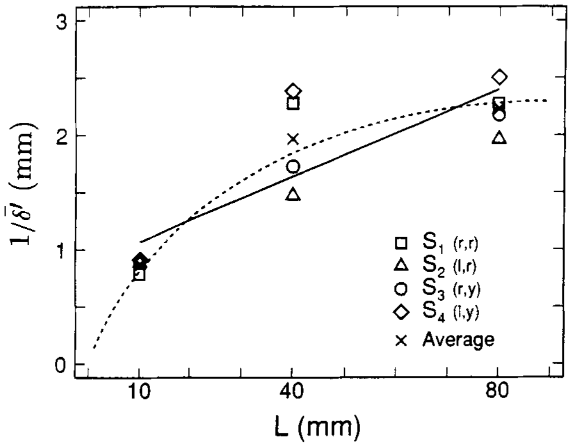

Figure 3. Dependence of $1/\bar{\delta}'$ on reference length L (Experiment 1A). The different symbols represent data for the different subjects and for the average over subjects. The straight-line fit is given by Equation 1 and the curvilinear fit by Equation 3.

be a slight tendency for resolution to be relatively poor at L = 40 mm. The best-fitting straight line to the average data is given by

$$1/\bar{\delta}' = 0.019L + 0.877 \tag{1}$$

and is shown in Figure 3. The RMS deviation of the average data from this line (along the ordinate) is 0.240.

Experiment 1B

In Experiment 1B, we examined the effect of maintaining a fixed finger span during the intertrial interval. Twenty-four runs (covering 3-5 sessions) were conducted, with successive runs alternating between Method B (bringing fingers together during the interstimulus interval, as was done in Experiment 1A) and Method H (holding finger span fixed during this interval). The value of L was held constant at 40 mm. The increment ΔL used for the Method H tests was always one-half that used for the Method B tests because sensitivity d' was found to be roughly twice as large for Method H when the same ΔL was used. For Subjects S_5 (r,r), S_6 (r,r), and S_7 (l,r), the values of ΔL were 1.0 mm (Method H test) and 2.0 mm (Method B test); for Subjects S_8 (r,r) and S_9 (r,r), they were 1.5 mm (Method H test) and 3.0 mm (Method B test).

The results of Experiment 1B are shown in Figure 4 and Table 1. Figure 4 shows, for each subject, the ratio $\delta'/\bar{\delta}'$ as a function of run number. The quantity $\bar{\delta}'$ represents the average of δ' over run number (with Method H and Method B runs treated separately). The values of $\bar{\delta}'$ for each method and each subject are shown in Table 1.

As seen in Table 1, resolution is substantially superior with Method H: $\bar{\delta}'$ (averaged over subjects) increased by roughly 50% when the subject was allowed to hold the finger span fixed between trials. Also, as seen in Figure 4, with the possible exception of Subject S_8, practice has at most only a minor effect.

Experiment 1C

In Experiment 1C, we explored the differences in resolution associated with the hand a subject used. The reference length was again held fixed at L = 40 mm. Five new subjects (S_{10}–S_{14}) were tested in two sessions with four runs per session. In Session 1, the sequence of tests was rlrl; in Session 2, it was lrlr. In both sessions, the first two runs used ΔL = 3 mm and the second two used ΔL = 2 mm.

Table 2 lists the results on $\bar{\delta}'$ as a function of whether the dominant or nondominant hand was tested, as well as the ratio $\bar{\delta}'$ (dominant)/$\bar{\delta}'$ (nondominant). Which of the two hands was dominant (r or l) is specified in the subject column. For the 5 subjects tested, the ratio varies between 1.0 and 1.5, and averages 1.2. The cause of the relatively poor performance of these subjects, even for the dominant hand, is unknown.

Reprocessing the Data from Experiment 1A Using a Log Scale

The choice of a linear scale for the variable L in the processing of the data is convenient but arbitrary. Moreover, the results on $1/\bar{\delta}'$ versus L using this scale are not particularly simple in that they are not strictly linear; the fit represented by Equation 1 is rather crude (see Figure 3). Since the logarithmic transformation has proved useful in other sensory dimensions, we reprocessed these data using a logarithmic scale.

