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Three-stimulus procedures in olfactory psychophysics: An experimental comparison of Thurstone-Ura and three-alternative forced-choice models of signal detection theory

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The experimental differences between the triangular method and three-alternative forced-choice (3-AFC) signal detection theory procedure are briefly reviewed, and the conditions to be met for a valid application of the respective procedures under the Thurstone-Ura and 3-AFC models are described. The assumptions of both of these unidimensional probabilistic models are given, and the functions relating the probability of a correct response to δ, a parameter of sensory stimulus distance, are specified. Since the two models both contain δ, the hypothesis of invariability of this parameter with the triangular method and the 3-AFC procedure was tested using olfactory stimuli. This hypothesis was strongly confirmed, since the functions relating δ to the logarithm of the ratio of the physical stimulus values were virtually identical for the two methods and their concomitant models. The results are important not only for the field of olfactory psychophysics; they also have wider methodological and psychophysical interest. The bias against the middle stimulus of a triangle in the triangular method observed when olfactory stimuli are used can be explained as the resultant of sensory adaptation. Finally, some practical implications of the present results for application of the triangular method are discussed.

When sensory discrimination between stimuli or sensitivity differences between subjects are investigated, a number of different methods can be used. The present paper is concerned with two of these: the triangle test, also called the triangular method, and the three-alternative forced-choice (3-AFC) procedure, both of which are tristimulus procedures.

The triangular method originates from the practical sensory evaluation of taste and odor of food (Amerine, Pangborn, & Roessler, 1965; Harrison & Elder, 1950; Peryam, 1958), which is still its main field of application. In the experimental procedure using this method, the subject is presented, on each trial, with three stimuli, two of which will have been obtained from the same sample of stimulus objects. The instructions tell the subject to select the odd stimulus and to guess if in doubt.

The triangular method has two distinct advantages over such other methods as, for example, the single-stimulus or paired comparison procedures. First, in the instructions, specification of a particular sensory attribute as a basis for differentiating between stimuli is not required. It is the subject's task to select the odd stimulus on the basis of whatever sensory attribute(s) he can. The second advantage is that a response can be classified unambiguously as correct or incorrect, using the origin of the stimulus selected as an external criterion; that is, no physical parameter is required for stimulus characterization. This methodological feature is especially attractive in food research because foods and drinks are often defined in terms of treatments rather than in physicochemical parameters.

The other procedure with which this paper is concerned is the 3-AFC task. This is a particular case of the multiple-AFC procedures that are used mainly in psychophysical experiments in audition and vision (Green & Swets, 1966; Swets, 1964). As in the triangular method, three stimuli are presented on each trial, two of them from the same sample. In contrast with the triangular method, however, in the 3-AFC procedure the subject is instructed *either* to select the strongest stimulus *or* to identify the weakest stimulus from each triplet presented. The use of a different mode of instruction is one of the two differences between the experimental procedures of the methods.

The second difference related to the experimental design is the specific tristimulus sets that are used. When the triangle test is applied, it is normal practice to present all of the six possible sequence types of three stimuli ("triangles"). Denoting the stimuli obtained from the two samples as S_i and S_j, respectively,

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these triangles are: $S_iS_iS_j$, $S_iS_jS_i$, $S_jS_iS_i$, $S_jS_jS_i$, $S_jS_iS_j$, and $S_iS_jS_j$. In the 3-AFC procedure, only the triplets $S_iS_iS_j$, $S_iS_jS_i$, and $S_jS_iS_i$ *or* the triplets $S_jS_jS_i$, $S_jS_iS_j$, and $S_iS_jS_j$ are presented in the same experiment.

The experimenter can choose to give trial-to-trial feedback in both methods, and this, indeed, is nearly always done with the 3-AFC procedure. However, in chemoreception research, the triangular method is most often applied without this form of feedback. For purposes of optimal comparison, the no-feedback procedures are used in the present paper.

