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Calculation of significance regions for multiple predictors by the Johnson-Neyman technique

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One advantage of multiple regression over traditional analysis of variance (ANOVA) is that continuous independent variables need not be artificially divided (say via median-split) prior to their analysis (cf. Cohen & Cohen, 1975). In practice, however, the researcher is likely to encounter some difficulties when more fine-grained analyses are desired. For example, consider the case in which a continuous independent variable (e.g., test anxiety) is investigated in an experiment along with a two-level manipulated independent variable (e.g., high vs. low task importance). In this case it is hypothesized that low test anxious subjects will outperform high test anxious subjects, particularly when the task is important (see Figure 1). This interaction is easily tested in multiple regression, and its significance would indicate that the regression of performance on test anxiety is significantly different between the two groups. The next step would be to determine the nature of this interaction more specifically. In ANOVA, this is accomplished by post hoc comparisons of means, perhaps using tests of simple main effects or pairwise comparisons. In the regression approach, the ability to compare points on the two regression lines is needed to determine when the lines diverge to a significant degree.

The solution provided by Johnson and Neyman (1936) establishes regions of significance for which two groups differ significantly (see also Kerlinger & Pedhazur, 1973; Potthoff, 1964). Recently, two computer algorithms for calculating the Johnson-Neyman significance regions have been described (Lautenschlager, 1987; Scialfa, 1987). Both provide the user with the points along a single predictor continuum (as in Figure 1) for which two groups differ significantly.

The present program extends calculation of the Johnson-Neyman significance regions to the case of one or more predictors. This program represents an advance over previous programs in that regression applications may involve the finding that two continuous predictors interact with a grouping variable. For example, if the curvilinear effect of a continuous variable is of interest, the multiple-predictor case is invoked (e.g., $\hat{Y} = b_0+b_1X+b_2X^2$). As an example, consider the hypothetical data in Figure 2, in which test anxiety is related to performance in a more complex fashion requiring two predictors (i.e., linear and quadratic components of test anxiety). Alternatively, the multiple predictors could be different variables (rather than a single variable and its square, cube, etc.): $\hat{Y} = b_0+b_1X+b_2Z$. This case is more difficult to graph because it requires a three-dimensional representation, but it is conceptually not different from the case illustrated in Figure 2. The research question for both cases is, What sets of fixed predictor values result in significant differences between the two groups on the criterion variable? In general, the goal is to define the boundaries of the r-dimensional significance region (for r predictors).

Abelson (1953) provided a general analytic solution to this problem. His solution represents a substantial increase in the flexibility of the Johnson-Neyman technique, but also involves a substantial increase in computational labor over the single-predictor case. The increased computational labor derives from the use of matrix operations, including calculation of inverses. Accordingly, Abelson's approach is prohibitively time-consuming without the aid of a computer. The present program provides the computational power to carry out Abelson's procedure efficiently.

Program Description

The program is written in Commodore BASIC (Version 7.0) for the Commodore C-128 computer. The program can be easily modified for use on other machines.

Input. The program requires as input several basic results from the multiple regression that the user will have run to determine if the groups differ in their regressions. The following are requested by the program: (1) predictor means, predictor standard deviations, and sample sizes for each group, (2) intercept and raw regression coefficients for each group, (3) interpredictor correlations for each group, (4) overall error variance (or pooled error variances from the separate regressions), and (5) the critical value of the F test. For a specific set of predictor values, the critical F is based on 1 and $N-k-1$ *df*, where N is the total sample size and k is the number of independent variables used in the total multiple regression analysis (or the number of degrees of freedom on which the error variance is based). For a simultaneous confidence region, the F value to be entered is $(r+1)F$, where r is the number of predictors within a given group and F is the critical value for $(r+1)$ and $N-k-1$ *df*. As Potthoff (1964) notes, the latter critical value allows the statement that the groups differ (with $1-\alpha$ confidence) for all points in the defined region. A prompt on the video monitor queries the user for each of these values. After these values are entered, a prompt asks the user to check that the input is correct.

Program computations. The program calculates the following test statistic derived by Abelson (1953):

$$TS = \left[\frac{DX'}{(XUX')^{1/2}}\right]^2,$$

where, for r predictors, D is a $1\times(r+1)$ matrix contain-

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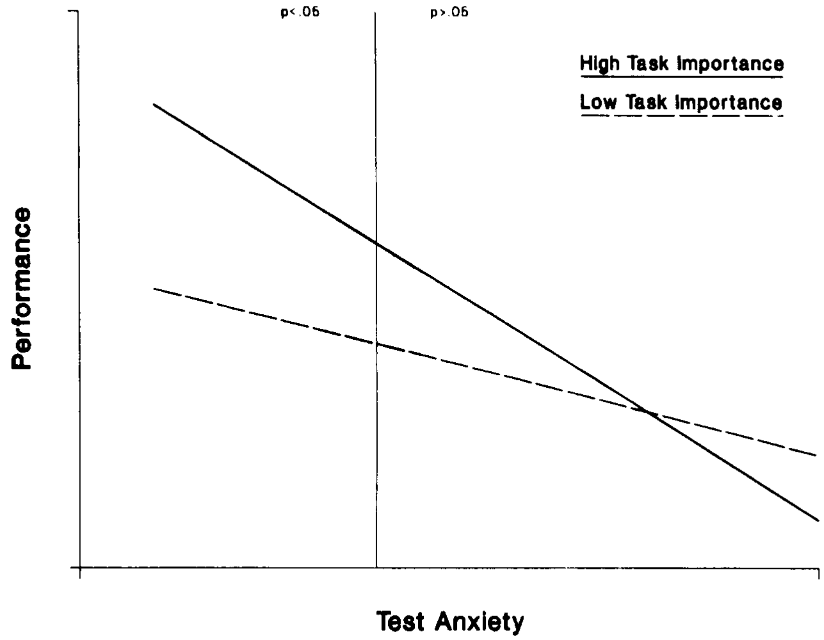