First, we replaced the original functions d' versus ΔL with new functions d' versus $10\log(1+\Delta L/L)$. These new functions, like the original ones, are well represented by straight lines through the origin and are summarized by the new slope constant $\tilde{\delta}' = d'/10\log(1+\Delta L/L)$ in place of the original slope constant $\delta' = d'/\Delta L$. As might

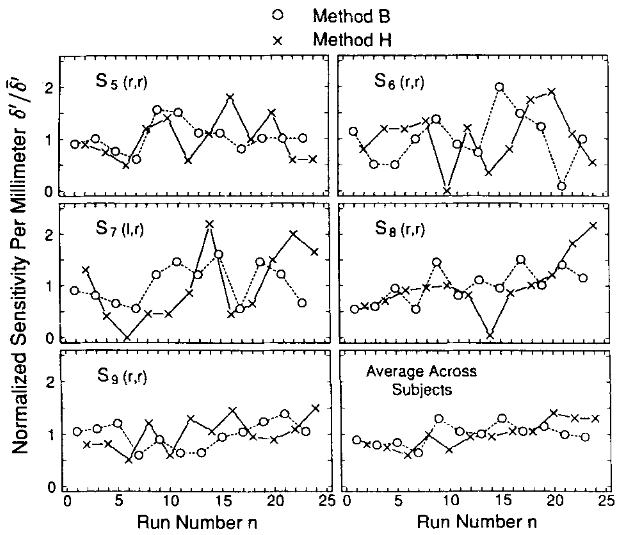

Figure 4. Dependence of discrimination performance on method and run number (Experiment 1B). The results for Method B are indicated by circles; those for Method H are indicated by crosses. The data are normalized separately for each method.

Table 1
Dependence of $\bar{\delta}'$ on Method (Experiment 1B)

Subject	Method H	Method B
S_5(r,r)	1.00	0.68
S_6(r,r)	0.75	0.40
S_7(l,r)	0.46	0.38
S_8(r,r)	0.94	0.83
S_9(r,r)	1.50	0.67
Average	0.93	0.59

Note—r and l denote right and left hand, respectively. The first entry in parentheses indicates the hand tested; the second entry indicates the subject's dominant hand.

Table 2
Dependence of $\bar{\delta}'$ on the Hand Used (Experiment 1C)

Subject	$\bar{\delta}'$ (dominant)	$\bar{\delta}'$ (nondominant)	$\bar{\delta}'$(dominant)/ $\bar{\delta}'$(nondominant)
S_{10}(r)	0.30	0.30	1.00
S_{11}(r)	0.48	0.33	1.46
S_{12}(r)	0.36	0.32	1.13
S_{13}(l)	0.44	0.32	1.38
S_{14}(l)	0.42	0.40	1.05
Average	0.40	0.33	1.21

Note—r and l refer to right and left hand, respectively, and indicate the subject's dominant hand.

be expected—since $\log(1+x)$ is proportional to x for small x—the quality of these new fits is roughly comparable to that of the original fits (the value of the ratio σ/M changes from 0.12 to 0.11).

Next, we replaced the function $1/\bar{\delta}'$ versus L, considered in Figure 3, by the function $1/\bar{\bar{\delta}}'$ versus $10 \log L$, shown in Figure 5. The quantity $1/\bar{\bar{\delta}}'$ decreases linearly with $10 \log L$ and is well represented by the straight line

$$1/\bar{\bar{\delta}}' = -0.025\,(10 \log L) + 0.598. \qquad (2)$$

The RMS deviation of the average data from this best-fitting line is only 0.003. In these coordinates, the departure from Weber's law is represented by the deviations of the data from a straight line of slope zero. (Given this departure from Weber's law, we are unable to attach any special meaning to the linearity of the dependence.)

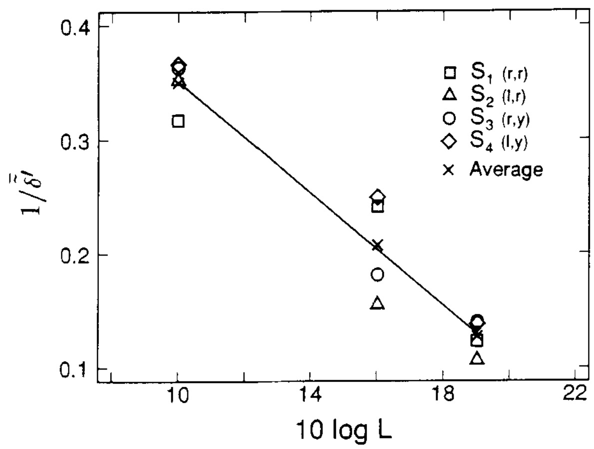

Figure 5. Dependence of $1/\bar{\bar{\delta}}'$ on reference length $10 \log L$ (Experiment 1A). These results are the same as those shown in Figure 3, except for the use of logarithmic increments and references in the data processing (see text for details). The straight-line fit is given by Equation 2.