In practical applications, experimental circumstances differ in the amount of information the subjects receive with respect to the nature and attributes of the stimuli to be presented. In the present paper, the procedures described assume that the subjects do not receive any information other than the instruction and a specification of the sensory modality by which the stimuli should be perceived. No stimuli are designated for the subjects as standards or variable stimuli, and no information is given about the composition of the three-stimulus set with which a subject is presented in a certain trial.

The final data obtained by both the triangular method and the 3-AFC procedure are proportions of correct responses for each two samples of stimulus objects from which the three-stimulus sets were prepared.

Several ways of handling such proportions have been proposed. For the triangle test, these include testing procedures such as the binomial test (e.g., Lockhart, 1951) and estimation of a dose-response relation using a model (Bock & Jones, 1968; Filipello, 1956) that was recently named the triangular constant method model (Frijters, 1979).

As is the case for a number of m-AFC procedures (Elliot, 1964; Green & Birdsall, 1964), models for the triangular method have been developed which specify the functional relationship between the probability of a correct response (P_c) and the distance (δ) between the stimuli on an underlying unidimensional sensory continuum (Ura, 1960; David & Trivedi, Note 1). In a recent paper (Frijters, 1979), one of these triangular method models was called the Thurstone-Ura model, and it was compared theoretically to a similar distance model for the 3-AFC procedure, called the normal 3-AFC SDT model. In the same paper, the logistic variants of these models were developed and compared with each other. It was observed that the shapes of the functions relating P_c to δ arising from the normal and the logistic Thurstone-Ura models were quite similar, and also that the pair of normal and logistic 3-AFC models had virtually identical functions.

The present experiment deals with these four models, and since they were extensively described in Frijters (1979), only a brief summary of their theory is given here, after which the aims of the experiment are described.

THEORY

The triangular method under a variant of the Thurstone-Ura model, or a 3-AFC procedure, can be used when the following experimental criteria are met:

Condition 1. There are $X_1 \ldots, X_j, X_s$ samples of stimulus objects.

Condition 2. The stimuli $S_1 \ldots, S_j, S_s$, drawn randomly from the samples $X_1 \ldots, X_j, X_s (S_1 \varepsilon X_1, S_j \varepsilon X_j, S_s \varepsilon X_s)$, differ only in magnitude of one particular sensory attribute.

Condition 3. Stimulus sets are composed of one standard stimulus, S_s, and two variable stimuli, S_j and S_j' ($s \neq j$), or of two standard stimuli, S_s, S_s', and one variable, S_j ($s \neq j$). (In order to avoid confusion, it should be noted that although the terms "standard" and "variable" are used here, these stimuli are not labeled as such for the subject.)

Only for the triangular method:

Condition 4a. The six sequence types of triangles are presented in random and balanced order under a forced-choice instruction to select the odd stimulus and to guess when in doubt.

And for the 3-AFC task:

Condition 4b. The three sequence types of triplets of one of the two combinations are presented in random and balanced order. The subject is instructed *either* to select the strongest stimulus *or* to select the weakest stimulus of each triplet presented.

The four models mentioned are derived from specific combinations of a number of assumptions, formulated below. As may be recalled, these are the normal Thurstone-Ura, the logistic Thurstone-Ura, the normal 3-AFC, and the logistic 3-AFC SDT models, which are specified after their assumptions are formulated.

Assumption A. The stimuli S_s, S_j, and S_j' (or S_s, S_s', and S_j) give rise to sensory responses of the magnitudes x_s, x_j, and x_j' (or x_s, x_s', and x_j), respectively; these sensory values are mutually independently distributed with the probability density functions $f(x_s)$, $f(x_j)$, and $f(x_j)$ [or $f(x_s)$, $f(x_s)$, and $f(x_j)$], respectively.

For the normal models:

Assumption B.1. The probability densities $f(x_s)$ and $f(x_j)$ are normal distributions; they have respective means μ_s and μ_j and equal variances, σ^2.

For the logistic models:

Assumption B.2. The probability densities $f(x_s)$ and $f(x_j)$ are logistic distributions with respective means μ_s and μ_j and equal variances, σ^2.

