Perception & Psychophysics
1983, 33 (2), 186-192

Notes and Comment

A model of perceived intersection of two converging line segments

LEON MITRANI and NAUM YAKIMOFF
Bulgarian Academy of Sciences, Sofia, Bulgaria

Several characteristic features of visual perception have been revealed in experiments on perceived orientation of line segments (Andrews, 1967; Blakemore, Carpenter, & Georgeson, 1970; Bouma & Andriessen, 1968, 1970; Lennie, 1971; Leushina & Pavlovskaia, 1976; Salomon, 1947) and on perceived intersection between a line segment and a long transversal line (Krantz & Weintraub, 1973; Weintraub & Krantz, 1971; Yakimoff, Mitrani, & Mateeff, 1977) or between two converging line segments (Weintraub & Virsu, 1971, 1972; Yakimoff, 1977). Many attempts to explain visual illusions evoked by different configurations of straight lines (the illusions of Müller-Lyer, Poggendorff, and Zöllner are prime examples) are grounded on the results of such experiments. These findings, however, are quite divergent. Several models based on the characteristics of and interactions between cortical receptive fields (detectors of orientation) have been developed (Andrews, 1967; Blakemore et al., 1970; Bouma & Andriessen, 1970; Carpenter & Blakemore, 1973; Lennie, 1971; Leushina & Pavlovskaia, 1976). Phenomenological regression models reasonably fitting the experimental data have also been proposed (Bouma & Andriessen, 1968; Weintraub & Virsu, 1972).

One of the methods used in studies on perceived orientation under different conditions consists of setting a dot on the apparent extension of a line segment. Extensive discussion of the line-dot method can be found in the papers of Bouma and Andriessen (1968), Emerson, Wenderoth, Curthoys, and Edmonds (1975), and Matin (1972). Despite some reservations, (Emerson et al., 1975), several findings support the idea that the dot-setting method provides valid information about perceived orientation. Dot alignment data on tilt illusion (Matin, 1974) are directionally consistent with data obtained with other techniques. The results from line-dot experiments (Bouma & Andriessen, 1968) are quite similar to the results obtained from direct estimation of line-segment orientation experiments (Leushina & Pavlovskaia, 1976). The idea that setting a dot to the imaginary extension of a line segment is determined by the perceived orientation of the segment is supported also by the fact that the mean values and the variances of the orientation estimates do not change with the distance between the segment and the dot (Bouma & Andriessen, 1968; Salomon, 1947).

We obtained similar results from a study on visual extrapolation of a line segment to its intersection with a straight transversal line (Yakimoff et al., 1977). We used 12 configurations of segment-line, varying the distance (2, 4, 8, and 16 cm) and the angle (30, 60, and 90 deg) between the line and the segment. Fifty subjects made one judgment of the intersection point position for the 12 configurations presented at about 40 cm from their eyes. The segments were 3 cm long, and the line was 28 cm. The position of the transversal line was fixed in the visual field. It was drawn at 83 deg with respect to the horizontal edge of the test field. The results were described by bundles of lines originating from the end of the segment that was closer to the transversal line. Each line in the bundle formed an angle $æ_i$ with the extension of the segment. It turned out that, for a fixed angle between the segment and the transversal line, the mean value $\overline{æ}$ of the angles $æ_i$ as well as their variances remained the same for the four distances used. The F ratios obtained from the analysis of variances were less than the corresponding critical values at a confidence level of $p = .05$. Furthermore, the distribution of the angles $æ_i$ could be considered normal, as the chi-square criterion provided no sufficient reasons for the rejection of this hypothesis at a confidence level of $p = .05$.

The present paper is a theoretical approach based on the results from line-segment extrapolation studies. A model of perceived orientation is proposed that suggests the possible principles underlying the position estimation of the intersection point between two converging line segments. Our considerations will be restricted to experiments and results obtained by extrapolation procedures, although they can be generalized to other cases and other experimental techniques.

Perceived Orientation of a Single Line Segment: A Definition

Let us consider a line segment (a,b) in the plane. We shall adopt the following definition: *The perceived orientation of the line segment (a,b) is represented by a bundle of straight lines (A) originating from the a end of the segment.* The relative density of the lines in bundle A (i.e., the number of lines per angle unit vs. the total number of lines) has a normal distribution (Figure 1). Each line of bundle A corresponds to one of the possible estimates for the orien-

The authors' mailing address is: Institute of Physiology, Bulgarian Academy of Sciences, P.O. Box 131, Sofia 1113, Bulgaria.

