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The effect of uncertainty on the detection of frequency modulation at low modulation rates

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Frequency modulation detection performance is compared in two conditions of stimulus variability. The data show that when the modulation rate is randomized within an experimental block, performance is somewhat poorer than when the modulation rate is fixed. The results are interpreted within the framework of a template matching model for modulation detection. They suggest that a template is acquired in part from experimental context and in part from a multistage template matching process.

The detection of frequency modulation (FM) at low modulation rates (modulation frequencies) has long been of interest in psychoacoustics. Elsewhere we suggested that a listener detects FM by comparing variations in the perceived stimulus pitch with a modulation template stored in memory. (Hartmann & Klein, 1980.) We will refer to that paper as A. Such a cross-correlation operation is known to be the optimum mechanism for a signal detection system (Schwartz, Bennett, & Stein, 1966).

In the present paper, we extend the study to FM detection by experiments in which the experimental context provides the listener with different degrees of uncertainty about the nature of the stimulus to be detected. The experimental results are of interest in themselves, but they have particular significance within the framework of a template matching model of the detection process. For the rest of the paper, we adopt that framework, without further apology.

In order to use a template in the detection process, a listener must first select or otherwise acquire a template for matching with the stimulus. Paper A left unspecified the means by which a template is acquired. Possibly, a template is acquired *only* from experimental context. The context is established by listening across many trials. On any given observation interval, the template used is simply selected at random from the context. Possibly, however, the template is acquired during a single observation interval. In such a case, template acquisition would be a multistage process in which cues obtained during an initial observation would restrict the range of templates to be used. Although information obtained from experimental context may aid this process, the multistage process is capable of acquiring templates that are more effective than templates selected at random from the context.

The question of template acquisition suggests experiments in which the listener must be, to a greater or lesser extent, uncertain about the nature of the stimuli. A large amount of uncertainty may prevent the efficient acquisition of a template and result in degraded detection performance. Zwicker (1962) performed FM detection experiments in which the carrier was a narrow band of noise. He found that modulation detection threshold, that is, the smallest detectable frequency excursion, was elevated by a factor of about 6 compared with threshold for a sinewave carrier. Presumably random fluctuations in the stimulus with the noise carrier prevent the efficient acquisition of a template. Zwicker's results, however, do not indicate the nature of the template acquisition process. Template acquisition from experience with experimental context and template acquisition during a single observation interval would both be disrupted by the randomness caused by the noiseband carrier.

To gain insight into the process of template acquisition, we need to establish uncertainty in a way that will distinguish between the two acquisition processes proposed. There must be stimulus variability among different observation intervals of an experiment, but there must be no stimulus variability within a single observation interval. Furthermore, the stimulus must vary in some property that is a significant template property. We believe that the most significant template property is the modulation rate. Other properties, initial modulation phase and modulation waveform, are of secondary importance.

We performed experiments with two conditions of subject uncertainty. In one condition (FIXED), the modulation rate and initial phase angle were always the same during an experimental run. The subject was told to expect this condition. Therefore, he could quickly choose a template for the stimulus from the experimental context. In the other condition (RANDOM), the modulation rate varied in a random way from trial to trial. We reasoned that if template acquisition is mediated largely by experimental context, then performance should be significantly poorer in the RANDOM condition than in the FIXED condition. By contrast, performance should not be sig-

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nificantly different in the two conditions if multistage template acquisition occurs during the course of a single observation.

There is an effect which tends to oppose the above reasoning, namely the FM adaptation effect discovered by Kay and Matthews (1972). These workers found that FM threshold is raised by prior exposure to an adapting FM with similar rate. Adaptation was observed for an adapting modulation rate within an octave of the probe modulation rate. However, the adaptors used by Kay and Matthews had very wide frequency excursions. There is no record of adaptation caused by FM with frequency excursions near the detection level. It was reasonable, therefore, to suppose that adaptation effects would not be a factor in our experiment.

