# Successiveness discrimination: Two models* 

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#### Abstract

Two models for successiveness discrimination, an attention-switching model and a duration-discrimination model, are described. Data are reported from a forced-choice successiveness discrimination task in which the standard stimulus assumed one of three values during a session. Of major interest is the ability of the models to account for the absence of observed variation in performance with changes in value of the standard. Conventional signal detection-type models or discrete state models would be unable to account for the data.


When two signals occur in close temporal succession, it is difficult to discriminate such a pattern from one in which the two signals actually occur simultaneously. The probability that two successive signals will be discriminated as successive rather than simultaneous is a function of the amount of time which separates them. Kristofferson and Allan (1973) briefly discussed two quantitative models for successiveness discrimination. The attention-switching model considers successiveness discrimination within the framework of Kristofferson's (1967a, 1970) time quantum theory and portrays the discrimination as being determined by the order of processing of discrete internal events. The onset-offset model, originally formulated by Allan, Kristofferson, and Wiens (1971) for duration discrimination, also contains a time quantum parameter, but it represents the 0 in a successiveness discrimination task as using duration information. In the present paper, we develop these models more fully, and present new data appropriate to the evaluation of the models.

## Attention-Switching Model

Kristofferson's time quantum theory postulates that the processing of information is sometimes under the control of an internal timing mechanism which generates a succession of equally spaced points in time. These points occur at the rate of one every q msec, and their occurrence is independent of the presentation of an external signal. The attention-switching model for successiveness discrimination (Kristofferson, 1967a) is a specific application of the time quantum theory. There are three basic assumptions. First, it is assumed that there are independent input channels. The channels are independent in the sense that at any point in time only one input channel can be attended to, and activity in one channel is not influenced by what activity occurs in the others or by when it occurs. Second, it is assumed that a signal which occurs in either an unattended or an attended channel can signal attention to switch channels. Third, it is assumed that when attention is signaled to

[^0]switch, its first opportunity to do so is at the next time point.

In order for two signals in independent input channels to be discriminated as successive, it is necessary to observe the occurrence of one, switch attention to the channel of the second, and then observe the occurrence of the second. If the second signal has already occurred by the time the switching operation is completed, then the two signals are equivalent to simultaneous signals. Thus, in order for two signals to be coded as successive, a time point must fall in the interval between them. The discrimination of two signals which are in independent channels as successive is limited by the time required before attention can switch from one channel to the other. The distribution of waiting times, $f(w)$, is uniformly distributed over a range from 0 to q msec.

$$
f(w)= \begin{cases}\frac{1}{q} & \text { if } 0 \leqslant w \leqslant q \\ 0 & \text { otherwise }\end{cases}
$$

It should be noted that the attention-switching model is applicable only to signals in independent channels. In its present form, the model makes no predictions about the discrimination of the successiveness of two signals when both occur in the same channel.

In a later paper, Kristofferson (1967b) formulated the two-state attention-switching model for successiveness discrimination. This is identical to the original model, except that the 0 is said to be in a 2 -quantum state on some proportion, $\mathrm{P}_{2}$, of the trials. In a 2 -quantum state, the distribution of waiting times is uniformly distributed over a range from 0 to 2 q msec . The predictions of the two-state attention-switching model for a two-alternative forced-choice successiveness discrimination task are presented in that paper. Each trial of the forced-choice task consisted of the presentation of two signal pairs, one after the other, each pair consisting of Signal A and Signal B. For one of the pairs, referred to as the standard and denoted by $\mathrm{S}_{\mathrm{s}}$, Signal A always occurred $\mathrm{t}_{\mathrm{s}} \mathrm{msec}$ before Signal B. For the other pair, referred to as the variable and denoted by $\mathrm{S}_{\mathrm{v}}$, Signal A always occurred $\mathrm{t}_{\mathrm{v}} \mathrm{msec}$ before Signal B. Thus, each trial consisted of an $\mathrm{S}_{\mathrm{v}} \mathrm{S}_{\mathrm{s}}$ or an $\mathrm{S}_{\mathrm{s}} \mathrm{S}_{\mathrm{v}}$ stimulus pattern. Ten different values of


Fig. 1. Forced-choice successiveness psychometric function predicted by the two-state attention-switching model.
$t_{v}$ were assigned randomly to trials, with the restriction that there be an equivalent number of each in every block of trials, and $t_{s}$ was constant through the experiment. The O was required to indicate whether it seemed more likely to him that Signal A occurred before Signal B in the first pair, an $\mathrm{R}_{1}$ response, or in the second pair, an $R_{2}$ response.