Finally, we converted the linear fit represented by Equation 2 to a curvilinear fit to the original function $1/\bar{\delta}'$ versus L. Since $\Delta L/L$ is small, $\log(1+\Delta L/L) \approx 0.43(\Delta L/L)$ and $\bar{\bar{\delta}}' \approx (L/4.3)\bar{\delta}'$. Substituting this value of $\bar{\bar{\delta}}'$ in Equation 2, we obtain

$$1/\bar{\delta}' = (L/4.3)[-0.250 \log L + 0.598]. \qquad (3)$$

As seen in Figure 3, Equation 3 provides a better fit than Equation 1.

EXPERIMENT 2: IDENTIFICATION

The identification experiments differed from the discrimination experiments only in the number of stimuli. The apparatus (Figure 1), the basic paradigm (single-interval forced choice with feedback), and the run length (50 trials) were the same. In all cases, the dominant hand was employed and the subject brought the two fingers together between trials (i.e., Method B was used).

The stimulus sets are shown in Figure 6. Sets A–G, each of which contained 10 stimuli with uniform spacing, were used to test Subjects S_9 (who had participated in Experiment 1B), S_{10} (who had participated in Experiment 1C), and S_{15} (a new subject) in Experiment 2A. The primary purpose of this experiment was to explore the dependence of identification performance on the range R of the stimulus set. Set A covers a range of 90 mm (corresponding to an interstimulus increment Δ of 10 mm); Sets B, C, and D each covers a range of 36 mm (Δ = 4 mm); and Sets E, F, and G each covers a range of 18 mm (Δ = 2 mm). Sets A, H, I, and J were used to test Subjects S_{16}–S_{18} (all new subjects) in Experiment 2B. The primary purpose of this experiment was to explore the effect of increasing the stimulus density or changing the spacing from linear to logarithmic. Thus, Sets A–H have 10 elements and Sets I and J have 19 elements; the spacing of Sets A–G and I is linear and that of Sets H and J is approximately logarithmic.

In each experiment, the stimulus sets were tested in random order without replacement (with a different random sequence for each subject). This procedure was repeated six times so that there were 300 trials in the confusion matrix for each stimulus set and each subject. These matrixes were processed to estimate the interstimulus sensitivity $d_i' = d'(L_i, L_{i+1})$ and the total sensitivity $D' = \sum_i d_i'$ (see Braida & Durlach, 1972).[3] In addition, we estimated the information transfer ψ. Finally, in order to examine practice effects, we computed percent correct P separately for each of the six runs that were cumulated to estimate d_i', D', and ψ.

Experiment 2A

The dependence of P on run number n for each stimulus set and each subject, as well as the average over the stimulus sets, is shown in Figure 7. Apparently, there was

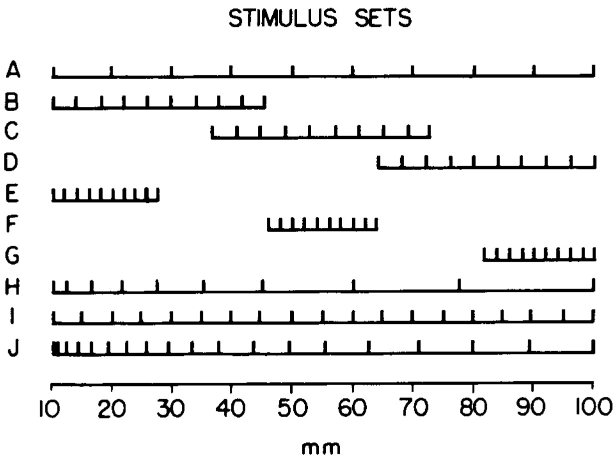

Figure 6. Stimulus sets used in the identification experiments (Experiments 2A and 2B). Sets A–G and I are linear; Sets H and J are approximately logarithmic. Each of Sets A–H have 10 elements; each of Sets I and J have 19 elements.

no practice effect, even though these subjects had no special training in identification and even though S_{15} had no previous experience in discrimination.

The interstimulus values d'_i for each stimulus set and each subject, as well as the average of d'_i over subjects, are shown in Figure 8. The results on $\bar{d}'_i$ (the average of d'_i over i), D' (the sum of d'_i over i), and ψ (the information transfer) for each of the stimulus sets A–G are shown in Table 3. Since the variation among subjects is small, attention will be focused on the average over subjects.

As expected, and as summarized in Table 3, when the range R of the stimulus set (or, equivalently, the interstimulus increment Δ) decreases, the value of $\bar{d}'_i$ decreases.[4] Thus, for example, in Stimulus Set A, one has $\bar{d}'_i \approx 2.0$, whereas for Sets E, F, and G, one has $\bar{d}'_i \approx 0.6$. Note also that for the large-range set A, d'_i is larger at the lower end than at the upper end of the range and d'_i tends to be larger at the edges of the range than in the middle (the *edge effect*). As the range R decreases, d'_i tends to flatten out across the range tested and to have a smaller average value $\bar{d}'_i$ for those sets at the upper end of the overall range. Thus, for example, in Sets E, F, and G, d'_i is roughly independent of i and $\bar{d}'_i$ is smaller for Set G than for Set E.