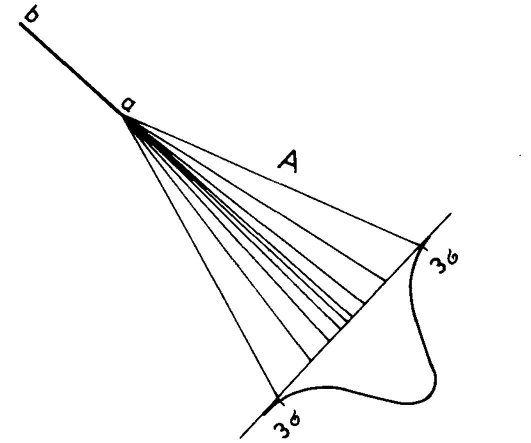

Figure 1. Schematic representation of the perceived orientation of a single line segment by means of a bundle of straight lines.

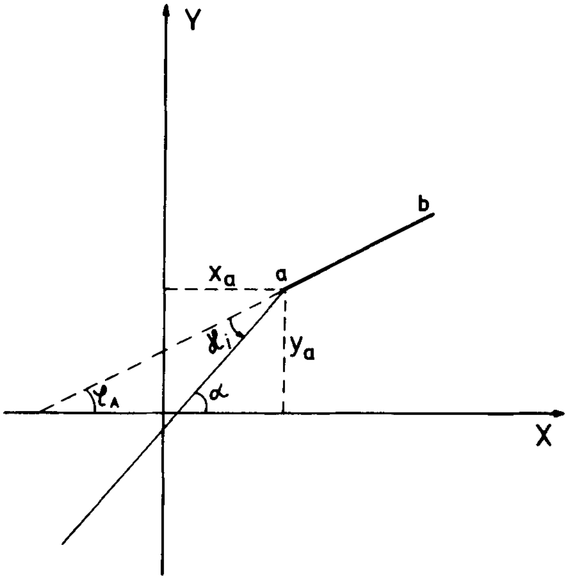

Figure 2. Description of the parameters in the equation of the bundle of straight lines (A) representing the perceived orientation of the line segment (a,b).

tation of the segment. The variance of the estimates determines the value of the angle within which the bundle lies.

In an arbitrarily chosen coordinate system [X,Y], the bundle of lines (A) is described by the following equation:

$$A\colon y - y_a = k_i(x - x_a),$$

where (x_a, y_a) are the coordinates of the a end of the segment, $k_i = tg\alpha_i$, $\alpha_i = \varphi_A + \ae_i$, φ_A is the angle determined by the segment (or by its extension) and the abscissa, and $\ae_i$ is the angle between the i^{th} line of the bundle and the extension of the segment (Figure 2). Angles $\ae_i$ are measured from the segment extension to the i^{th} line, the sign of $\ae_i$ depending on the position of the i^{th} line with respect to the extension of (a,b). The values of $\ae_i$ are normally distributed, and their variance can be used to determine the angle within which bundle A lies.

The definition adopted and the experimental data (Yakimoff et al., 1977) suggest that the estimated position of the point of intersection between a line segment and a transversal line is determined by the perceived orientation of the segment.

Estimating the Position of the Intersection Point of Two Converging Line Segments

Let us consider two line segments (a,a′) and (b,b′) in an arbitrarily chosen coordinate system [X,Y]. Let the extensions of both segments cross at point 0 (Figure 3). The orientations of the two segments are described by the bundles of lines:

$$A\colon y - y_a = k_i(x - x_a)$$

and

$$B\colon y - y_b = l_j(x - x_b),$$

where (x_a, y_a) and (x_b, y_b) are the coordinates of the segment ends that are closer to the intersection point. Coefficients k_i and l_j are of the type:

$$k_i = tg(\varphi_A + \ae_i^A)$$

and

$$l_j = tg(\varphi_B + \ae_j^B). \qquad (1)$$

Designations and angle measurement are as in Figure 2. The coordinates of the intersection point of