METHOD

Two subjects, the authors, participated in a 2IFC method-of-constant-stimuli task. A trial consisted of a sequence of two tones, each lasting 1 sec and separated by 250 msec. One tone was a sine-wave carrier tone frequency-modulated by a sine waveform beginning and ending at a positive-going zero crossing. The other tone had a constant frequency equal to the center frequency of the FM tone. Both tones were presented at 75 dB SPL. The center frequency of the FM tone varied randomly on each trial within a 10-Hz region centered at 800 Hz. The order of the two tones within a trial was random. The subjects chose the interval containing the frequency-modulated tone. There was no time limit for making the choice. Each subsequent trial began 1 sec after the subjects had made a response. No feedback was given.

Three-point psychometric functions were generated by finding the percent correct at each of three FM widths (±Δf) for various modulation rates (f_m). On half of the trials (FIXED condition), the modulation rate used during a single experimental run remained constant while the modulation width varied. For the other half of the trials (RANDOM condition), both modulation width and modulation rate varied within an experimental run. Experimental runs for each condition were generally run in blocks to reinforce the nature of the particular condition. To minimize transient effects associated with the establishment of a context (Durlach & Braida, 1969), 10 or 12 practice trials were included at the beginning of each experimental run to demonstrate each rate-width combination for that run.

The experiment was performed in two contexts, characterized by the number of different modulation rates, N. For $N=4$, modulation rates of 2, 4, 6, and 10 Hz were used. Each experimental run included 120 trials. In the RANDOM condition there were 10 trials at each of the 12 width-rate combinations, presented in random order. In the FIXED condition there were 40 trials at each of the three values of the width. From eight RANDOM runs and two FIXED runs, we collected 80 decisions for each data point on the psychometric functions.

For $N=10$, modulation rates of 1, 2, 3, 4, 5, 6, 8, 10, 12, and 15 Hz were used. Each experimental run included 90 trials. Each RANDOM run included each of the 10 rates nine times in random order. In a FIXED run there were 30 trials at each of the three values of the width. From 30 RANDOM runs and 3 FIXED runs we collected 90 decisions for each data point on the psychometric functions.

Experiments in the two contexts were entirely separate. There was, in fact, a 16-month gap between runs with the $N=4$ context and runs with the $N=10$ context.

The three values of Δf, the modulation width, were the same for all values of f_m and for both values of N. The Δf values were different for the two subjects because of differences in absolute sensitivity. For Subject M, the three values were ±.94, ±1.56, and ±2.19 Hz. For Subject W, the values were ±.56, ±.94, and ±1.31 Hz.

Apparatus

The tones were generated by a Wavetek voltage-controlled oscillator, VCG116, with frequency modulation control voltages produced by a microcomputer. The frequency modulation waveform was a digitally computed 12-bit 256-sample sine waveform. The modulation width was modified by rescaling the sample values, and the modulation rate was manipulated by changing the sampling rate. The subjects heard the tones diotically through Beyer DT-48 headphones while seated in a soundproof room. The microcomputer controlled the experimental sequencing and collected the response data.

RESULTS

Psychometric functions for detection in FIXED and RANDOM conditions are shown for contexts of $N=4$ and $N=10$ modulation rates in Figures 1 and 2, respectively. There is evidence for superior performance in the FIXED condition. For $N=4$