The fact that $\bar{d}'_i$ tends to decrease with R is to be expected because of the corresponding decrease in the interstimulus increment Δ. Also, the fact that d' tends to be greater at the lower end of the range than at the upper end when R is large (Stimulus Set A) can be explained, at least qualitatively, by the increase in basic sensitivity at the lower end observed in the discrimination experiments (see Figure 3). Finally, it should be noted that the edge effect seen in the results for the large-R case (Stimulus Set A), as well as the reduction of this effect as R is decreased, is similar to the results observed in work on auditory intensity perception (e.g., see Berliner, Durlach, & Braida, 1977; Braida et al., 1984).

To separate out the first two of these factors and to make the data obtained with the different stimulus sets more comparable, we have normalized the results shown in Figure 8 by transforming the data on $d'_i = d'(L_i, L_{i+1})$ to data on the quantity

$$\eta_i = \frac{d'_i}{d'_{\text{disc}}(L_i, L_{i+1})} . \tag{4}$$

The numerator d'_i in this fraction is the value of d'_i obtained between L_i and $L_{i+1} = L + \Delta$ in the identification experiment, whereas the denominator $d'_{\text{disc}}(L_i, L_{i+1})$, is the value of d' between L_i and L_{i+1} for a discrimination experiment. The quantity $d'_{\text{disc}}(L_i, L_{i+1})$ is estimated from the relation $d'_{\text{disc}}(L_i, L_{i+1}) = \delta'[(L_i + L_{i+1})/2]\Delta$ shown to be valid in discrimination and from the results on $1/\bar{\delta}'$ versus L displayed in Figure 3 (specifically, the curvilinear fit). Comparisons of the results on d'_i and on η_i (averaged over subjects) are shown in Figure 9, and comparisons of $\bar{d}'_i$ and $\bar{\eta}_i$ (the averages over i) are shown in Table 3. Note that the results on η_i and $\bar{\eta}_i$ would be identical to those shown if the logarithmic rather than the linear processing were used (see Figure 3) because $d'_{\text{disc}}(L_i, L_{i+1})$ is an empirical quantity that is independent of this processing.

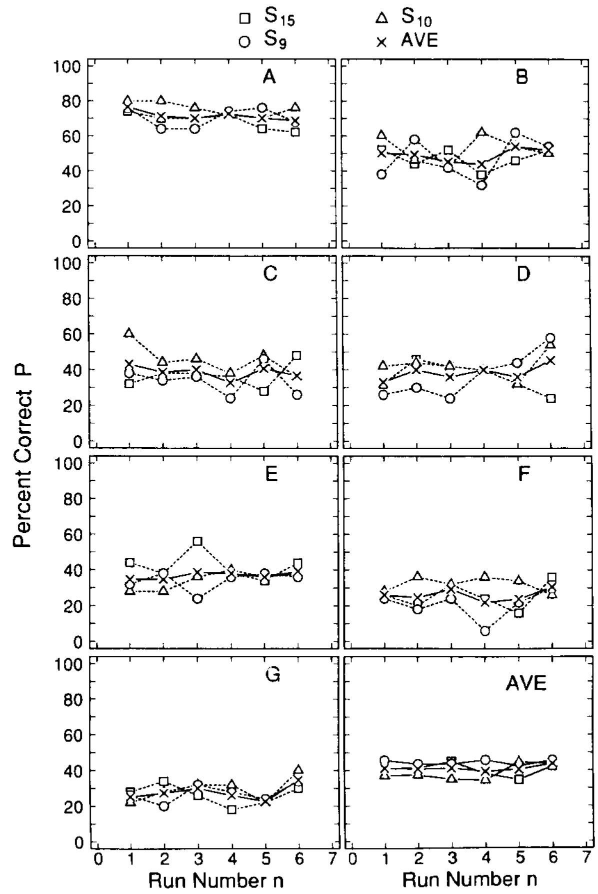

Figure 7. Dependence of percent correct P in identification on run number (Experiment 2A). The different panels show the results for different stimulus sets; the different curves within each panel indicate the data for the different subjects.