In the case of the triangular method:

Assumption C.1. On a particular observation, a correct response will be produced if (1) $| x_j - x_j' | < | x_j - x_s |$ *and* $| x_j - x_j' | < | x_j' - x_s |$, when the triangle presented is composed of the stimuli S_s, S_j, and S_j', or if (2) $| x_s - x_s' | < | x_s - x_j |$ *and* $| x_s - x_s' | < | x_s' - x_j |$, when the triangle contains the stimuli S_s, S_s', and S_j.

In case of the 3-AFC procedure:

Assumption C.2. A correct response will be produced (1) when the instruction says to select the *weakest* stimulus, if (a) $x_s < x_j$ *and* $x_s < x_j'$, given $\mu_s < \mu_j$ and one of the triplets $S_sS_jS_j'$, $S_jS_sS_j'$, and $S_jS_j'S_s$ was presented, or if (b) $x_j < x_s$ *and* $x_j < x_s'$, given $\mu_j < \mu_s$ and one of the triplets $S_jS_sS_s'$, $S_sS_jS_s'$, and $S_sS_s'S_j$ was presented, and (2) when the instruction says to select the *strongest* stimulus, if (a) $x_s > x_j$ *and* $x_s > x_j'$, given $\mu_s > \mu_j$ and the subject was presented with one of the triplets $S_sS_jS_j'$, $S_jS_sS_j'$, and $S_jS_j'S_s$, or if (b) $x_j > x_s$ *and* $x_j > x_s'$, given $\mu_j > \mu_s$ and the subject was presented with one of the triplets $S_jS_sS_s'$, $S_sS_jS_s'$, and $S_sS_s'S_j$.

As previously stated, these assumptions form the basis from which the functional relationships between the probability of a correct response (P_c) and the unidimensional sensory stimulus distance (δ) are derived. The equations obtained are:

Normal Thurstone-Ura model for the triangular method (Assumptions A, B.1, C.1):

$$P_c = 2\int_0^{\infty}\{\Phi[-u\sqrt{3}+\delta\sqrt{(2/3)}] + \Phi[-u\sqrt{3}-\delta\sqrt{(2/3)}]\}\frac{e^{-\frac{1}{2}u^2}}{\sqrt{2\pi}}du. \quad (1)$$

Logistic Thurstone-Ura model for the triangular method (Assumptions A, B.2, C.1):

$$P_c = -1+2\gamma\left[\frac{\gamma^2-2\gamma\ln\gamma-1}{(\gamma-1)^3}\right] - 2\gamma\int_0^{\infty}\frac{1}{(1+\gamma u)^2}\left[\frac{u}{(u+1)^2}\ln\frac{u}{(u+1)^2} - \frac{u(u-1)}{(u+1)^2} - \frac{(u-1)\sqrt{u}}{(u+1)^2} \times (\pi/2 - 2\arctan\sqrt{u})\right]du. \quad (2)$$

Normal 3-AFC SDT model (Assumptions A, B.1, C.2):

$$P_c = \int_0^{\infty}[\Phi^2(u+\delta)+\Phi^2(-u+\delta)]\frac{e^{-\frac{1}{2}u^2}}{\sqrt{2\pi}}du. \quad (3)$$

Logistic 3-AFC SDT model (Assumptions A, B.2, C.2):

$$P_c = \gamma\left[\frac{\gamma^2-2\gamma\ln\gamma-1}{(\gamma-1)^3}\right]. \quad (4)$$

In Equations 1 and 3, $\Phi(u)$ and δ are respectively given by:

$$\Phi(u) = 1/\sqrt{2\pi}\int_{-\infty}^{u}e^{-\frac{1}{2}t^2}dt \quad (-\infty < u < +\infty) \quad (5)$$

and,

$$\delta = \frac{\mu_2-\mu_1}{\sigma}. \quad (\mu_2 > \mu_1) \quad (6)$$

In Equations 2 and 4, γ is:

$$\gamma = \exp\left(\delta\frac{\pi}{\sqrt{3}}\right), \quad (7)$$

in which the parameter δ is equal to Equation 6.