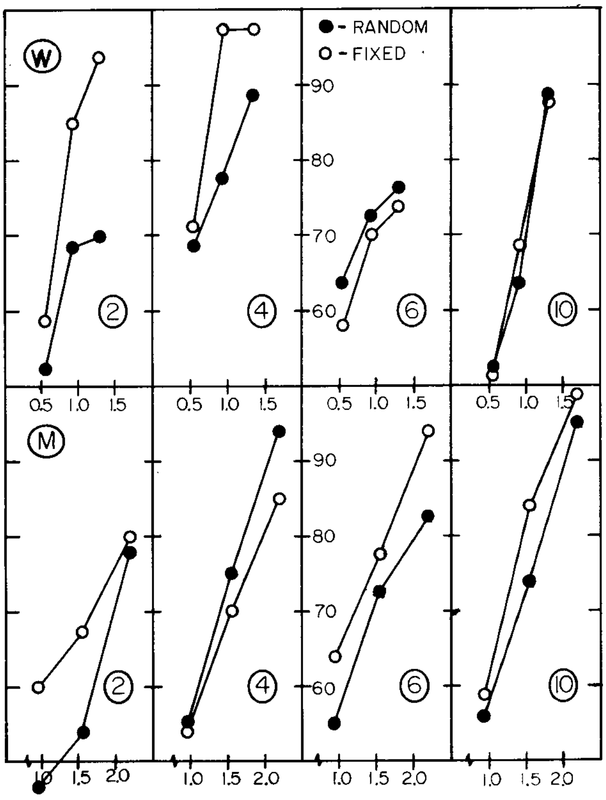

Figure 1. The panels show the three-point psychometric functions for subjects W and M for FM detection at 2, 4, 6, and 10 Hz modulation ratio (N = 4). The vertical axis shows percent correct; the horizontal axis shows the modulation frequency peak excursion in hertz. Each point is based upon 80 trials.

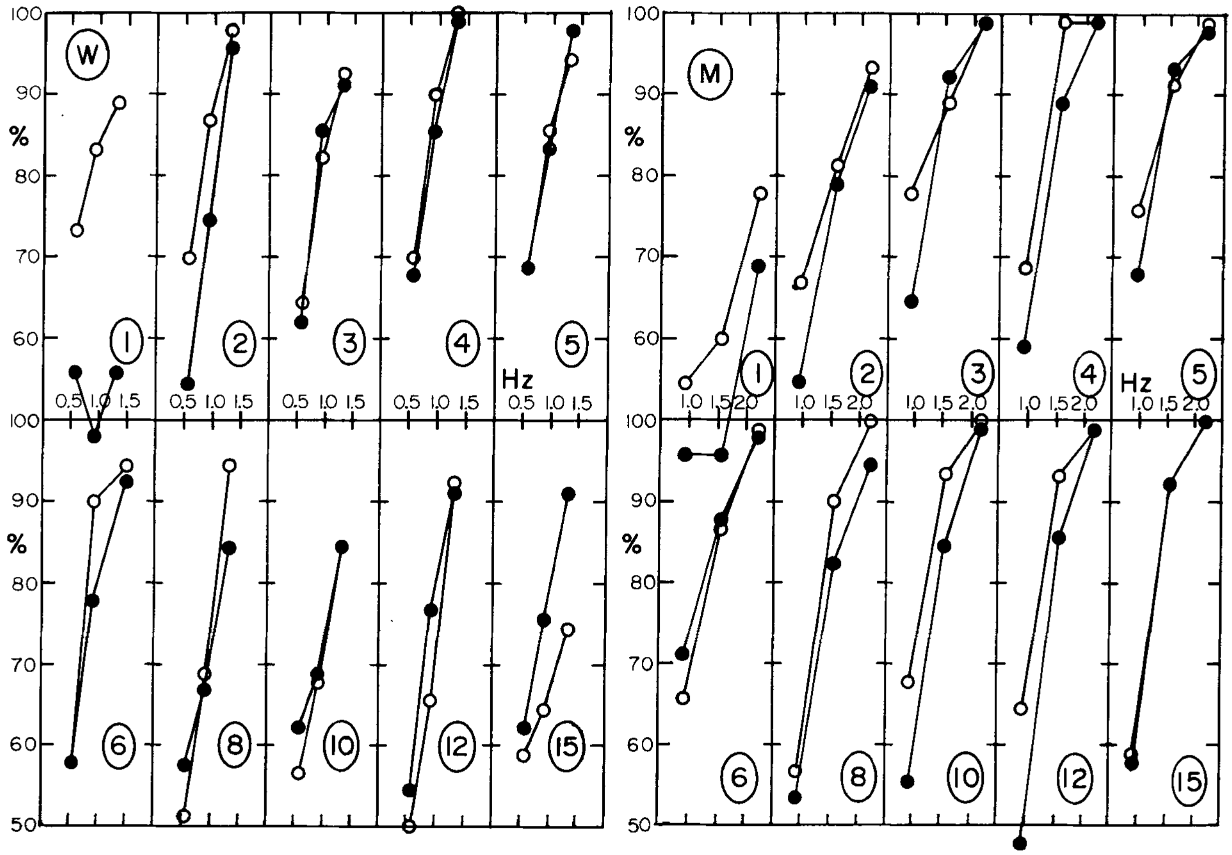

Figure 2. Psychometric functions as in Figure 1, except that there are 10 different modulation rates. Each point is based upon 90 trials.