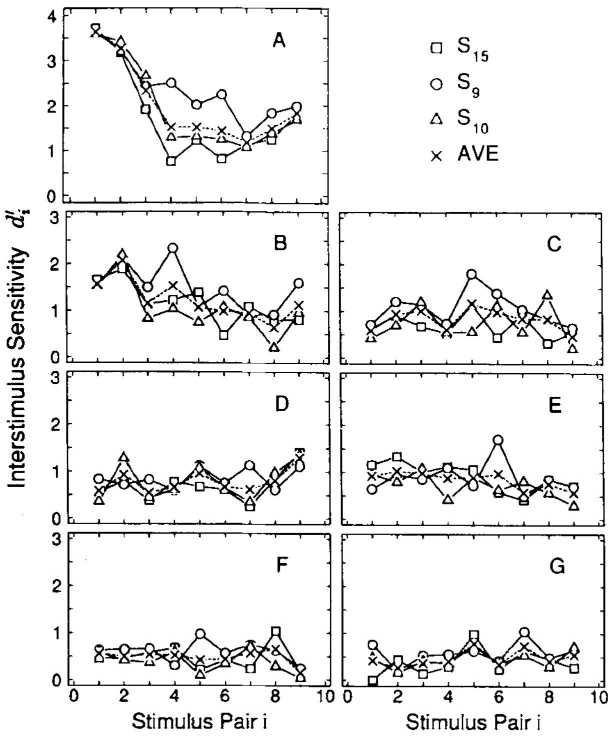

Figure 8. Dependence of interstimulus sensitivity $d'_i = d'(L_i, L_{i+1})$ on the stimulus-pair index i (Experiment 2A). The different panels show data for the different stimulus sets; the different curves within each panel show data for the different subjects.

On the whole, the normalized results suggest that once the effect of variations in Δ and of the dependence of d'_{disc} on L have been eliminated, the main effect of the variation in stimulus set is a tendency for $\bar{\eta}_i$ to increase with a decrease in R (see Note 4). (The edge effect, clearly evident for Stimulus Set A in the unnormalized results, appears much less pronounced in the normalized results.) The tendency for $\bar{\eta}_i$ to increase with a decrease in R is roughly similar to the same tendency observed in the identification of auditory intensity (Braida & Durlach, 1972).

Note also that since the sensory factors are essentially the same in both the identification task and the discrimination task, the amount by which $\bar{\eta}_i$ falls below unity measures the increased central-processing load (e.g., the increased requirements on short-term memory) in the identification task. This issue is considered further below.

Experiment 2B

The dependence of percent correct P on run number n is shown in Figure 10. Again, as in Experiment 2A, there is no indication of a practice effect. Apparently, this result is quite robust.

Graphs of the interstimulus d'_i for each stimulus set and each subject, as well as the average over subjects, are shown in Figure 11. Results on d'_i, D', and ψ are shown in Table 3.[5] Again, as with Experiment 2A, and despite the considerable intersubject variation for Stimulus Set A shown in Figure 11, attention will be focused on the averages. Furthermore, to reduce the noisiness of the data for Stimulus Sets I and J (which were tested with the same number of trials as Sets A–H despite the greater number of stimuli in these sets), the results for these two sets have been smoothed by averaging over adjacent pairs (i.e., d'_1 and d'_2 are averaged to form a new d'_1; d'_3 and d'_4 are averaged to form a new d'_2; etc.). The resulting average values of d'_i, together with the corresponding values of η_i (the normalized results), are shown in Figure 12. The average values of η_i are shown in Table 3.

According to these results, for a fixed large-range R, neither modest changes in stimulus spacing nor modest changes in stimulus density have a large effect on the normalized sensitivity parameter η_i.[6] Note also that (as in Experiment 2A) the edge effect appears much less pronounced when the results are normalized. Further experiments (with more trials and greater variation in density and spacing) are needed to characterize the effects of these parameters precisely.