Equations 1 to 4 have been solved numerically, and, for all models, δ has been tabulated for values of P_c ranging between .3333 and .99 (Frijters et al., 1979).

Note that all four equations given above contain the same parameter, δ, which represents the sensory distance between the two particular stimuli S_s and S_j. Theoretically, the value of δ must be invariant with the procedure by which it is measured, by the triangular method or by the 3-AFC procedure, and it should make no difference whether the normal or the logistic model is used for this purpose. Therefore, the main aim of the present experiment was to test the hypothesis of invariance of δ with these two procedures. This was made possible by using an experimentally obtained proportion of correct responses as the estimate for the probability of a correct response.

The second aim of the experiment was to investigate a positional response bias against the middle stimulus of a triangle, reported in a previous triangle-method investigation (Frijters, 1977).

EXPERIMENT

Method

Subjects. The subjects were 18 employees of the Spelderholt Institute for Poultry Research who were divided randomly into two groups of 9 subjects each.

Stimuli. Seven concentrations of linalylacetate diluted in diethylphtalate were used as odorants (Table 1, col. 2). These concentrations were identical to those used by Frijters (1977). Both chemicals were of high odor purity (I.F.F. brand). To prepare the stimuli, the top (~30 mm) of an odor paper strip (145 × 7 mm) was dipped into one of the dilutions and then exposed to air (~20°C). Samples were assessed at least 10 min and at most 40 min after dipping. The middle concentration of the range was used as the standard stimulus, S_s, but was not so labeled for the subjects.

Procedure. Triangles were composed under the following factorial combinations: (1) Types of triangles—All six sequence types, $S_sS_sS_j$, $S_sS_jS_s$, $S_jS_sS_s$, $S_jS_jS_s$, $S_jS_sS_j$, and $S_sS_jS_j$, were used with equal frequencies. (2) Concentration level—The levels denoted as j = 1 . . . 6 were used. For their molarities, see Table 1, columns 1 and 2. (3) Type of instruction—On three separate occasions, both groups received each of the following instructions: Type O—Which is the odd stimulus? (Thurstone-Ura model). Type S—Which is the strongest stimulus? (3-AFC SDT model). Type W—Which is the weakest stimulus? (3-AFC SDT model). This factorial design resulted in 1,944 presentations (6 triangle types × 6 concentration levels × 3 instruction levels × 2 groups × 9 subjects). Additionally, 324 triangles, composed of 3 standard stimuli ($S_sS_sS_s$) were presented, 6 per factorial combination of instruction type × subjects (3 × 18).

Under each of the three separate instructions, 42 triangles were presented in random order to each of the subjects. These were 36 triangles resulting from triangle type × concentration level and 6 triangles of the type $S_sS_sS_s$. This series of 42 was split so that, in one session, 21 triangles were presented to a subject.

For both groups, the instruction type O preceded the other two types, S and W. Subsequently, Group 1 was subjected to the order S-W and Group 2 to the order W-S.

The stimuli of a triangle were presented in a line, with distances of 150 mm between the strips. The subjects were instructed to smell the stimuli from the left to the right. Reuse of a stimulus after its termination was not allowed. Intratriangle intervals and duration and manner of sniffing were not controlled. Intertriangle intervals were constant at 1 min.

Results

From the data obtained under Instruction Type O, the proportion of correct responses was calculated for each concentration level and for Groups 1 and 2 separately (Table 1, cols. 4 and 5). These proportions were converted into corresponding values of δ (Table 1, cols. 8 and 9 for the normal and cols. 12 and 13 for the logistic Thurstone-Ura model), using tables of δ (Frijters et al., 1980).