there are 21 width-rate combinations in which performance differs in the two conditions. Of these, performance is better in the FIXED condition in 16 cases. For N = 10 performance differs for 43 width-rate combinations and is better in the FIXED condition for 31 of these. The results therefore suggest that template acquisition is, to some extent, dependent upon experimental conditions. However, the effect is not a dramatic one; it is much smaller than Zwicker's factor of 6. Apparently, a significant amount of template acquisition can be done during an observation interval.

Three other features of the data are presented, although they are not directly germane to the point of this paper. First, FM detectability is maximum at a modulation rate of 4 Hz, in agreement with the data of Zwicker (1952).

Second, the data for Subject M for the FIXED condition in the N = 10 context show a regular increase in performance as the modulation rate increases from 1 to 4 Hz. This result was predicted by paper A. Because our stimulus tones were of constant duration (1 sec), stimuli at low modulation rates have fewer cycles than stimuli at higher rates. For $f_m = 1$ Hz, there is only a single cycle. Paper A incorporated the effect of a small number of cycles by a decision variable that increases with increasing number of cycles up to a maximum of 4 cycles. The data for Subject W in the FIXED condition do not show as significant an effect at low rates. The effect of stimulus duration on *relative* performance in the two conditions will be discussed in the Comparison section.

Third, psychometric functions appear to have positive curvature near 50% correct. There are 19 psychometric functions with two points below 75% correct. Of these, 16 have positive curvature. This effect was noted by Jesteadt and Sims (1975) and is predicted by the theory of paper A.

MODEL

The experimental results imply that the template used in FM detection is, to some extent, established by experimental context, but that the context dependence may not be a strong one. This interpretation has no quantitive meaning unless we have some estimate of the degree to which performance in the FIXED and RANDOM conditions *ought* to depend upon those conditions. In this section, we present a model for FM detection performance based upon template matching. The model allows for the effect of experimental context on performance, but the model makes no provision for an active multistage process by which a template might be acquired during a single observation interval. The model will make a prediction for relative performance in the FIXED and RANDOM conditions that can be compared with the experimental data. If the model agrees exactly with the data, we will conclude that template acquisition is established entirely by context. If the model disagrees with the data, then one or both of two things are true: either the model is no good or a template is to some extent acquired during a single observation interval.

Suppose that there is some measure of cross correlation between a stimulus waveform with modulation rate f_m and a template with rate f_T. Let this function be $C_m(f_T)$. This function has its maximum value, 1.0, if the subject applies a template with a rate equal to the rate of the modulation. If the template rate is close to the modulation rate, the correlation should be close to 1.0. The cross correlation is a linear function of the modulating waveform. Therefore, the overlap between modulation with peak frequency excursion Δf and the template is given by $\Delta f C_m(f_T)$.

If the subject is uncertain about which template to apply, then he will apply templates with a certain probability density, $p(f_T)$. The detectability of modulation with given rate f_m is therefore proportional to

$$D_m = \Delta f \int df_T C_m(f_T) p(f_T). \tag{1}$$

The integrand contains two factors, the cross-correlation function and the template density. We will consider these two factors in turn.

There are many possible choices for the cross-correlation function, $C_m(f_T)$. Some considerations are as follows: (1) We prefer a form for C which does not depend upon initial phase angle. Although initial phase angle is fixed in our experimental context, template matching may occur many times during a 1-sec stimulus. For most of these matches, initial phase information is lost. Therefore, our cross-correlation function will be randomized over initial phase angles. (2) It is far from clear whether C should be nonnegative. Quite possibly a "wrong" template can actually make a negative contribution to the detectability integral in Equation 1. However, general forms for C which can be negative tend to vanish when one averages over relative phase angles between m and T, as suggested by (1) above.