Table 3
Results of Experiments 2A and 2B

Stimulus Set	$\bar{d}'_i$	D'	ψ	$\bar{\eta}_i$	$\bar{d}'_i$	D'	ψ	$\bar{\eta}_i$	$\bar{d}'_i$	D'	ψ	$\bar{\eta}_i$	$\bar{d}'_i$	D'	ψ	$\bar{\eta}_i$
	Experiment 2A															
	S_{15}				S_9				S_{10}				Average			
A (90 mm)	1.76	15.79	2.10	.30	2.36	21.21	2.38	.43	1.96	17.64	2.16	.34	2.02	18.21	2.21	.36
B (36 mm)	1.16	10.47	1.71	.41	1.49	13.41	1.89	.54	1.05	9.45	1.50	.43	1.24	11.11	1.70	.46
C (36 mm)	0.69	6.17	1.20	.32	1.06	9.57	1.69	.51	0.74	6.68	1.24	.36	0.83	7.47	1.38	.40
D (36 mm)	0.70	6.32	1.16	.39	0.89	7.97	1.37	.49	0.79	7.13	1.30	.44	0.79	7.14	1.28	.44
E (18 mm)	0.92	8.27	1.47	.54	0.89	8.05	1.47	.54	0.72	6.46	1.23	.42	0.84	7.59	1.39	.50
F (18 mm)	0.47	4.27	0.83	.46	0.60	5.43	1.09	.58	0.38	3.45	0.72	.37	0.49	4.38	0.88	.47
G (18 mm)	0.38	3.44	0.81	.44	0.58	5.26	1.02	.66	0.45	4.04	0.82	.51	0.47	4.25	0.88	.54
	Experiment 2B															
	S_{16}				S_{17}				S_{18}							
A (90 mm)	2.85	25.68	2.60	.53	2.09	18.79	2.21	.30	1.70	15.31	2.04	.38	2.21	19.93	2.28	.40
H (90 mm)	2.63	23.71	2.48	.51	2.16	19.42	2.24	.42	2.16	19.46	2.31	.42	2.32	20.86	2.34	.45
I (90 mm)	1.25	22.52	2.66	.45	1.03	18.57	2.27	.35	0.96	17.29	2.34	.35	1.08	19.46	2.42	.38
J (90 mm)	1.16	20.81	2.63	.40	0.82	14.74	2.21	.35	1.00	18.15	2.46	.27	0.99	17.90	2.43	.34

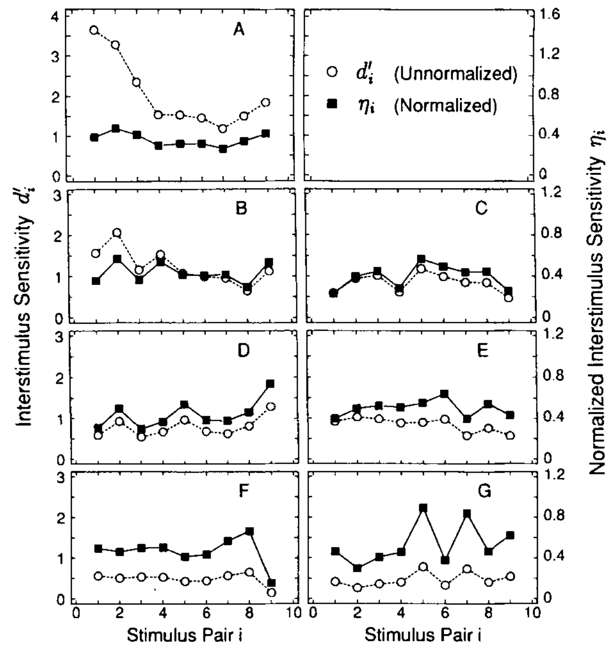

Figure 9. Comparison of interstimulus sensitivity d'_i and normalized interstimulus sensitivity η_i (Experiment 2A). The different panels show the results for the different stimulus sets. Both d'_i and η_i have been averaged over subjects.

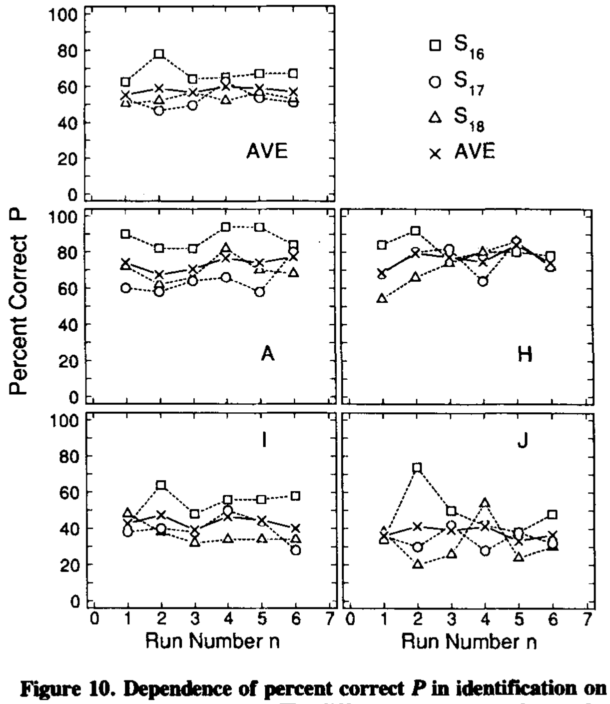

Figure 10. Dependence of percent correct P in identification on run number (Experiment 2B). The different panels show the results for the different stimulus sets; the different curves within each panel show the data for the different subjects.

