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A guide to LISREL-type structural equation modeling

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Recently, researchers in psychology have achieved the statistical sophistication necessary for building and testing LISREL-type causal models. However, the literature is devoid of any description of how to proceed in the LISREL modeling process. Using reading test scores obtained from empirical studies, causal models were constructed and tested. The guide resulted from these analyses and features descriptions of (1) a goodness-of-fit test, (2) the use of multiple indicator variables, (3) evaluation of any possible causal relationship, (4) a "residual" analysis for detecting measurement problems, (5) a "derivative" analysis for correlated measurement error, and (6) estimation of the amount of unexplained variability for each dependent latent variable.

The investigation of formal statistical models of causality has always been a major focus of science. The notion of causal modeling has existed since the formulation of path analysis by Wright (1918, 1921, 1934). Although the value of causal modeling as a nonexperimental technique has been known for a number of years, it is only a recent development that researchers in psychology have achieved the statistical sophistication necessary for the implementation and testing of causal models (Bentler, 1980).

A causal model typically consists of a theoretical structure involving the relationships among unobservable, hypothetical constructs referred to as latent variables. A latent variable may be designated as an independent variable, a dependent variable, or both, depending upon the specified structure. Each latent variable, although not directly measurable, can be evaluated by one or more observable indicator variables. In utilizing only a single indicator variable to measure each latent variable, as is actually the case in path analysis, considerable measurement error can be introduced into the causal model. Such a problem can be alleviated through the use of multiple indicators of each latent variable. Through some combination of the indicators for a particular latent variable, in a factor-analytic sense, one may obtain better measurement of the latent variables.

Causal modeling as defined and developed by Jöreskog (1973, 1977, 1978) is known as the linear structural relationship model, or LISREL. LISREL consists of the structural equation model and the measurement model. The structural equation model describes the theoretical causal relationships among the latent variables via a set of general linear equations. The measurement model describes the measurement of the latent variables by the observable indicator variables and allows evaluation of the measurement properties of such measures.

The LISREL system of causal modeling is presently implemented in the LISREL IV (Jöreskog & Sörbom, 1978) computer program. Since 1972, when LISREL was first made available to the research community, several applications of the technique have appeared in the psychological literature. However, the literature is devoid of any description of how to proceed in the LISREL-type causal modeling process. Although Maruyama and McGarvey (1980) discuss the evaluation of causal models via the LISREL technique, they fail to aid the practitioner in two important areas. First, they do not describe in adequate detail the theory underlying the LISREL model, especially with respect to the following: the estimation both of parameters and of the population variance-covariance matrix, the identification problem, and the sequential testing of models in a confirmatory sense. Second, although the application presented appears to be sufficient, Maruyama and McGarvey do not tell the reader how to proceed in the modeling process. If a researcher's initial model does not yield an acceptable fit to the data, what should the next step be? The purpose of this paper is to (1) describe in detail the theory underlying the LISREL model and (2) construct a guide to model fitting for prospective LISREL users. The guide is supported by empirical work done on a reading comprehension model developed by Lomax (1980, Note 1).

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THE THEORY UNDERLYING THE LISREL MODEL

The theory supporting the general LISREL model (see also Jöreskog, 1977, 1978) is as follows.

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The Model

Consider, first, the structural equation model. Let $\eta(m \times 1)$ and $\xi(n \times 1)$ be random vectors of the latent dependent and independent variables, respectively, so that a system of linear structural equations follows

$$B\eta = \Gamma\xi + \zeta, \tag{1}$$

where $B(m \times m)$ and $\Gamma(m \times n)$ are matrices of coefficients, and $\zeta(m \times 1)$ is a random vector of residuals due to equation errors (e.g., misspecification error). Assume that $E(\eta) = E(\zeta) = 0$ and that $E(\xi) = 0$. Assume also that ζ is uncorrelated with ξ and that B is nonsingular (i.e., invertible).

Now consider the measurement model. Since the vectors η and ξ are unobservable, let $y(p \times 1)$ and $x(q \times 1)$ be vectors of the observable indicator variables, so that

$$y = \Lambda_y \eta + \epsilon \tag{2}$$

and

$$x = \Lambda_x \xi + \delta, \tag{3}$$

where $\epsilon(p \times 1)$ and $\delta(q \times 1)$ are vectors of the errors of measurement in y and x, respectively. The vectors y and x are taken to be measured as deviations from their respective means. Let $\Lambda_y(p \times m)$ and $\Lambda_x(q \times n)$ be regression matrices of y on η and of x on ξ, respectively. Assume that the errors of measurement ϵ are uncorrelated with the errors of measurement δ. Also assume that the errors of measurement, ϵ and δ, are uncorrelated with η, ξ, and ζ.