The forced-choice, successiveness psychometric function predicted by the two-state attention-switching model is shown in Fig. 1. For $t_{s}=x, t_{v} \geqslant x$ and for stimulus patterns which occur with equal frequency, the function rises from $P(C)=0.5$ when $t_{v}=x$ to $P(C)=1.0$ when $t_{v}=x+2 q$ as two linear segments, each segment spanning $q \mathrm{msec}$. The parameter x represents the difference between the two input channels in transmission times from the periphery to the first processing stage. When Signal A precedes Signal B by x msec, the two signals arrive simultaneously at the first stage. Thus, while a $0-\mathrm{msec}$ interval is simultaneity in the signal pair, $x$ is simultaneity in the arrival times, a positive value of $x$ implying slower transmission in the Signal A channel than in the Signal B channel. The parameter $q$ represents the period of the internal timing mechanism.

It should be noted that the psychometric function shown in Fig. 1 is based on the assumption that the 0 always attends to the $A$ channel when a signal pair is presented. If the $O$ were attending to the $B$ channel when a signal pair was presented, then it would be necessary to switch to the A channel and then 'rack again to the $B$ channel during the interval between the signals in order to discriminate them as successive. Two time points would be needed if the $O$ were in a 1-quantum state, and four time points if he were in a 2-quantum state.

We have now determined the forced-choice psychometric functions predicted by the two-state attention-switching model for

$$
\begin{gathered}
0 \leqslant \mathbf{P}_{\mathbf{A}} \leqslant 1 \\
0 \leqslant \mathrm{P}_{2} \leqslant 1 \\
\mathrm{x}-\mathrm{q} \leqslant \mathrm{t}_{\mathrm{s}} \leqslant \mathrm{x} \\
\mathrm{x}-\mathrm{q} \leqslant \mathrm{t}_{\mathrm{v}}
\end{gathered}
$$

where $P_{A}$ represents the probability that the $O$ is attending to the Signal $A$ channel when a signal pair is presented. Note that $t_{i}<0$ ( $i$ equal to $v$ or $s$ ) indicates that Signal A follows Signal B in stimulus time.

Since the pair of signals may fall anywhere on the time continuum with respect to the time points, the probability that one point will fall between the internal signals will be zero when the external signals are separated by $x$ msec, and it will be one when they are separated by $x+q$ msec. In general, the probability that a time point will fall between the internal signals is

$$
\begin{aligned}
& \int_{0}^{t_{i}-x-n q} f(w) d w \quad \text { for } x+n q \leqslant t_{i} \leqslant x+(n+1) q \\
& \int_{0}^{-\left(t_{i}-x\right)-n q} f(w) d w \quad \text { for } x-n q \leqslant t_{i} \leqslant x
\end{aligned}
$$

for $n$ equal to a nonnegative integer, and $i$ equal to $v$ or s.

As a psychophysical model, the attention-switching model is a simple three-state model. It provides a mechanism for generating three states, a simultaneous state, denoted by O, and two types of successive states, Signal A followed by Signal B, denoted by AB, and Signal B followed by Signal A, BA. It says that these are in fact states, and that there are not degrees of simultaneity. That is, when attention switches to the channel of the second signal, and if that signal has already occurred, it does not matter how long ago that happened. The trace of the signal conveys no useful temporal information.