CONCLUDING REMARKS

Our results on the just noticeable difference $(\Delta L)_0 = 1/\delta'$ in length L are roughly consistent with the results of previous studies. For example, our straight-line fit to the dependence of $(\Delta L)_0$ on L is given by $0.019L + 0.877$, whereas Stevens and Stone (1959) report $0.029L + 0.8$.

That the dependence of $(\Delta L)_0$ on L does not satisfy Weber's law [i.e., that $(\Delta L)_0$ is not proportional to L] is not surprising; the quantity L is only remotely related to the relevant sensory quantities concerning the muscles, joints, or skin. Even if Weber's law were valid for such quantities (e.g., for the jnd in joint angle), it would probably not be valid for the quantity L.

To test hypotheses about underlying physical mechanisms, it is necessary to model not only the transformation from the sensed quantity to the variable L, but also the statistical variability of the sensed quantity. Such modeling has been performed with considerable success in the area of audition (e.g., Colburn, 1973; Siebert, 1968); it has been largely absent, however, in the tactual-kinesthetic area.

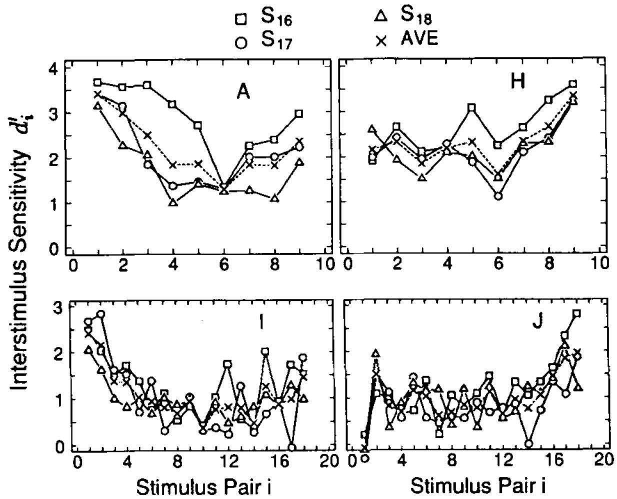

Figure 11. Dependence of interstimulus sensitivity $d'_i = d'(L_i, L_{i+1})$ on the stimulus-pair index i (Experiment 2B). The different panels show the data for the different stimulus sets; the different curves within each panel show the data for the different subjects.

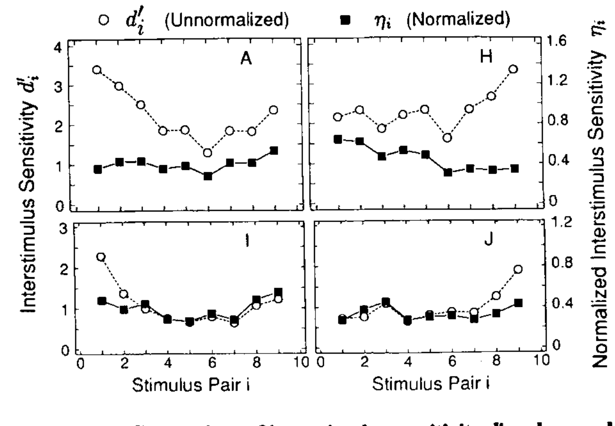

Figure 12. Comparison of interstimulus sensitivity d'_i and normalized interstimulus sensitivity η_i (Experiment 2B). The different panels show the results for the different stimulus sets. Both d'_i and η_i have been averaged over subjects.

Our results on the identification of length L and its relation to the jnd $(\Delta L)_0$ in L are roughly similar to those found for other dimensions and other modalities. As usual, resolution in large-range identification corresponds to an information transfer of roughly 2 bits and is much poorer than would be expected if the only limitations were those that constrain the difference limen.

Based on our previous work in the area of unidimensional identification (e.g., Durlach & Braida, 1969), we would expect that: (1) resolution in discrimination is limited primarily by sensory noise; (2) the difference between the resolution achieved in discrimination and that achieved in large-range identification is due mainly to the presence of substantial amounts of memory noise in the identification task (specifically, context-coding noise); and (3) the average normalized sensitivity $\bar{\eta}_i$ in the identification task is given by $\bar{\eta}_i = \sigma^2_{sen}/(\sigma^2_{sen}+\sigma^2_{mem})$, where σ^2_{sen} and σ^2_{mem} are the variances of the sensory noise and memory noise, respectively. According to our results on $\bar{\eta}_i$ (Table 3), the magnitudes of these two variance terms are roughly equal ($\bar{\eta}_i$ is roughly one-half) and the relative effect of the memory noise decreases as the range of the stimulus set decreases ($\bar{\eta}_i$ increases as R decreases). According to both our model and to our data on sound intensity (Pynn, Braida, & Durlach, 1972), if R is made sufficiently small, the effect of the memory noise becomes negligible, $\bar{\eta}_i$ approaches unity, and the classical discrepancy between discrimination and identification (e.g., Miller, 1956) disappears. It should be realized, however, that our results from Experiment 1B (on the effect of maintaining a fixed finger span between trials) suggests that the value of d'_{disc} used in computing η_i for the identification experiments is itself limited by memory noise. In other words, the above conclusion concerning the relative sizes of σ_{sen} and σ_{mem} in the identification task should be altered from $\sigma_{mem} \approx \sigma_{sen}$ to $\sigma_{mem} \geq \sigma_{sen}$.