The raw data from Groups 1 and 2 obtained under Instruction Type S were combined, and the same was done for the Instruction Type W data. From the two data sets thus obtained, the proportions of correct responses were calculated for each concentration level (Table 1, cols. 6 and 7). In these calculations, only half of the data were taken into account; that is, from the data obtained under Instruction Type S, only the responses from the triplets $S_sS_s'S_j$, $S_sS_jS_s'$, and $S_jS_sS_s'$ for j = 4, 5, 6 and those from $S_jS_j'S_s$, $S_jS_sS_j'$, and $S_sS_jS_j'$ for j = 1, 2, 3 were used. Because of symmetry under Instruction Type W, only the responses from the triplets $S_sS_s'S_j$, $S_sS_jS_s'$, and $S_jS_sS_s'$ for j = 1, 2, 3 and from $S_jS_j'S_s$, $S_jS_sS_j'$, and $S_sS_jS_j'$ for j = 4, 5, 6 were used for calculation of the proportions of correct responses. [Only half of the data were taken into consideration because disregarded responses cannot be assessed as correct or incorrect: Which response is correct or incorrect in a triplet, as for example $S_1S_1S_s$, presented under the instruction to select the weakest stimulus? It should be stressed that all six triplets were presented under Instruction Types S and W to ensure that the experimental designs of these conditions were *exactly* identical to that used in combination with the (triangular method) Instruction Type O design. Not taking this precaution would introduce an unwanted difference between experimental conditions.]

The values of δ for the normal and logistic 3-AFC model corresponding to the calculated proportions were obtained from the tables of δ (Frijters et al., 1980). These values are given in Table 1, in columns 10 and 11 and in columns 14 and 15, respectively. It should be noted that in this conversion the value of δ was taken to be negative if the corresponding value of log (Conc. S_j/Conc. S_s) < 0 and positive if log (Conc. S_j/Conc. S_s) > 0. Using the log of the ratio of the eight concentrations S_j and S_s (Table 1, col. 3) as the independent variable, eight functions of the type $\delta = B \log (\text{Conc. } S_j/\text{Conc. } S_s)$ were calculated. These are given in Table 2. The correlation coefficients between δ and B log (Conc. S_j/Conc. S_s) were calculated; none of the eight coefficients is lower than .98, so all are highly significant [r(.005) = .92, one-tailed].

Table 1
Molarities for Six Concentration Levels, Proportions of Correct Responses Obtained Under the Three Instruction Types, and Their Corresponding δ Values for the Normal and Logistic Models

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	.127	−.4983	.537	.574	.796	.907	−1.66	−1.84	−1.63	−2.28	−1.54	−1.73	−1.56	−2.25
2	.202	−.2967	.500	.444	.667	.667	−1.47	−1.16	−1.12	−1.12	−1.37	−1.07	−1.05	−1.05
3	.282	−.1518	.352	.370	.426	.537	− .46	− .64	− .32	− .68	− .42	− .59	− .29	− .63
4	.555	.1422	.444	.426	.593	.500	1.16	1.05	.86	.56	1.07	.97	.81	.52
5	.741	.2678	.426	.389	574	.630	1.05	.80	.80	.99	.97	.73	.75	.93
6	1.177	.4687	.593	.556	.777	.722	1.94	1.75	1.55	1.32	1.82	1.64	1.48	1.25

Note—Column 1 = Concentration Level S_j; Column 2 = Concentration S_j (S_s = .40 Mol/l); Column 3 = log (Concentration S_j/Concentration S_s); Columns 4, 5, 6, and 7 = proportions of correct responses for Instruction Types $(O)_1$, $(O)_2$, (S), and (W), respectively (each proportion is based on 54 observations); Columns 8, 9, 10, and 11 = δ values for Normal Models $(O)_1$, $(O)_2$, (S), and (W), respectively; Columns 12, 13, 14, and 15 = δ values for Logistic Models $(O)_1$, $(O)_2$, (S), and (W), respectively.

Table 2
Psychophysical Functions for the Normal and Logistic Thurstone-Ura and 3-AFC SDT Models

	Normal Models			Logistic Models		
	Regression Equation	S_e	S_B	Regression Equation	S_e	S_B
			Thurstone-Ura Model			
Instruction Type (O): Group 1	$\delta = 4.01 \log(S_j/S_s)$	.34	.41	$\delta = 3.73 \log(S_j/S_s)$	.31	.38
Instruction Type (O): Group 2	$\delta = 3.79 \log(S_j/S_s)$	.25	.31	$\delta = 3.53 \log(S_j/S_s)$	.23	.28
			3-AFC SDT Model			
Instruction Type (S): Groups 1 + 2	$\delta = 3.36 \log(S_j/S_s)$	.20	.25	$\delta = 3.19 \log(S_j/S_s)$	.19	.24
Instruction Type (W): Groups 1 + 2	$\delta = 3.77 \log(S_j/S_s)$	.27	.33	$\delta = 3.63 \log(S_j/S_s)$	.29	.35

Note—The standard error of regression (S_e) and the standard error of the regression coefficient (S_B) are also given.