There are eight possible state combinations on any trial: $(A B, O),(0, A B),(B A, 0),(O, B A),(A B, B A)$, $(B A, A B),(B A, B A),(0,0)$. The following decision rule is assumed:

$$
\begin{aligned}
P\left[R_{1} \mid(A B, O) \cup(O, B A) \cup(A B, B A)\right] & =1 \\
P\left[R_{1} \mid(O, A B) \cup(B A, O) \cup(B A, A B)\right] & =0 \\
P\left[R_{1} \mid(O, O)\right] & =\gamma \\
P\left[R_{1} \mid(B A, B A)\right] & =\gamma^{\prime}
\end{aligned}
$$

It is also assumed that on any trial the 0 attends to the same channel when the second signal pair is presented as he did when the first was presented. The psychometric functions generated by the model are presented in

Table 1
Predicted Psychometric Functions for AttentionSwitching Model

| 1 | $t_{v}$ |
| :--- | :--- |
| $x-q \leqslant t_{v} \leqslant x$ | $\frac{\left(t_{v}-t_{g}\right)\left(1-P_{A}\right)\left(1-.5 P_{2}\right)}{2 q}+.5$ |
| $x \leqslant t_{v} \leqslant x+q$ | $\frac{\left(P_{A}\left(t_{v}+t_{x}-2 x\right)+x-t_{s}\right]\left(1-.5 P_{2}\right)}{2 q}+.5$ |
| $x+q \leqslant t_{v} \leqslant x+2 q$ | $\frac{\left(1-P_{A}\right)\left(1-P_{2}\right)\left(t_{v}-x\right)\left(t_{s}-x\right)}{2 q^{2}}+\frac{\left(1-P_{A}\right)\left(1-.5 P_{2}\right)\left(t_{v}-2 t_{s}+x\right)}{2 q}+$ |
| $x+2 q \leqslant t_{v} \leqslant x+4 q$ | $\frac{P_{2}\left(1-P_{A}\right)\left(2 q+t_{s}-t_{v}\right)}{4 q}+\frac{P_{2} P_{A}\left(t_{v}-x-2 q\right)}{4 q}+P_{A}$ |
|  |  |

Table 1. For a fixed set of values of $\mathrm{P}_{\mathrm{A}}, \mathrm{P}_{2}$, and $\mathrm{t}_{\mathrm{s}}$, the function relating $P(C)$ to $t_{v}$ consists of a series of linear segments, each segment spanning one quantum. Representative functions for $\mathrm{q}=50 \mathrm{msec}$ and $x=10 \mathrm{msec}$ are presented in Fig. 2. Figures 2c, 2d, and 2e illustrate the manner in which the functions change as $P_{A}$ varies between 0 and 1 for three different $t_{s}$ values and $P_{2}=0$. For $\left(2 x-t_{s}\right)>t_{v}$, performance improves as $P_{A}$ decreases from 0 to 1 , while for $t_{v}>\left(2 x-t_{s}\right)$ performance improves as $P_{A}$ increases. Figure $2 b$ directly compares performance for three $t_{s}$ values for $\mathrm{P}_{\mathrm{A}}=.5$ and $\mathrm{P}_{2}=.7$, showing performance to improve as $\mathrm{t}_{\mathrm{s}}$ becomes more negative. For all values of $\mathrm{P}_{\mathbf{A}} \neq 1, \mathrm{P}(\mathrm{C})$ will be an increasing function of $t_{s}$ for

$$
x-q \leqslant t_{s} \leqslant x
$$

For $\mathrm{P}_{\mathbf{A}}=1$, the expressions in Table 1 simplify to those presented by Kristofferson (1967b). It is important to note that the functions for $\mathrm{P}_{\mathrm{A}}=1$ are invariant for all values of $t_{s}$ between $(x-q)$ and $x$ msec (Fig. 2a). Thus, if the O always attends to Channel A when a signal pair is presented, his successiveness discrimination functions will be identical for all values of $t_{s}$ between $(x-q)$ and x msec. Performance should be the same when Signal A follows Signal B by $(x-q)$ msec in the standard signal pair as when it precedes Signal B by $x$ msec. This is because all standard pairs between ( $\mathrm{x}-\mathrm{q}$ ) and x msec have either zero or one time point between them and


Fig. 2. Attention-switching model: representative functions.

Table 2
Predicted Psychometric Functions for the Onset-Offset Model

since the first internal signal for the intervals in this range is $B$, two points are required for the two signals to be judged successive if the 0 is in a 1 -quantum state and four if he is in a 2 -quantum state. Thus, all standard intervals between ( $x-q$ ) and $x$ msec will be coded as simultaneous.