A form for C which we find attractive is that of the correlated-differencing model of paper A. In this model, we supposed that the listener takes the difference between pitches of the stimulus, at times separated by half the period of the template. The absolute value of this difference serves as a decision variable in the detection operation. In paper A, this variable was called $|z|$. We found statistical distributions for $|z|$ under conditions of modulation-plus-internal-noise and internal-noise-alone to predict psychometric functions. When the modulation rate and the template rate can be different, as for the present paper, the complete statistical treatment becomes unwieldy. We therefore choose, for C, the mean value of the distribution of $|z|$ essentially by setting the perceived pitch equal to the instantaneous frequency of the modulation.

The model then reduces to a simple deterministic form. The subject is listening to a modulation waveform $\sin(2\pi f_m t)$. If he is using a template with period τ_T (rate $f_T = 1/\tau_T$), he takes the difference,

$$|\sin 2\pi f_m t - \sin 2\pi f_m (t + \tau_T/2)|. \tag{2}$$

Because the subject has no phase information, he is equally likely to take the difference at any time t of the modulation cycle, between $t = 0$ and $t = \tau_m$, where τ_m is the period of the modulation. Therefore, the cross-correlation function is

$$C_m(f_T) = \pi/4\tau_m \int_0^{\tau_m} dt \, |\sin 2\pi f_m t - \sin 2\pi f_m (t + \tau_T/2)|. \tag{3}$$

The prefactor normalizes C to unity for $f_m = f_T$. The only way that the template enters the cross-correlation function in this model is through the time delay between the two samples in the differencing operation.

The template density $p(f_T)$ describes the subjective uncertainty about the stimulus, involving both stimulus variability and imperfect memory. We use subscripts F and R for the two stimulus conditions. For the FIXED rate condition, we assume that there is no uncertainty at all; $p_F(f_T)$ is zero except where $f_T = f_m$. For the RANDOM condition, our density, p_R, includes all the assumptions of the *context coding* model of Durlach and Braida (1969). The allowable range of density is limited to the range of modulation rates in the stimulus context. The template density is supposed to be unaffected by order of presentation or decay of memory between experimental runs.

The template density p_R may be discrete or continuous. If the context has only a few well-separated rates, the template density may be discrete, with nonzero density only at those few rates. If the context has many rates within a range, the template density may be finite for all rates within the range. In the case of extreme uncertainty, the template density is constant within the range. The subject is equally likely to apply a template with a rate of 4.5 Hz and a template with a rate of 4.0 Hz, despite the fact that the stimulus sets include only integer value rates.

Note that the operational nature of p_R is left ambiguous. It may represent a density for template scanning in a single observation, or it may be a density for single template matching across all observations. We are unable to distinguish between these two interpretations. What is important about p_R, however, is that it is not a function of the stimulus rate f_m. That is, it does not include any dependence upon the current stimulus. Dependence of p_R on the current stimulus is expected only in models of a multistage process.

COMPARISON OF EXPERIMENT AND MODEL

Experimental

In order to compare the experimental data with the template model calculations, we express the data in terms of a performance ratio. The performance ratio measures performance in the RANDOM (R) condition relation to that in the FIXED (F) condition. We obtain the performance ratio from the values of the frequency excursion at the 75% correct points on the psychometric functions in Figures 1 and 2. The performance ratio, Δf(F)/Δf(R), averaged over the two subjects, is shown by open circles in Figures 3 and 4.

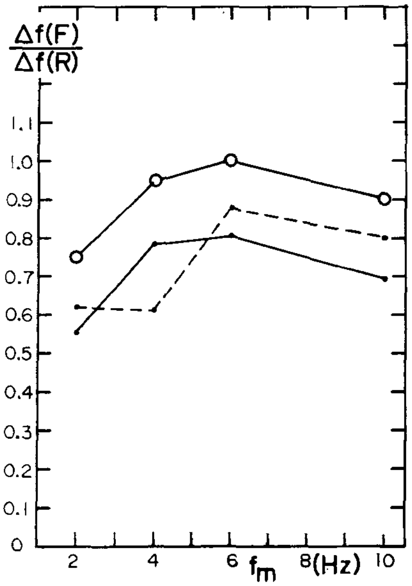

Figure 3. Circles show experimental values of the ratio of thresholds for FIXED and RANDOM conditions as determined from the 75% correct points of the psychometric functions for N = 4. Straight lines connect the circles to aid the eye. Dots connected by dashed lines show model calculations for a discrete template density. Dots connected by solid lines show model calculations for a continuous template density.