The slopes of the psychophysical functions (Table 2) from the logistic models all appear to be lower than those from the corresponding normal models obtained under identical experimental conditions. For example, for Group 1 under Instruction Type O these are 3.73 and 4.01 for the logistic and normal Thurstone-Ura models, respectively. Since each such pair of functions was derived from the same set of proportions, a statistical test of the difference between the two regression coefficients is not appropriate due to lack of stochastic independence. However, with regard to the magnitudes of the standard errors of the regression coefficients, the differences between the related functions from the normal and logistic models seem to be of minor significance. For example, the above regression coefficient, from the functions for Group 1 and obtained under Instruction Type O are 4.01 and 3.73 and their respective 95% confidence intervals range between 2.96 and 5.05 and between 2.75 and 4.71.

For the remaining pairs of functions, a similar result can be shown; so it can be concluded that, under the same experimental conditions, the normal and logistic models give virtually identical results. The psychophysical functions of Groups 1 and 2 obtained under the instruction to select the odd stimulus are statistically not significantly different. For both the normal and logistic model, the test of the difference between the regression coefficients is insignificant [$t(10) = .428$, $p > .05$, and $t(10) = .424$, $p > .05$, respectively]. Since this condition was considered to be the control, it can be concluded that the combination of the group data obtained under the two other instruction types was justified.

There is also no difference between the psychophysical functions obtained with the 3-AFC procedure under the instruction to select the weakest stimulus and under the instruction to select the strongest. This can be concluded from the statistical tests of the difference between the slopes from the two normal model functions and from the two logistic model functions, which gave values of $t = .990$ ($df = 10$, $p > .05$) and $t = 1.037$ ($df = 10$, $p > .05$), respectively.

Finally, the most important conclusion to be drawn is that the psychophysical functions obtained under the Thurstone-Ura and 3-AFC SDT models show no significant difference. Testing was done on the mean regression coefficients as follows. The mean slope of the Group 1 and Group 2 functions obtained under Instruction Type O are 3.90 and 3.63 for the normal and logistic Thurstone-Ura models, respectively. The corresponding regression coefficients obtained by pooling are .26 and .23, respectively. The means of the slopes of the Instruction Type S and Instruction Type W functions are 3.56 and 3.41 for the normal and logistic 3-AFC SDT models, respectively, with corresponding standard errors of the regression coefficients of .21 and .21. The t tests of the respective differences between 3.90 and 3.56 and between 3.63 and 3.41 are insignificant [$t(22) = 1.039$, $p > .05$, and $t(22) = .697$, $p > .05$, for the normal and logistic models, respectively].

Results with respect to the spatial positions of the stimuli selected from the sets $S_sS_sS_s$ are given in Table 3. The chi-square test showed that each of the three distributions was significantly different from a random distribution. Table 3 shows that, under the instruction to select the odd stimulus, the middle stimulus was the least chosen. Under the instruction to select the weakest and under the instruction to select the strongest, it is the last and the first stimulus, respectively, that was most often chosen.

DISCUSSION

The present findings confirm the empirical inter-

Table 3
Positions of the Stimuli Selected from the Sets S_{s_1} S_{s_2} S_{s_3}

	F	M	L	N	χ^2
Which is the odd one? Group 1	24	12	18	54	
Which is the odd one? Group 2	23	14	17	54	
Which is the odd one?*	47	26	35	108	6.17
Which is the weakest?	23	37	48	108	8.72
Which is the strongest?	46	40	22	108	8.67

*Note—F = first; M = middle; L = last. For all chi-square values given, $p < .05$. *Groups 1 and 2* changeability of the Thurstone-Ura and 3-AFC models; the close similarity of the psychophysical functions resulting from the respective models is striking. Among other factors, such as the range of the physical stimulus values, the fact that use has been made of a repeated measurement design must have contributed to the goodness of the present results.