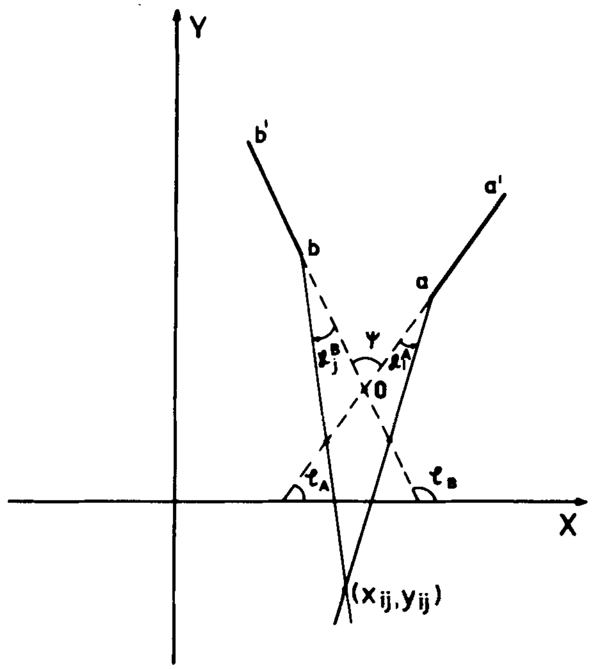

Figure 3. Description of the parameters in the equations of bundles A and B representing the perceived orientations of two converging segments (a,a′) and (b,b′).

the i^{th} line from bundle A and the j^{th} line from bundle B are given by the following expressions:

$$x_{ij} = \frac{(k_i x_a - y_a) - (l_j x_b - y_b)}{k_i - l_j}$$

and (2)

$$y_{ij} = \frac{(k_i x_a - y_a) - k_i(l_j x_b - y_b)}{k_i - l_j}.$$

The points (x_{ij},y_{ij}) are distributed within the area of overlap between bundles A and B. From Equations 1 and 2, it is clear that this area is defined by: (1) distances Oa and Ob, (2) angles φ_A and φ_b or angle ψ between the two segments (Figure 3), and (3) the parameters of the distributions of angles æ_i^A and æ_j^B.

Let us consider the distribution of the intersection points (x_{ij},y_{ij}) within the area of intersection between bundles A and B. Let both segments lie at equal distances Oa = Ob from their intersection point (Figure 4). Let the mean values $\overline{\text{æ}}^A = \overline{\text{æ}}^B = 0$ and the variances $\sigma_{\text{æ}^A} = \sigma_{\text{æ}^B}$. The symmetric unimodal distributions of the relative density of the lines forming bundles A and B determine the equal number of intersection points (x_{ij},y_{ij}) in subareas PVOT, VQNO, ONRM, and TOMS. While areas VQNO and TOMS are equal, areas PVOT and ONRM might be rather different. Consequently, the density (number per unit area) of the intersection points (x_{ij},y_{ij}) in the latter two areas might be quite unequal. These considerations led us to the formulation of a basic assumption about the estimation of a two-segment-intersection point: *The probability of obtaining an estimate about the position of the intersection point of two converging line segments is proportional to the density of the intersection points between the straight lines forming the bundles describing the perceived orientation of the two segments.* It has to be emphasized that for a given configuration of segments (a,a′) and (b,b′), the density distribution of the points (x_{ij},y_{ij}) does not depend on the number of lines that form bundles A and B.

When the density of points (x_{ij},y_{ij}) in area ONRM is greater than the density in area PVOT (i.e., when the surface of PVOT is larger than the surface of ONRM), a displacement of the estimates inside angle ψ formed by the two segments is to be expected. Such a displacement may be interpreted as an overestimation of the angle between the two segments. Areas PVOT and ONRM become equal for angles ψ of about 120 deg. Therefore, an overestimation of angles, in the above sense, might be expected for acute and obtuse (up to 120 deg) angles as well. Such an overestimation has been produced by Weintraub and Virsu (1971, 1972) and by us (unpublished data).