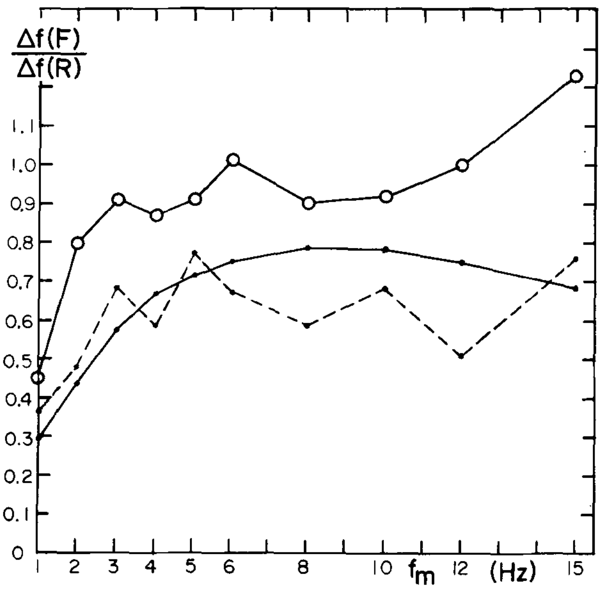

Figure 4. As in Figure 3 for N = 10.

Most of the performance ratios lie below the value 1.0, indicating that detection performance is less good in the RANDOM condition than in the FIXED condition. For a rate of 1 Hz, the performance ratio is unusually small. However, most points are not greatly below 1.0, indicating that the difference in performance is not a large one.

One might initially expect that the performance ratio would be lower for context N = 10 than for context N = 4, because there is more uncertainty about the RANDOM stimulus for N = 10. However, we are unable to observe any significant experimental difference in performance ratio for corresponding rates in the N = 10 and N = 4 contexts.

We are completely unable to account for experimental values of the performance ratio that are greater than 1.0. There are two such points in the average data. If they are not due to experimental error, then they must reflect some form of adaptation process in the FIXED condition, a mechanism that is outside the scope of this paper.[1]

Theoretical

To apply the model to the experiment, we consider the situation when the modulation is equally detectable in both FIXED and RANDOM conditions. Using subscripts F and R for FIXED and RANDOM conditions, we have

$$D_{mF} = D_{mR}, \qquad (4)$$

or, from Equation 1,

$$\Delta f(F) \int df_T C_m(f_T) p_F(f_T) = \Delta f(R) \int df_T C_m(f_T) p_R(f_T). \qquad (5)$$

Quantities $\Delta f(F)$ and $\Delta f(R)$ are the peak frequency excursions for which detection performances in the two conditions are equal. The FIXED and RANDOM contexts enter the integrals only through the template densities. We assume that $p_F(f_T)$ is zero unless $f_T = f_m$. Then, because $C_m(f_m) = 1$, the left-hand side of Equation 5 is simply $\Delta f(F)$. Therefore, we find

$$\frac{\Delta f(F)}{\Delta f(R)} = \int df_T C_m(f_T) p_R(f_T). \qquad (6)$$

The left-hand side of Equation 6 is the performance ratio. In theory, this ratio should be independent of actual performance, that is, should have the same value for 50% correct or for 100% correct. In fact, we know that the correlated differencing model fails when performance becomes significantly greater than 75% correct, because the assumption that the template is not phase locked with the stimulus becomes invalid. Therefore, we compare the performance ratio from Equation 6 with the experimental performance ratio at 75% correct.