In future work, we intend to measure length resolution in regions of small L ($L < 10$ mm), extend our study of the relationship between discrimination performance and identification performance, and examine resolution along other stimulus dimensions, including those related to object compliance as well as shape.

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NOTES

1. Unlike the parameters related to detailed shape characteristics, overall object extent becomes immediately apparent when the object is first grasped.

2. In future work, we intend to include some of these deaf-blind individuals, as well as normal subjects, in our research on manual sensing of object properties.

3. In some of the identification experiments with a 90-mm range, the number of errors for stimuli at or near the end of the range was too small to permit finite estimates of d'_i. In these cases, we altered the confusion matrix by inserting some additional "artificial" errors. In the worst case (Experiment 2B, Stimulus Set A, Subject S_{16}), seven such errors were inserted. This modification corresponded to taking 7 correct responses (out of 257) and replacing them with responses that deviated from the correct response by one stimulus step. This modification reduced the information transfer ψ from 2.74 to 2.60. In all other

cases, the number of inserted errors was less than or equal to four (and usually only one or two). No such artificial errors were inserted for Stimulus Sets B-G or J.

4. The range factor R was found to be significant at the 0.001 level in ANOVAs performed for both the $\bar{d}'_i$ results and the $\bar{\eta}_i$ results. Also, post hoc pairwise comparisons among the data for each pair of ranges support the conclusion that $\bar{d}'_i$ decreases with a decrease in range and $\bar{\eta}_i$ increases with a decrease in range. (In the six pairwise tests performed, the null hypothesis was rejected at the 0.001 level in five cases and at the 0.01 level in the remaining case.)

5. In considering the results on information transfer for Experiment 2B, it should be noted that estimates of ψ tend to be biased toward the high side and that, for a fixed number of trials, the larger the stimulus set, the greater the bias (see Houtsma, 1983). Thus, our estimates of ψ for Stimulus Sets I and J, each of which contained 19 elements, should be more biased than our estimates for Sets A-H, each of which contained only 10 elements.

6. In considering these results, it should be noted that our methods of data processing (estimation of d'_{disc} from $d'_{disc} = \delta'\Delta$ and alteration of the confusion matrices as described in Note 3) tend to suppress the values of η_i slightly when the corresponding values of d'_i are large. If this suppression were eliminated, it would increase the values of η_i at the lower end of the range for Stimulus Sets A and I and at the upper end for Sets H and J. This would not only reduce the tendency for η_i to decrease with increasing i for Stimulus Set H, but would also increase the edge effect for the various stimulus sets (particularly Sets A and H).

APPENDIX

Small Intrarun Practice Effect

We examined in Experiment 1A how our results on d' would have changed if we had included in our estimates of d' the first 10 trials of each run as well as the last 50 trials. Specifically, for each of the 12 runs in the first two sessions and for each of the 4 subjects, we computed the ratio $r = d'$ (50 trials)/d' (60 trials). This ratio was then averaged over the six runs in each session for each subject, leading to eight values of the average ratio (one for each subject and session). We found that all eight values fell within 10% of unity and that the overall average differed from unity by less than 2%. In other words, it doesn't matter whether the first 10 trials are regarded as practice or are included in the estimates of sensitivity.

Small Response Bias

Again using the results obtained in the first two sessions of Experiment 1A, we examined the size of the response bias β. We found that, for each subject, the average value of β (averaged over all 12 runs in the first 10 sessions) always fell in the interval $(-0.2, 0.0)$. The overall average (averaged over subjects as well as runs) was given by $\beta = -0.08$. The negligible values of β obtained in this analysis convinced us that β was an uninteresting parameter in these experiments and could be ignored.

Small Deviation Between Averaged d' and Pooled d'

We examined the difference between (1) computing d' for each run and then averaging d' across runs (averaged d') and (2) pooling the results from all runs by adding the matrixes and then computing d' for the pooled results (pooled d'). These two methods of integrating results across runs were compared for four cases (each of which include eight runs, one for each session). Overall, the pooled d' is less than the averaged d' by only about 10% (consistent with the small values of bias obtained in the individual runs).

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