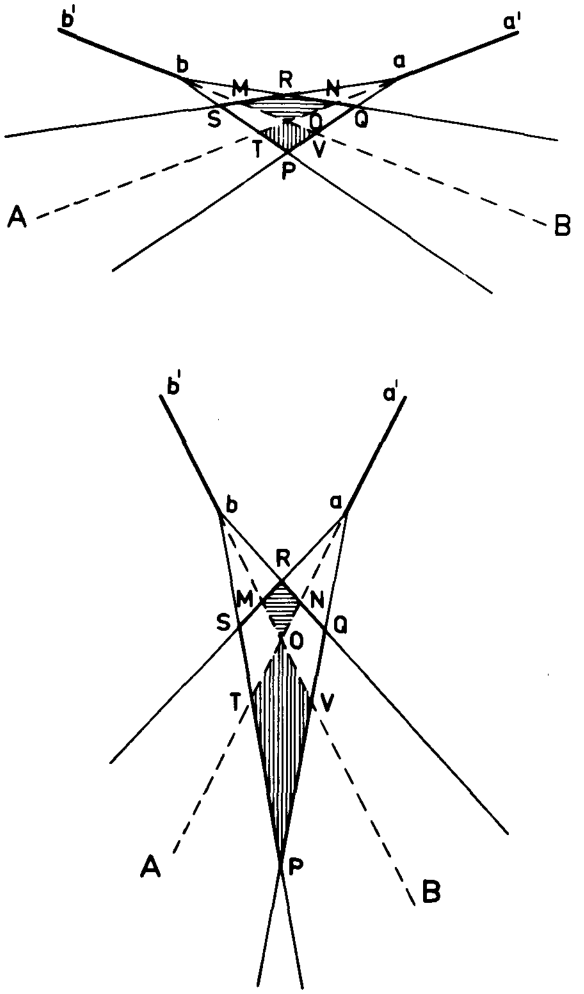

Figure 4. Two examples of the area of intersection between bundles A and B describing the perceived orientation of the segments.

Our basic assumption predicts a two-dimensional distribution of the estimates for the position of a two-segment-intersection point. This distribution is restricted to the area of overlap between bundles A and B. The form of this area will change considerably with changes in distances Oa and Ob and of angle ψ formed by the segments. Experiments on the estimation of single-line-segment orientation have revealed that the variances of the estimates depend on the position of the segment in the visual field. The smallest variances have been obtained for vertical and horizontal line segments (Andrews, 1967; Bouma & Andriessen, 1968; Leushina & Pavlovskaia, 1976). Differences in estimation variances will also alter the form of the overlapping area between bundles A and B. The three examples in Figure 5 illustrate these alterations for different configurations of line segments.

The examples in Figure 5 show that due to the alterations of area PQRS, much different distributions of the intersection points (x_{ij},y_{ij}) can be ex-

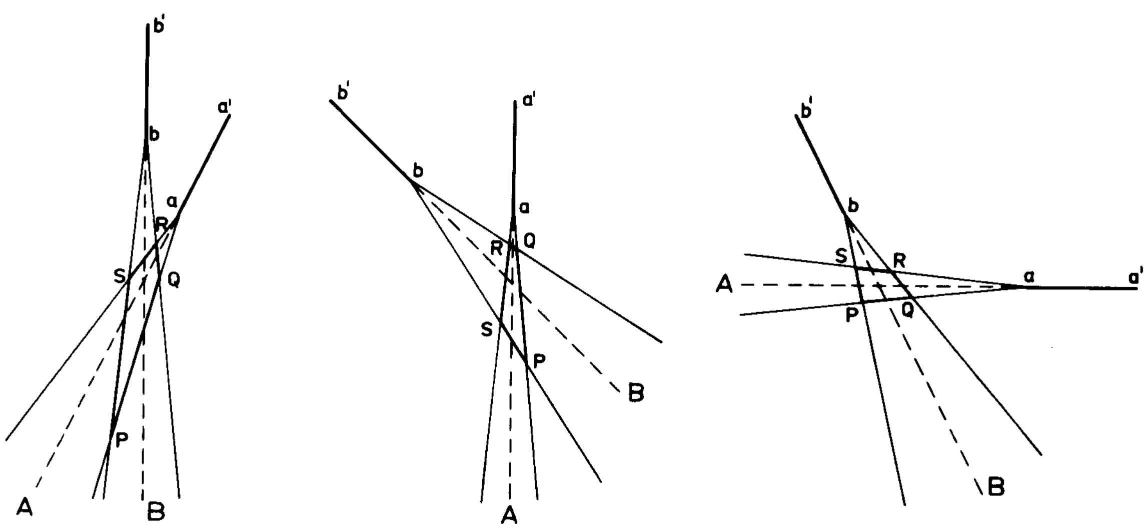

Figure 5. Alterations in the form of the intersection area between the bundles describing the perceived orientation of the segments in different configurations of two segments.