To compute the right-hand side of Equation 6, we use $C_m(f_T)$ from Equation 3 and try two forms of the template density, $p_R(f_T)$, discrete and continuous. In the discrete condition, density p_R has strength equal to 1/N at each of the N values (N = 4 or 10) of the modulation rate of the context, and is otherwise zero. In the continuous form, p_R is a rectangular density between the lowest and highest rates of the context. Model predictions are plotted on Figures 3 and 4. The performance ratio calculated with the discrete model is not a smooth curve; it reflects certain symmetries associated with the integer values of the frequencies, which make a number of entries in the stimulus-template matrix equal to 1.0 or 0. The two-model curves are intertwined and it may be difficult to distinguish between their predictions.

There a number of points for comparison between model and experiment:

(1) The model calculations predict a peak in the performance ratio for modulation rates in the center of the range. This occurs because, on the average, the templates of the range provide better fits to stimuli with intermediate rates than to stimuli with extreme rates of the range. The experimental values of the performance ratio show a peak for N = 4; experimental values for N = 10 might show a peak if it were not for the anomalous points at $f_m = 12$ and 15 Hz.

(2) The model curves for the performance ratio predict that overall performance in the N = 10 context should be comparable to that in the N = 4 context. The model peaks in the two contexts have the same value. (Peaks occur at rates in the center of the range which is higher for the N = 10 context.) This, somewhat surprising, result occurs because "wrong" templates, can, nonetheless, lead to rather large values of our cross-correlation function. This model prediction appears to be in agreement with the experiment. In fact, the experimental performance ratios for N = 10 and N = 4 are comparable.

(3) As f_m decreases from 3 to 1 Hz in the N = 10 context, the experimental performance ratio decreases more rapidly than does the predicted ratio. Our explanation for this result involves the template density. The predicted performance ratios were calculated from template densities that were uniform. The predicted ratio curve could be brought into better agreement with the shape of the experimental curve by using, instead, a nonuniform density, in which the listener is less likely to apply templates at the extreme low rates. Such a nonuniform density seems plausible; template matching for low-rate templates is more time consuming than matching for high-rate templates because the subject has to wait longer to take the correlated difference. It seems quite probable that the nonuniformity of template density depends upon stimulus duration. If the stimulus were very long, there would be no disadvantage to lengthy templates. One might expect performance in both FIXED and RANDOM contexts to improve, and, more to the point, the performance ratio of these conditions should not decrease so rapidly as the modulation rate becomes small.

(4) Most obviously, the model considerably underestimates the performance ratio for all values of f_m. No change in the random template density can entirely correct for this discrepancy. Changing the template density can raise the predicted performance ratio for some rates, but only at the expense of decreasing the performance ratio at other rates. Because our model calculation assumes that templates are acquired only through experimental context, we regard the discrepancy between model predictions and experimental data as evidence that a template may, to some extent, be acquired on a single observation interval.

DISCUSSION

Experimentally, we found that increased uncertainty, resulting from RANDOM as opposed to FIXED conditions, had only a small effect upon detection. This result is similar to the effects of uncertainty on the detection of sine tones. Green (1961) found a surprisingly small dependence upon uncertainty in sine-tone frequency, and Egan et al. (1961) found a similarly small dependence upon presentation time. Green suggested that the effect of stimulus uncertainty is small because subjects are always uncertain about the nature of the stimulus, even in minimal-uncertainty conditions.

We have suggested that the relatively small effect of stimulus uncertainty in our experiments, especially in comparison with our model calculations, indicates that subjects are able to acquire a template during a single interval. The alternative possibility that our data can be fitted by adding a plausible amount of uncertainty in the FIXED conditions needs to be checked. We therefore relaxed the assumption that the template density is infinitely sharp in the FIXED condition. Instead, we introduced rectangular densities of various widths centered about f_m. With a rectangular density having a full width of 6 Hz, truncated as needed to eliminate templates with negative rate, the model eliminates about 75% of the discrepancy between calculations and the data for $N = 10$ in the region between 2 and 8 Hz, while making little change for higher values of f_m. But even an uncertainty as large as 6 Hz does not account for all the discrepancy, and 6 Hz seems to us to be an implausibly large uncertainty for the FIXED condition. We conclude that one cannot completely account for our data by introducing model uncertainty in the FIXED condition.