## Onset-Offset Model

Allan et al (1971) developed a model for duration discrimination which specifies that signal duration is mapped into continuous internal duration with a quantal mechanism being involved in the mapping. It is assumed that all of the variability in the internal durations which are produced by repeated presentation of a fixed stimulus duration is the result of variation in the times at which the internal duration begins and ends. For any signal duration, the time lag preceding the internal begin and end points, the perceptual onset and offset latencies, are each independently and uniformly distributed over a range of q msec . This results in a triangular distribution of internal durations, spanning 2 q msec , which has a mean equal to the physical duration of the signal and a variance which is independent of signal duration. Thus, while the variability in internal duration is caused by a quantal process, internal duration is not quantized, but varies continuously.

Kristofferson and Allan (1973) extended the onset-offset model for duration discrimination to successiveness discrimination. If it is assumed that the perceptual latencies associated with Signals A and B are independently and uniformly distributed over a range of q msec, then the internal intervals separating the two signals will have a triangular distribution spanning 2 q msec . The distribution will have a mean equal to the stimulus interval between the two signals (positive when

A precedes B , and negative when B precedes A ) when the afferent latencies of A and B are equal, and a variance that is independent of the interval. They suggested that in the forced-choice successiveness discrimination task, the 0 adopts a criterion interval size, C , and makes an independent decision regarding the successiveness of the two signals after each stimulus-pair presentation, and that his response on each trial is based on these two decisions.

Specifically, on each forced-choice trial, each signal pair is coded as either successive, AB, or simultaneous, 0 , depending upon whether the internal interval is greater or less than C , respectively. The probabilities of an $A B$ code, given an $S_{i}$ signal pair (for $i=v$ or $s$ ), are

$$
P\left(A B \mid S_{i}\right)=\int_{C}^{t_{i}+q} f_{i}(I) d I,
$$

where $f_{i}(I)$ represents the distribution of internal intervals;

$$
f_{i}(I)= \begin{cases}\frac{q+t_{i}-I}{q^{2}} & \text { if } t_{i} \leqslant I \leqslant t_{i}+q \\ \frac{q-t_{i}+I}{q^{2}} & \text { if } t_{i}-q \leqslant I \leqslant t_{i} \\ 0 & \text { otherwise }\end{cases}
$$

The two signal pairs on each trial can be coded in four ways: $(A B, A B),(A B, 0),(0, A B),(0,0)$. The following decision rule is assumed:


Fig. 3. Onset-offset model: representative functions.

$$
\begin{aligned}
\mathrm{P}\left[\mathrm{R}_{1} \mid(\mathrm{AB}, \mathrm{O})\right] & =1 \\
\mathrm{P}\left[\mathrm{R}_{1} \mid(\mathrm{O}, \mathrm{AB})\right] & =1 \\
\mathrm{P}\left[\mathrm{R}_{1} \mid(\mathrm{AB}, \mathrm{AB})\right] & =\gamma \\
\mathrm{P}\left[\mathrm{R}_{1} \mid(0,0)\right] & =\gamma^{\prime}
\end{aligned}
$$

This model can be thought of as a two-state psychophysical model in which the distributions of I and the placement of the criterion generate the state probabilities. For stimulus patterns which occur with equal frequency, the psychometric function is

$$
P(C)=\frac{1}{2}\left[\int_{C}^{t_{v}+q} f_{v}(I) d I+\int_{t_{s}-q}^{C} f_{s}(I) d I\right]
$$

The shape of the function depends upon the placement of the criterion. These functions are presented in Table 2 and in Fig. 3 for $q=50 \mathrm{msec}$. It should be noted that if the O's criterion is greater than the largest internal value generated by the largest value of $\mathrm{t}_{\mathrm{s}}$, for example $\mathrm{C}=$ 60 msec , then the psychometric function will be invariant over all smaller values of $\mathrm{t}_{\mathrm{s}}$ (Fig. 4d).