pected. The unimodal distribution of the relative density of the lines in bundles A and B determine the very low density of the points (x_{ij},y_{ij}) at the periphery of PQRS. Hence, the probability for obtaining an estimate in the periphery will also be very low. This probability can be expressed by the relative density Π of the points (x_{ij},y_{ij}) in PQRS. $\Pi = \Delta n/N$, where Δn is the number of dots per unit area and N is the whole number of points (x_{ij},y_{ij}); $N = i \cdot j$. Here we shall make an additional assumption: *The estimation of the position of a two-segment-intersection point is accompanied by rejection of part of the overlapping area between the two bundles describing the orientation of the segments in which the relative density of the points (x_{ij}, y_{ij}) is less than some* threshold *value.*

For a more clear presentation, we used a computer to find a numerical expression for the distribution of the relative density of the points (x_{ij},y_{ij}). Bundles A and B were represented by 100 straight lines, and the number of intersection points between these lines per unit area (1 mm²) was calculated for two examples.

Example 1. $\psi = 30$ deg, $Oa = 2Ob = 4$ cm, $\overline{\text{æ}}^A = \overline{\text{æ}}^B = 0$, $\sigma_{\text{æ}^A} = 1.6\sigma_{\text{æ}^B} = 3.7$ deg. Such values of variances $\sigma_{\text{æ}^A}$ and $\sigma_{\text{æ}^B}$ can be expected if, for example, the segment (a,a′) is vertical. The distributions of angles æ in this example were normal ones. They are illustrated in Figure 6. Figure 7 shows the results of the computations—the distribution of the relative density (Π) of the points (x_{ij},y_{ij}). Black squares of different area correspond to different values of Π. It can be seen that our suppositions lead to clear predictions about the outcome of a real experiment. After the rejection of the area with low relative density of the points (x_{ij},y_{ij}) (e.g., below a threshold value of $\Pi = .005$), the area of the expected estimates distribution becomes elliptiform. Figure 8 permits a comparison between the theoretical distribution of the relative density of the points (x_{ij},y_{ij}) in Example 1 and the results of a real experiment performed with 50 subjects with the same configuration of line segments (Yakimoff, 1977). Each segment in the experiment was 3 cm long, and the segment (a,a′) was vertical in the visual field.

Example 2. $\psi = 30$ deg, $Ob = 2Oa = 4$ cm, $\overline{\text{æ}}^A \neq \overline{\text{æ}}^B \neq 0$. The distributions of angles æ_i^A and æ_j^B in

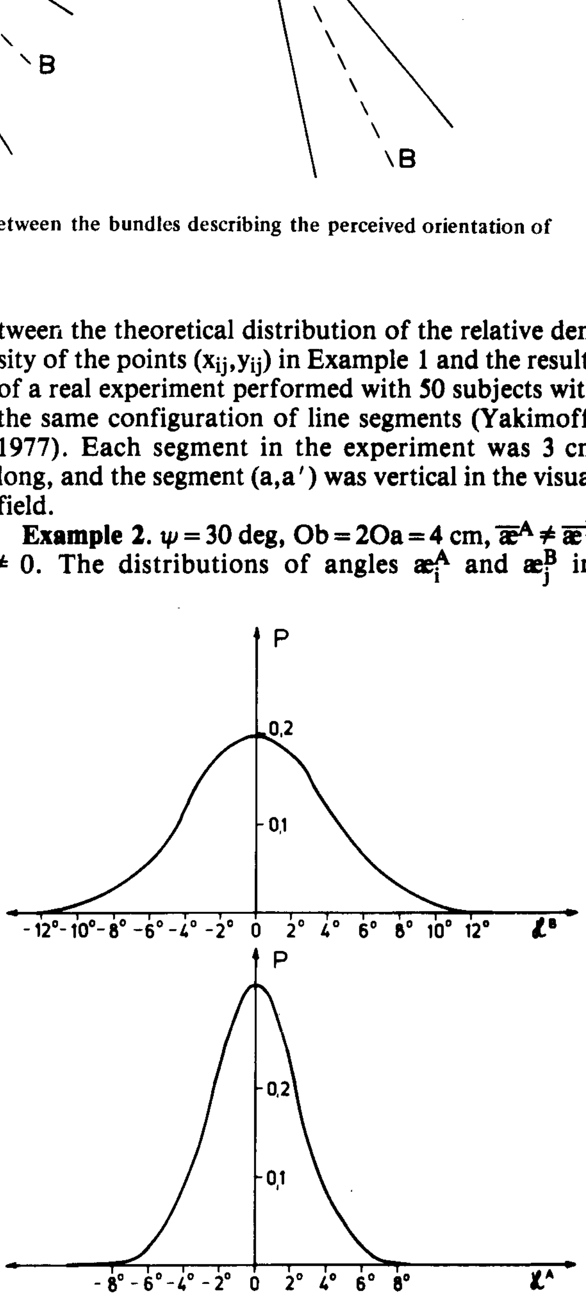

Figure 6. Example 1: Distribution of angles æ^A and æ^B in the two bundles describing the perceived orientation of the segments.