Figure 1. Hypothetical example of an interaction between a grouping variable and a continuous variable.

ing group differences for the intercept and regression coefficients:

$$D = [b1_0 - b2_0,\ b1_1 - b2_1,\ \ldots].$$

The X matrix is a $1 \times (r+1)$ matrix containing specific predictor values with unity as the first element:

$$X = [1,\ X_1,\ X_2,\ \ldots]$$

The U matrix is a $(r+1) \times (r+1)$ matrix calculated as a function of predictor variances and covariances. U is calculated as the sum of two matrices (P_1^{-1} and P_2^{-1}) each of which is $(r+1) \times (r+1)$ and based on a different group. The elements of each P matrix are as follows:

$$P = \begin{bmatrix} n & \Sigma X_1 & \Sigma X_2 \ldots \\ \Sigma X_1 & \Sigma X_1^2 & \Sigma X_1X_2 \ldots \\ \Sigma X_2 & \Sigma X_1X_2 & \Sigma X_2^2 \ldots \\ \vdots & \vdots & \vdots \end{bmatrix}$$

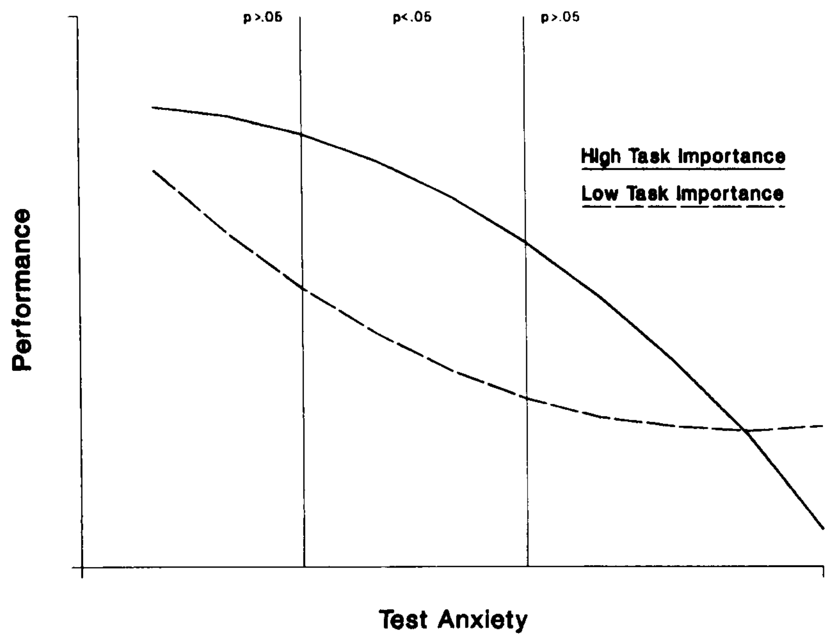

Figure 2. Hypothetical example of an interaction requiring more than one continuous variable (i.e., linear and quadratic components).

All elements of the U matrix are multiplied by the error variance from the complete multiple regression. Computation of the U matrix requires obtaining the inverse of each P matrix. This is accomplished in the program by the method of pivotal condensation (see Tatsuoka, 1971, Appendix B). The resulting test statistic (*TS*) is compared to the critical value of F for the test of significance.

Output. The user is queried for the choice of output. A particular combination of predictor points can be specified, and the difference between regression estimates for those points can be tested for significance. Alternatively, a range of values for each predictor can be specified and all possible combinations of those values will be tested for significance. If the range option is chosen, a prompt asks the user to specify the upper and lower limit for each predictor, as well as the size of the increment to be used. Output is routed to the screen in 80-column format. With program modification, the output can be routed to a printer or formatted for a 40-column screen display (e.g., for use with the Commodore C-64).

Availability

A listing of the program is available from the author. Alternatively, a formatted disk with an appropriate return envelope may be sent to the author and the program will be copied on the disk and returned.

REFERENCES

ABELSON, R. P. (1953). A note on the Neyman-Johnson technique. *Psychometrika*, **18**, 213-218.

COHEN, J., & COHEN, P. (1975). *Applied multiple regression/correlation analysis for the behavioral sciences*. Hillsdale, NJ: Erlbaum.

JOHNSON, P. O., & NEYMAN, J. (1936). Tests of certain linear hypotheses and their applications to some educational problems. *Statistical Research Memoirs*, **1**, 57-93.

KERLINGER, F. N., & PEDHAZUR, E. J. (1973). *Multiple regression in behavioral research*. New York: Holt.

LAUTENSCHLAGER, G. J. (1987). JOHN-NEY: An interactive program for computing the Johnson-Neyman confidence region for nonsignificant prediction differences. *Applied Psychological Measurement*, **11**, 174, 194.

POTTHOFF, R. F. (1964). On the Johnson-Neyman technique and some extensions thereof. *Psychometrika*, **29**, 241-256.

SCIALFA, C. T. (1987). A BASIC program to determine regions of significance using the Johnson-Neyman technique. *Behavior Research Methods, Instruments, & Computers*, **19**, 349-352.

TATSUOKA, M. (1971). *Multivariate analysis: Techniques for educational and psychological research*. New York: Wiley.

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