It should be noted that the above conclusion is based upon the comparison of psychophysical functions and not on the comparison of single values of δ, the distributions of each of which varied over about the same range (see Table 1).

Inspection of Table 1 shows that, for the same concentration level, there are certainly differences between the individual estimates of δ obtained by various procedures. However, since the psychophysical functions are not significantly different, and, in addition, since these have similar magnitudes of standard regression errors, it can be concluded that these differences must be mainly the result of error variance.

The conclusion of invariance of δ with the triangular method under the Thurstone-Ura model and 3-AFC procedures is important for olfactory psychophysics, but it also has wider methodological significance: It is another piece of evidence for the method invariance of δ. The invariability of δ with the number of alternatives in forced-choice procedures has been demonstrated in previous experiments (Green & Birdsall, 1964; Swets, 1959). More recently, it was found that δ is also invariant in a variety of two-response discrimination paradigms (Creelman & Macmillan, 1979). The present experiment shows that this measure of sensory distance is also independent of the instruction given when the appropriate experimental design and adequate model is used.

The second conclusion was that the normal and logistic pair of Thurstone-Ura models and the two variants of the 3-AFC SDT model give identical results. This conclusion deserves some additional comment. In the analysis of the data, each particular proportion of correct responses was used to estimate the value of δ for the normal as well as the concomitant logistic model. Since the functions relating the probability of a correct response to δ are virtually identically shaped (Frijters, 1979), these two values of δ cannot be very different. Therefore, the psychophysical functions arising from a normal model and its concomitant logistic model must be more or less identical. However, it should be noted that, for the logistic models, these theoretical functions are slightly less steep up to the middle region (see Figure 2, Frijters, 1979). This accounts for the fact that the slopes of all the present functions are slightly lower than those obtained from the corresponding normal models (see Table 2).

An unexpected finding is that under the two different modes of instruction, using the appropriate 3-AFC procedure, very similar psychophysical functions were obtained. In order to explain these findings, certain facts should be considered first.

In a number of studies using chemical stimuli, the effect of triangle composition on the subject's performance has been investigated (Frijters, 1977; Hopkins, 1954). In these experiments, it was observed that a higher proportion of correct responses was obtained from triangles composed of two low and one higher concentration than from triangles containing two high and one lower concentration. This phenomenon was explained as being the consequence of sensory adaptation. Indeed, on a particular trial, a greater loss of sensitivity will occur with the first type of triangle than with the second type, all other things being equal. Since sensory adaptation is independent of the instruction given, it follows that the same effect should be observed in the 3-AFC SDT procedure. Thus, in the latter procedure, a higher proportion of correct responses should be found under the instruction to select the strongest stimulus (in which only triplets of two weak and one stronger stimulus are presented) than under the instruction to select the weakest stimulus (in which triplets consist of two strong plus one weaker stimulus). Since higher proportions correspond to greater absolute values of δ, a steeper psychophysical function will be the final result. The psychophysical functions resulting from the two different instruction types and the corresponding experimental designs under which the 3-AFC procedure was applied are almost identical. Since this is contrary to the "differential adaptation explanation" given for the triangular method results, another explanation must be found. Probably, the estimation of each particular δ value is too imprecise to allow the variance caused by differences in adaptation to be separated from the variance resulting from sampling errors.

The last conclusion from the present experiment concerns the positions of the stimulus selected from the sequential sets $S_SS_SS_S$ (denoted $S_{S_1}S_{S_2}S_{S_3}$ in the remainder of this paper).