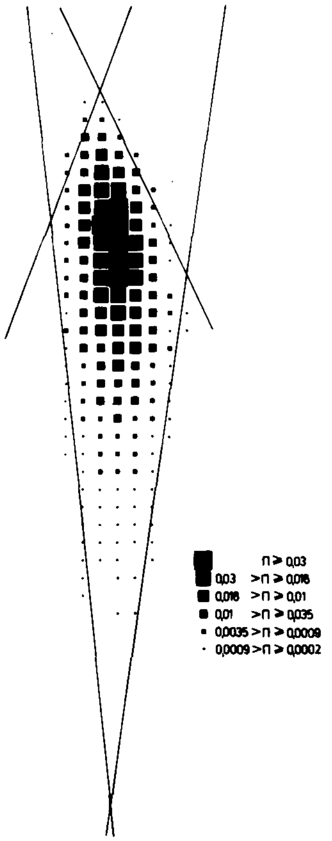

Figure 7. Example 1: Distribution of the relative density of the intersection points between the lines of bundles A and B within the area of overlap between the two bundles.

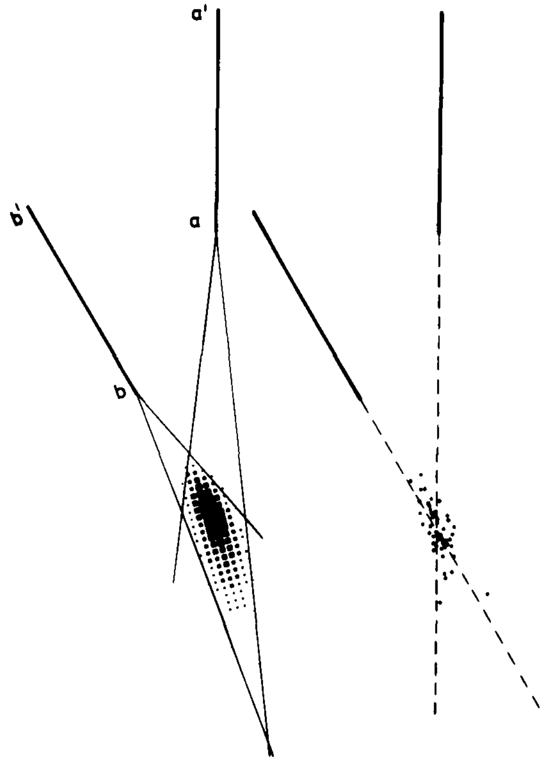

Figure 8. Example 1: Comparison of the theoretical distribution of the intersection points (Example 1, Figure 7) with the experimentally determined distribution of the estimates for the position of the intersection point for the same configuration of two segments.

this case (Figure 9) were unimodal but asymmetrical, and both modes were displaced from the "zero direction." The results of the computations (Figure 10) show once more that after the rejection of the areas with small values of Π, an elliptiform distribution of the estimates in a real experiment is to be expected. Figure 11 makes it possible to compare the theoretical and the experimental results (Yakimoff, 1977).

The elliptiform two-dimensional distribution of the estimates for the position of the intersection point between the extensions of two converging line segments can be considered experimentally confirmed. Figure 12 shows another three examples taken from our previous study (Yakimoff, 1977). The elliptiform character of the estimates distribution is clearly seen. Similar distributions were obtained in the experiments of Weintraub and Virsu (Note 1). These authors, however, considered only the mean estimated position of the vertex of two segments. They proposed a four-factor regression model describing *the tilt error* of each individual line segment. Weintraub and Virsu's (1972) model is a very good approximation of the experimental data they obtained with configurations of line segments lying at equal distances from their intersection point. That is why the model does not account for the possible influence of the dis-