## Evaluations of the Models

Kristofferson and Allan (1973) presented the data from the Kristofferson (1967b) study in terms of both the attention-switching model and the onset-offset model. In that study, two light-tone pairs were presented on each trial and the 0 had to indicate whether the offset of the light (Signal A) occurred prior to the offset of the tone (Signal B) in the first or second pair. There were 10 values of $\mathrm{t}_{\mathrm{v}}$ ( 30 to 120 msec in $10-\mathrm{msec}$ steps), one value of $t_{s}(20 \mathrm{msec})$, and 13 Os . They found that both models provided an excellent account of the data. They also reported data from another experiment, which included five values of $\mathrm{t}_{\mathrm{v}}(10,30,50,70$, and 90 msec$)$ and three values of $t_{s}(0,-10$, and -25$)$, positive values indicating that the offset of the light occurred before the offset of the tone, negative values that tone offset occurred before light offset. The different experimental conditions were randomly intermixed in a block of trials. The data averaged over six Os (Fig. 2 in Kristofferson \& Allan, 1973) indicated that the psychometric function was not affected by the variations in $t_{s}$ that were used. The analysis of the data in terms of the two models led to the tentative conclusion that the attention-switching model provided a better representation of the performance than the onset-offset model. The conclusion was tentative
$S_{v} S_{s}$ Trial


RESPONSE PERIOD

FEEDBACK TONE

Fig. 4. Sequence of events on each trial.

because the amount of data per $O$ was small, the within- 0 variability quite large, and $\mathrm{P}_{\mathrm{A}}$ was arbitrarily set equal to 1 . We will now present data which permit a more adequate evaluation of the two models.

## METHOD

The sequence of events on each trial is shown in Fig. 4. A two-alternative, forced-choice paradigm was used in which each trial consisted of the presentation of two light-tone pairs, one after the other. For the variable pair ( $\mathrm{S}_{\mathrm{v}}$ ), the light was always terminated before the tone ( $\mathrm{t}_{\mathrm{v}}$ was positive). For the standard pair ( $\mathrm{S}_{\mathrm{s}}$ ), the light was terminated either simultaneously with the tone or after the tone ( $\mathrm{t}_{\mathrm{s}}$ was zero or negative). On half the trials, the variable was presented first, followed by the standard ( $\mathrm{S}_{\mathbf{v}} \mathrm{S}_{\mathrm{s}}$ trials); on the remaining trials, the standard occurred first, followed by the variable ( $\mathrm{S}_{\mathrm{s}} \mathrm{S}_{\mathrm{v}}$ trials). The O responded during a 3 -sec response period by pushing one of two buttons on the arm of his chair. He was instructed to decide whether the light terminated before the tone in the first signal pair (an $\mathrm{R}_{1}$ response) or in the second signal pair (an $\mathrm{R}_{2}$ response). On $\mathrm{S}_{\mathrm{v}} \mathrm{S}_{\mathrm{s}}$ trials, a $100-\mathrm{msec}, 500-\mathrm{Hz}, 68-\mathrm{dB}$ gated tone was presented at the end of the response period. The next trial always began 1.1 sec after the end of the response period.

For a variable pair, the tone was $\mathrm{t}_{\mathrm{v}} \mathrm{msec}$ longer than 2 sec in duration and the light was 2 sec . For a standard pair, the light was $t_{s}$ msec longer than 2 sec in duration and the tone was 2 sec . The interval between the offset of the light in the first pair and the onset of the light in the second pair was 2 sec .

Data from six paid Os will be reported. Each O was seated in an IAC sound-attenuated auditory chamber, 66 cm from the visual display. The chambers were illuminated by a $40-\mathrm{W}$ bulb.

Glow modulator bulbs (Sylvania R1131C), driven by an Iconix power supply (Model 6195-4), were used to generate the light signal. The bulb was enclosed in a metal box with an aperture of 4 mm in diam subtending a visual angle of 21 min . The aperture was covered on the inside with a Kodak No. 96 neutral density 2.00 filter and then translucent milk glass. The luminance of the visual signal was constant at 50 fL , as measured by 150 UB Photo Research Corporation photometer. The auditory signal was a $2,000-\mathrm{Hz}, 68-\mathrm{dB}$ gated tone, delivered through earphones. The presentation and timing of the signals and the recording of the responses were under the control of a PDP computer.