Table 3 shows that the middle stimulus was less frequently selected as the odd one than was the first or the last stimulus. This was also found to be the case in a previous triangular method experiment (Frijters, 1977), in which the same procedure, the same odorant, and the same concentrations were used. From an inspection of the present 3-AFC task data, it becomes clear that this bias against the middle stimulus is the result of adaptation processes. Theoretically, the mean sensory response at each of the three stimuli for the stimulus set of the type $S_{S_1}S_{S_2}S_{S_3}$ is identical, that is, $\mu_{S_1} = \mu_{S_2} = \mu_{S_3}$. However, in each particular trial, because of the self-adapting properties of the odorant, a loss of sensitivity will occur as the subject proceeds through the sequence of the three stimuli (reuse of a stimulus was not allowed). For a given number of trials, the result is that $\mu_{S_1} > \mu_{S_2} > \mu_{S_3}$, so that, in a 3-AFC trial under the instruction to select the strongest stimulus, the probability that $x_{S_1} > x_{S_2} > x_{S_3}$ is greater than each of the two probabilities that $x_{S_2} > x_{S_1} > x_{S_3}$ and $x_{S_3} > x_{S_1} > x_{S_2}$. That means that the first stimulus (left) has the greatest probability ($p > .33$) of being selected as the strongest. Similarly, under the instruction to select the weakest stimulus, the third stimulus (right) has the greatest probability of being selected, because

$$\Pr(x_{S_3} < x_{S_2}, x_{S_1}) > \Pr(x_{S_2} < x_{S_3}, x_{S_1})$$

and

$$\Pr(x_{S_3} < x_{S_2}, x_{S_1}) > \Pr(x_{S_1} < x_{S_2}, x_{S_3}).$$

Table 3 shows that, under the instruction to select the strongest stimulus, the respective percentages of choice of the first, second, and third stimuli are 42.6%, 37.0%, and 20.4%, respectively. Under the instruction to select the weakest stimulus, these percentages are 21.3%, 34.3%, and 44.4%, respectively. These patterns agree with the self-adaptation explanation given.

Similarly, under the instruction to select the odd stimulus, the middle stimulus has the smallest probability of being selected because it can be shown that:

$$\Pr(|x_{S_1} - x_{S_3}| < |x_{S_2} - x_{S_3}|, |x_{S_1} - x_{S_2}|)$$
$$< \Pr(|x_{S_2} - x_{S_3}| < |x_{S_1} - x_{S_2}|, |x_{S_1} - x_{S_3}|)$$

and

$$\Pr(|x_{S_1} - x_{S_3}| < |x_{S_2} - x_{S_3}|, |x_{S_1} - x_{S_2}|)$$
$$< \Pr(|x_{S_1} - x_{S_2}| < |x_{S_2} - x_{S_3}|, |x_{S_1} - x_{S_3}|).$$

Since the percentages of choice in Table 3 are 45.5% 24.1%, and 32.4% for the first, second, and third stimuli, respectively, it can be concluded that the bias against the middle stimulus is entirely the result of self-adaptation within each triangle presented.

Finally, a few practical notes must be made.

In a number of established triangular method applications (e.g., Dravnieks & Prokop, 1975), in each trial two physically weak (or blank) stimuli plus one stronger one are presented under the instruction to select the odd stimulus. The present experiment justifies the recommendation that all six types of triangles be used, or the instruction should be modified so that the subject is requested to select the strongest stimulus. In the first case, the data can be handled by the Thurstone-Ura model; in the second case, a 3-AFC SDT model would be appropriate.

Also, in light of the present theories and results, the A.S.T.M. recommendation (A.S.T.M., Note 2, p. 26) to use, in the triangular method, only triangles composed of two physicochemically weak stimuli plus one stronger one, if possible, needs revision.

The last note refers to the traditional triangular method instruction itself: Which of the three is the odd one?

The Thurstone-Ura models are probabilistic models; that is, they consider sensory responses to be continuously distributed. This implies that the probability that two physicochemically identical stimuli are perceived as exactly identical is negligible, and therefore, strictly, the instruction given is not appropriate. Investigators who have had experience with the triangular method will confirm that subjects often report that all three stimuli of a triangle are perceived as different. Therefore, the present author suggests modification of the instruction for the triangular method into: Which two of the three stimuli are most similar?

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