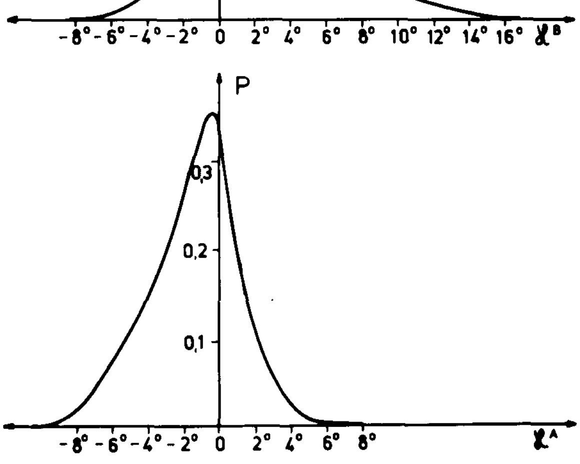

Figure 9. Example 2: The same as in Figure 6.

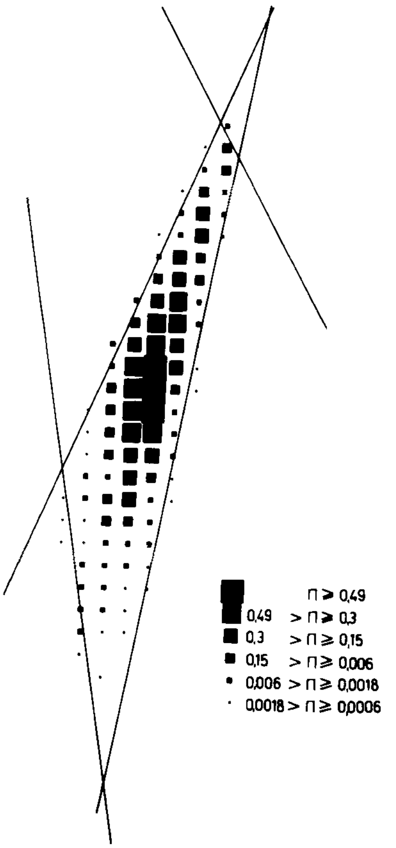

Figure 10. Example 2: The same as in Figure 7.

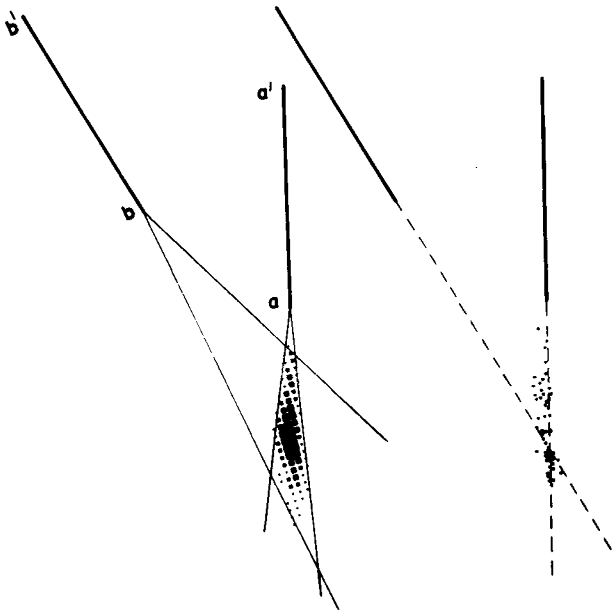

Figure 11. Example 2: The same as in Figure 8.

tance between the segments and the intersection point. The first term in the model accounts for the perceived orientation of a single-line segment described earlier by Bouma and Andriessen (1968). The remaining three terms are added to include the additional factors determining the tilt errors of each line segment in the vertex estimation experiment. With our model, these additional factors might find a natural explanation. The estimation of the position of the intersection point between the extensions of two converging line segments is based upon an estimation of the relative density of the intersection points between the straight lines representing the perceived orientation of the segments as well as upon the rejection of the areas where the relative density has less than a threshold value.