Three Os were run simultaneously. For Os J.P., P.S., and P.C., there were 10 values of $\mathrm{t}_{\mathbf{v}}$ ( 10 to 100 msec in $10-\mathrm{msec} s t e p s$ ) and three values of $t_{s}(0,-10$, and -25$)$. During each session, the 3 values of $t_{s}$ and 5 values of $t_{v}$ (either $10,30,50,70$, and 90 or $20,40,60,80$, and 100 ) were presented. The 15 conditions were intermixed randomly in each block of 90 trials, with the restriction that each condition occur six times, three times on an $S_{v} S_{s}$ trial and three times on an $S_{s} S_{v}$ trial. Three blocks of 90 trials were run during a session. J.P. participated in 28 sessions ( 15 in the $10-90$ range, 13 in the $20-100$ range), P.S. in 34 ( 17 and 17), and P.C. in 28 (14 and 14). The first four sessions were considered as practice sessions, and the data from these sessions were not included in the analysis.

For Os O.C., M.F., and C.C., there were five values of $\mathrm{t}_{\mathrm{v}}(10$, $30,50,70$, and 90 msec$)$ and three values of $\mathrm{t}_{\mathrm{s}}(0,-10$, and -25 msec ). Again, the 15 conditions were intermixed randomly in each block of 90 trials. Each of these Os participated in 24 sessions, the first 4 not being included in the analysis.

## RESULTS

The probability of a correct response, $\mathrm{P}(\mathrm{C})$,

Fig. 5. $P(C)$ as a function of $t_{v}$ for each of the three values of $t_{\mathbf{s}}$, for each 0 .









$$
P(C)=1 / 2\left[P\left(R_{1} \mid S_{v} S_{s}\right)+P\left(R_{2} \mid S_{s} S_{v}\right)\right]
$$

under each experimental condition is shown separately
for each 0 in Fig. 5 and in Table 3. It is clear that for O.C. successiveness discrimination performance improves considerably as $t_{s}$ becomes more negative.

Table 3
P(C) for Each Experimental Condition

| $\mathrm{t}_{\text {S }}$ | Os | $t_{v}$ |  |  |  |  | $\mathrm{t}_{\mathrm{v}}$ |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | 10 | 30 | 50 | 70 | 90 | 20 | 40 | 60 | 80 | 100 |
| 0 | J.P. | . 542 | . 537 | . 652 | . 828 | . 888 | . 574 | . 596 | . 717 | . 843 | . 888 |
|  | P.S. | . 515 | 595 | . 685 | . 774 | . 893 | . 566 | . 707 | . 760 | . 856 | . 918 |
|  | P.C. | . 438 | . 564 | . 668 | . 798 | . 902 | . 520 | . 606 | . 724 | . 798 | . 910 |
|  | O.C. | . 575 | . 660 | . 754 | . 863 | . 911 |  |  |  |  |  |
|  | M.F. | . 503 | . 539 | . 670 | . 742 | . 850 |  |  |  |  |  |
|  | C.C. | . 513 | . 555 | . 644 | . 820 | . 950 |  |  |  |  |  |
| Average |  | . 514 | . 575 | . 679 | . 804 | . 899 | . 553 | . 636 | . 734 | . 832 | . 905 |
| -10 | J.P. | . 513 | . 594 | . 687 | . 789 | . 914 | . 485 | . 544 | . 697 | . 858 | . 909 |
|  | P.S. | . 604 | . 680 | . 733 | . 826 | . 907 | . 566 | . 644 | . 737 | . 900 | . 944 |
|  | P.C. | . 516 | . 630 | . 610 | . 782 | . 886 | . 568 | . 612 | . 716 | . 843 | . 902 |
|  | O.C. | . 669 | . 703 | . 822 | . 869 | . 928 |  |  |  |  |  |
|  | M.F. | . 530 | . 600 | . 653 | . 778 | . 856 |  |  |  |  |  |
|  | C.C. | . 500 | . 553 | . 678 | . 848 | . 942 |  |  |  |  |  |
| Average |  | . 555 | . 627 | . 697 | . 815 | . 906 | . 540 | . 600 | . 717 | . 867 | . 918 |
| -25 | J.P. | . 530 | . 524 | . 686 | . 806 | . 897 | . 503 | . 612 | . 783 | . 854 | . 918 |
|  | P.S. | . 596 | . 689 | . 730 | . 848 | . 904 | . 630 | . 689 | . 802 | . 892 | . 922 |
|  | P.C. | . 512 | . 542 | . 656 | . 826 | . 868 | . 556 | . 552 | . 710 | . 824 | . 938 |
|  | O.C. | . 721 | . 783 | . 875 | . 908 | . 944 |  |  |  |  |  |
|  | M.F. | . 530 | . 570 | . 689 | . 791 | . 859 |  |  |  |  |  |
|  | C.C. | . 503 | . 561 | . 647 | . 803 | . 956 |  |  |  |  |  |
| Average |  | . 565 | . 612 | . 714 | . 830 | . 905 | . 563 | . 618 | . 765 | . 857 | . 926 |