The model predicts that uncertainty is most effective in *degrading* performance at extreme modulation rates of the range. This effect appears in the data for the $N = 4$ context and at the low f_m edge for the $N = 10$ context. This situation can be contrasted with the resolution edge effect in discrimination and absolute identification results, where performance is *best* at the edges of the range (cf. Berliner et al., 1977). This contrast could be predicted. According to our model, detection performance in the RANDOM condition is relatively poor for modulation rates near the edges of the range because there are relatively fewer templates available which correlate well with the modulation. If we now suppose that the same cross-correlation functions are used in discrimination and identification experiments for FM rate, then performance should be good if cross-correlation functions for different values of f_m are significantly different. Because the detection performance ratio is a measure of the cross-correlation, one might expect that discrimination and identification performance should follow the absolute value of the derivative of the detection curve, taken with respect to modulation rate. As can be seen in Figures 3 and 4, this quantity is largest near the edges of the range. Therefore, discrimination and identification ought to be best near the edges of the range.

The trouble with the above argument is that discrimination and identification experiments are normally done well above threshold. Application of the present model to hypothetical FM rate discrimination and identification experiments, which to our knowledge has not been done, is questionable because we believe that the correlated differencing model is valid only near detection threshold. Possibly a revised cross-correlation model incorporating phase locking between template and stimulus would make similar predictions for resolution edge effects.

Alternatively, the processes of discrimination and identification well above threshold might not be based upon the cross-correlation function at all but, rather, be based upon some internal representation of the rate itself. We note that our discrete and rectangular template densities correspond to extreme assumptions about the subject's memory for the modulation rate context. The discrete template density corresponds to perfect memory, without error, bias, or drift. The rectangular density corresponds to no memory at all for boundaries between FM rate categories, although there is perfect memory for the edges of the range. These two different assumptions would result in greatly different predictions for a model process of discrimination/identification based upon rate itself. By contrast, these two assumptions, within the correlated differencing model, lead to predictions for detection that are almost identical. In this case, discrimination/identification performance should be weakly coupled, at best, with detection performance.

For our detection experiments with two conditions of stimulus variability, we have concluded that the templates used in the correlated differencing model must be acquired, to some extent, on a single observation interval. The acquisition process must be an active multistage process in which the listener ultimately arrives at a template that is more effective in detecting the modulation than is a template picked at random from the context.

There are a variety of quite different candidates for the multistage process. Simplest is a process in which the subject applies a variety of templates, in sequence, with modulation rates of the range, and selects the one that provides the largest value of the decision variable, D_{mR}. Alternatively, the subject may select a template based upon the time interval between perceived pitch fluctuations, observed in a preliminary scan. As a subject listens to a tone, there are times when the pitch can be heard as unusually high or low. The time interval between two high points suggests a template period; the time between high and low points suggests the half-period. On occasion, the subject may sense a sequence of pitches in a pattern that resembles a portion of the modulating waveform, for example, an upper or lower turnaround. These too provide timing information which the subject could use to establish a trial template half-period. In these preliminary scans, the subject acquires not only modulation rate information but potential phase information as well. The correlated differencing model assumes that the subject uses the rate information but ignores the phase information, that is, that once the subject acquires a rate template, he searches, at any and all times, for a pitch pattern conforming to the appropriate rate.

We note that it is difficult to imagine a multistage single-interval template acquisition process in which the subject derives *no* benefit from familiarity with the context. Our model calculations correspond to a situation in which template acquisition is mediated *only* by context. This does not imply that the alternative, template acquisition on a single interval, is free of context effects. The shape of our model curves for performance ratio tended to correspond with the shape of the data. This does not necessarily imply that subjects ever resort to the random selection process of that (context only) model. It does imply that there are context effects in the template selection process. The substantial underestimate by the model of the performance ratio suggests that subjects acquire templates by a single-interval process which is more efficient than random selection from a context.

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NOTE

1. There is only one value of f_m where the performance ratio is significantly different, greater than 1, namely at $f_m = 15$ Hz. This modulation rate and also $f_m = 12$ Hz fall outside the regime one would normally consider low modulation rates. We included them in the study to expand the set of rates to 10, but we question whether the correlated differencing model can properly be applied to these high rates. It is not surprising to find anomalous performance for these rates.

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