Conclusions

The starting point of our considerations about the perceived intersection of the extensions of two converging line segments is the definition of the perceived orientation of a single line segment. Given two converging segments, the bundles of straight lines representing their perceived orientation intersect and form the area containing the possible estimates of the intersection point position. Our basic assumption is

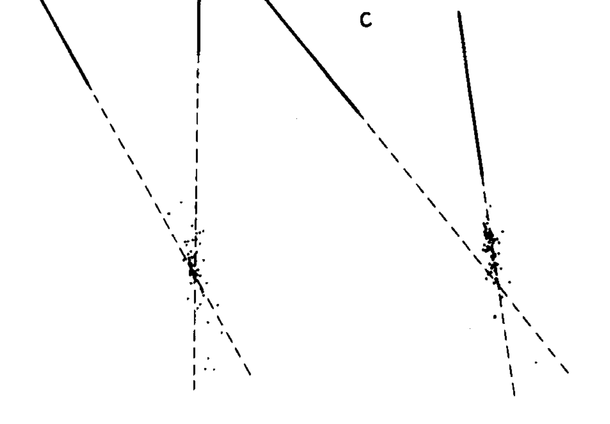

Figure 12. Results from an experiment on estimation of the position of the intersection point in different configurations of two converging line segments.

that the probability for obtaining a particular estimate is determined by the relative density of the intersection points between the lines of the bundles and by the existence of a fixed threshold for rejection of the areas with low intersection-point relative density.

The definition adopted and our basic assumption lead to the prediction of the results from a real experiment: an elliptiform two-dimensional distribution of the estimates that, depending on the parameters of the line-segment configuration, can be displaced as a whole with respect to the position of the real intersection point between the two segments. It is difficult to argue whether or not such a displacement of the estimates can be interpreted reliably as angle misperception.

The experimental data available are consistent with the predictions based on our considerations. They show that our ideas have a good chance of obtaining further confirmation.

REFERENCE NOTE

1. Weintraub, D. J. Personal communication, September 1980.

REFERENCES

ANDREWS, D. P. Perception of contour orientation in the central fovea. Part 1: Short lines. *Vision Research*, 1967, **7**, 975-997.

BLAKEMORE, C., CARPENTER, R. H. S., & GEORGESON, M. A. Lateral inhibition between orientation detectors in the human visual system. *Nature*, 1970, **228**, 37-39.

BOUMA, H., & ANDRIESSEN, J. J. Perceived orientation of isolated line segments. *Vision Research*, 1968, **8**, 493-507.

BOUMA, H., & ANDRIESSEN, J. J. Induced changes in the perceived orientation of line segments. *Vision Research*, 1970, **10**, 333-349.

CARPENTER, R. H. S., & BLAKEMORE, C. Interaction between orientations in human vision. *Experimental Brain Research*, 1973, **18**, 287-303.

EMERSON, P., WENDEROTH, P., CURTHOYS, I., & EDMONDS, I. Measuring perceived orientation. *Vision Research*, 1975, **15**, 1031-1035.

KRANTZ, D. D., & WEINTRAUB, D. J. Factors affecting perceived orientation of the Poggendorff transversal. *Perception & Psychophysics*, 1973, **14**, 511-517.

LENNIE, P. Distortions of perceived orientation. *Nature*, 1971, **233**, 155-156.

LEUSHINA, L. I., & PAVLOVSKAIA, M. B. [On the estimation of line orientation and its possible neurophysiological mechanisms]. *Human Physiology*, 1976, **2**, 685-692. (In Russian)

MATIN, E. Light adaptation and the dynamics of induced tilt. *Vision Research*, 1974, **14**, 255-265.

MATIN, L. Eye movements and perceived visual direction. In D. Jameson & L. M. Hurvich (Eds.), *Handbook of sensory physiology VII/4: Visual psychophysics*. Berlin: Springer, 1972.

SALOMON, A. D. Visual field factors in the perception of direction. *American Journal of Psychology*, 1947, **60**, 68-88.

WEINTRAUB, D. J., & KRAHTZ, D. H. The Poggendorff illusion: Amputations, rotations and other perturbations. *Perception & Psychophysics*, 1971, **10**, 257-264.

WEINTRAUB, D. J., & VIRSU, V. The misperception of angles: Estimating the vertex of converging line segments. *Perception & Psychophysics*, 1971, **9**, 5-8.

WEINTRAUB, D. J., & VIRSU, V. Estimating the vertex of converging lines: Angle misperception? *Perception & Psychophysics*, 1972, **11**, 277-283.

YAKIMOFF, N. Visual extrapolation of two converging nonintersecting line segments. *Acta Physiologica et Pharmacologica Bulgarica*, 1977, **3**, 58-64.

YAKIMOFF, N., MITRANI, L., & MATEEFF, S. Visual extrapolation of a line segment to the point of its intersection with a straight line—I. *Acta Physiologica et Pharmacologica Bulgarica*, 1977, **3**, 13-20.

(Manuscript received December 4, 1981;
accepted for publication October 29, 1982.)