Fig. 6. Best-fitting functions (line), averaged over Os, predicted by the attention-switching model. Data points are mean $\mathbf{P ( C )}$ values.

There is the suggestion of a similar trend for P.S., especially for the 10.90 sessions. The other four Os do not show any systematic changes in performance with variations in $\mathbf{t}_{\mathbf{s}}$. For these four Os, simultaneous offsets of the light and tone is equivalent to the offset of the tone preceding the offset of the light by 25 msec .

We shall now examine the data in terms of the two models discussed earlier.

## THEORETICAL ANALYSIS

## Attention-Switching Model

For Os J.P., P.S., and P.C., two sets of the four parameters, $q, x, P_{A}$, and $P_{2}$, were independently estimated, one set for the sessions during which $t_{v}$ varied in $20-\mathrm{msec}$ steps between 10 and 90 msec , and the other set for the sessions during which $\mathrm{t}_{\mathrm{v}}$ varied in $20-\mathrm{msec}$ steps between 20 and 100 msec . The parameters $\mathbf{P}_{\mathrm{A}}$ and $P_{2}$ were varied between 0 and 1 in .01 steps. The parameter x was varied in millisecond steps for

$$
0 \leqslant x \leqslant q-25
$$

The above restriction on x was placed since the predicted psychometric functions presented in Table 2 were determined assuming

$$
x-q \leqslant t_{s} \leqslant x
$$

and $t_{s}$ varied between 0 and -25 msec in the experiment. Since we (Kristofferson, 1967b; Kristofferson \& Allan, 1973) had previously found that on the average q was about 50 msec , the parameter q was varied in both directions, starting at 50 msec , in $1-\mathrm{msec}$ steps. For each O , the two sets of four values which yielded a minimum sum of squared deviations (SS) between the 15 obtained and predicted values of P(C) were determined. For Os O.C., M.F., and C.C., a single set of four parameters was estimated in the same way. The predicted functions averaged over Os are presented in Fig. 6. For each O, the proportion of the total variance in $\mathrm{P}(\mathrm{C})$ accounted for by the model was

Table 4
Parameter Values for the AttentionSwitching Model

| Os | $\mathrm{t}_{\mathrm{v}}$ : $10-90$ |  |  |  |  |  | $\mathrm{t}_{\mathrm{v}}$ : 20-100 |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | q | x | $\mathrm{P}_{\mathbf{A}}$ | $\mathrm{P}_{2}$ | SS | Proportion | q | x | $\mathrm{P}_{\text {A }}$ | $\mathrm{P}_{2}$ | SS | Proportion |
| J.P. | 53 | 27 | . 91 | . 46 | . 0067 | . 979 | 55 | 28 | . 93 | . 49 | . 0119 | . 965 |
| P.S. | 42 | 3 | . 69 | . 91 | . 0087 | . 960 | 44 | 5 | . 80 | 1.00 | . 0074 | . 970 |
| P.C. | 46 | 18 | 1.00 | 1.00 | . 0162 | . 950 | 51 | 15 | . 99 | 1.00 | . 0101 | . 965 |
| O.C. | 38 | 9 | . 42 | . 35 | . 0047 | . 974 |  |  |  |  |  |  |
| M.F. | 47 | 22 | . 85 | . 88 | . 0027 | . 988 |  |  |  |  |  |  |
| C.C. | 66 | 25 | . 98 | . 15 | . 0043 | . 990 |  |  |  |  |  |  |

Fig. 7. Best-fitting functions (line) averaged over Os predicted by the onset-offset model. Data points are mean $P(C)$ values.

calculated. These proportions, as well as the values of the four parameters and the values of SS are presented in Table 4 for each O . It is clear that the model provides an excellent account of the observed variation in $\mathrm{P}(\mathrm{C})$ for each of the six Os. The average value of $\mathrm{q}, 49.1 \mathrm{msec}$ (standard deviation, 7.84), is in agreement with the average value of 48.1 reported by Kristofferson (1967b) for $\mathrm{t}_{\mathrm{s}}=20 \mathrm{msec}$.

In general, for those Os for whom two independent sets of parameters were estimated, the two sets are in good agreement. Only P.S. shows a large discrepancy in his two values of $\mathbf{P}_{\mathbf{A}}$. However, this is not troublesome for the model. One would expect q and x to be independent of $t_{v}$, but not necessarily $P_{A}$ and $P_{2}$. What is somewhat troublesome is that for each of the three $\mathrm{Os}, \mathrm{q}$ is always slightly larger for the 20.100 range than for the $10-90$ range.

## Onset-Offset Model

For Os J.P., P.S., and P.C., two sets of the two Table 5

| Parameter | Values | for the | Onset-Offset | Model |
| :---: | :---: | :---: | :---: | :---: |
| Os | q | C | SS | Proportion |
| $\mathrm{t}_{\mathrm{v}}: 10-90$ |  |  |  |  |
| J. P. | 70 | 62 | . 0072 | . 977 |
| P.S. | 95 | 43 | . 0091 | . 958 |
| P.C. | 71 | 65 | . 0153 | . 953 |
| O.C. | 74 | 19 | . 0161 | . 911 |
| M.F. | 89 | 65 | . 0036 | . 985 |
| C.C. | 55 | 61 | . 0023 | . 994 |
| $t_{v}: 20-100$ |  |  |  |  |
| J.P. | 71 | 66 | . 0170 | . 950 |
| P.S. | 90 | 50 | . 0094 | . 962 |
| P.C. | 78 | 67 | . 0064 | . 978 |

parameters q and C were independently estimated. Both parameters were varied in $1-\mathrm{msec}$ steps. For Os O.C., M.F., and C.C., a single set was estimated. The predicted functions averaged over Os are presented in Fig. 7. In Table 5, the values of the two parameters, the values of SS, and the proportion of the total variance in $P(C)$ accounted for by the model are presented. The onset-offset model also provides reasonable account of the variation in $\mathrm{P}(\mathrm{C})$ for all Os except O.C. The average value of $q$ for the onset-offset model is 77 , the standard deviation is 12 .

## CONCLUDING COMMENTS

The attention-switching model represents successiveness discrimination as based upon the relative times of occurrence of discrete internal events. The O is unable to attend to two independent inputs simultaneously, and a mechanism is postulated which controls when it is possible to switch attention from one input to the other. The longer the temporal separation between the two inputs, the greater the probability that the second input will provide the necessary order information. The variability in waiting times is the limiting factor in successiveness discrimination. When $P_{A}=1$, the distribution of internal events determining the probability that two stimulus events will be coded as successive is a mixture of two uniform distributions, one spanning $q \mathrm{msec}$, the other $2 q \mathrm{msec}$, where $\mathrm{P}_{2}$ is the mixture parameter. The value of $\mathrm{P}_{\mathbf{A}}$ adopted by the O determines the effect of variations in $\mathrm{t}_{\mathrm{s}}$.

The onset-offset model also represents successiveness discrimination as based upon the relative times of occurrence of discrete internal events. Here, however, it
is the variability in perceptual latencies which determines the internal order of the two events. The triangular distribution of internal events determining the probability that two stimulus events will be coded as successive is the convolution of two uniform distributions, each spanning q msec. The value of C adopted by the $\mathbf{O}$ determines the effect of variations in $t_{s}$.

Both models can account for the major features of successiveness performance observed in the present experiment. Of major interest is the success of the models in representing the differential effects variations in $t_{s}$ had on the performance of the six Os. We are unaware of any other models for successiveness discrimination which can successfully account for such data. Conventional signal detection-type models or discrete state models applied to successiveness discrimination would predict improved performance as $t_{s}$ becomes more negative, and would be unable to represent the performance of most of the $O s$ in the present experiment. Thus, while the data we report are not very
sensitive for discriminating between the two models, they are powerful as far as rejecting traditional models is concerned.

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