Next consider combining aspects of the structural equation and measurement models. Let $\Phi(n \times n)$ and $\Psi(m \times m)$ be the variance-covariance matrices of ξ and ζ, respectively, and let $\Theta_\epsilon(p \times p)$ and $\Theta_\delta(q \times q)$ be the variance-covariance matrices of ϵ and δ, respectively. From the above assumptions, it follows that the variance-covariance matrix $\Sigma\ [(p + q) \times (p + q)]$ of $z = (y',x')'$ is

$$\Sigma = \begin{bmatrix} \Lambda_y(B^{-1}\Gamma\Phi\Gamma'B'^{-1} + B^{-1}\Psi B'^{-1})\Lambda_y' + \Theta_\epsilon & \Lambda_y B^{-1}\Gamma\Phi\Lambda_x' \\ \Lambda_x\Phi\Gamma'B'^{-1}\Lambda_y' & \Lambda_x\Phi\Lambda_x' + \Theta_\delta \end{bmatrix}$$

$$= \begin{bmatrix} \Sigma_{yy} & \Sigma_{yx} \\ \Sigma_{xy} & \Sigma_{xx} \end{bmatrix}. \tag{4}$$

The variance-covariance matrix Σ is a supermatrix constructed from the four submatrices. The diagonal submatrices Σ_{yy} and Σ_{xx} consist of both variance and covariance terms and reflect the relationships within each of the y and x indicator variable sets of η and ξ, respectively. The off-diagonal submatrices $\Sigma_{xy} = \Sigma_{yx}'$ consist only of covariance terms and reflect the relationships between the y and x indicator variable sets. The elements of Σ, then, are obviously functions of the elements of the matrices Λ_y, Λ_x, B, Γ, Φ, Ψ, Θ_ϵ, and Θ_δ. Elements of these eight matrices are of three types: (1) fixed parameters that have been assigned certain values a priori, (2) constrained parameters that are unknown but equal to one or more of the other parameters, and (3) free parameters that are unknown and unconstrained.

The measurement model can be written in a factor-analytic sense as:

$$z = \Lambda f + e, \tag{5}$$

where $z = (y',x')'$, $f = (\eta',\xi')'$, $e = (\epsilon',\delta')'$, and

$$\Lambda = \begin{bmatrix} \Lambda_y & 0 \\ 0 & \Lambda_x \end{bmatrix}.$$

Thus, z is a vector of observed indicator variable measurements, Λ is a matrix of factor loadings on the latent variables, f is a vector of unobserved latent variables or common factors, and e is a vector of measurement errors or unique factors. From this, one can see that the LISREL model is simply a restricted factor-analysis model (Jöreskog, 1969) in which the factors η and ξ satisfy a linear structural equation system of the form in Equation 1. The restriction is that the observables are permitted to load only on specified elements of f, which necessitates fixing the majority of the elements in Λ to zero. The restrictions are derived from the hypothesized measurement model. However, there is no requirement that $m < p$, that $n < q$, and that Θ_ϵ and Θ_δ be diagonal, as there is in traditional factor analysis. The only requirements are that Σ as defined in Equation 4 is nonsingular (i.e., invertible) and that the model is identified.

Identification and Estimation

Prior to the application of estimation procedures to the model, one needs to assess the identification problem. Identification depends on the specific LISREL model under consideration and on the specification of fixed, constrained, and free parameters. A specified structure of Λ_y, Λ_x, B, Γ, Φ, Ψ, Θ_ϵ, and Θ_δ generates one and only one Σ, although several structures may generate the same Σ. If more than one structure generates the same Σ, the structures are said to be equivalent. If a parameter has the same value in all equivalent structures, then that parameter is identified. If all of the parameters of the model are identified, a consistent estimator of it can be found. If a parameter is not identified, it will not be possible to find a consistent estimator of it. Thus, a model is identified when all parameters are uniquely estimable from the data, and not identified when there is not enough information to uniquely estimate certain parameters.

Jöreskog (1977, 1978) assumes that the distribution of the observed indicator variables can be sufficiently well described by the first- and second-order moments so that information contained in higher order moments can be ignored. This assumption will be upheld if the distribution is multivariate normal. With the mean vector unconstrained, the distribution of $z = (y', x')'$ is described by the independent parameters in Λ_y, Λ_x, B, Γ, Φ, Ψ, Θ_ϵ, and Θ_δ. Let θ be a vector of Order s of all of the independent, free, and constrained parameters (i.e., each distinct constrained parameter is counted only once). For each fixed parameter (that is, for each parameter set equal to a constant, typically 0 or 1), one less parameter need be identified and estimated. The identification problem, then, is whether or not θ is determined by Σ.

For a specific LISREL model, the identification problem can be evaluated by considering the upper triangular portion of Σ. There are $\frac{1}{2}(p + q)(p + q + 1)$ equations in Σ and s unknown elements in θ. Therefore a necessary, but not sufficient, condition for the identification of all of the parameters is that

$$s < \tfrac{1}{2}(p + q)(p + q + 1). \tag{6}$$

A parameter θ is identified if that parameter can be determined from Σ; otherwise, the parameter is not identified. In some cases, a parameter can be determined from Σ in different ways, such that the parameter is said to be overidentified.

Since it has already been assumed that the distribution of the observed indicator variables is sufficiently described by the variance-covariance matrix, the estimation problem involves the fitting of Σ as described by the LISREL model to the sample variance-covariance matrix S. The fitting function used by Jöreskog (1977, 1978) is

$$F = \log |\Sigma| + \mathrm{tr}(S\Sigma^{-1}) - \log |S| - (p+q), \tag{7}$$

which is minimized with respect to θ. Minimizing F is equivalent to (and more convenient than) maximizing the logarithm of the likelihood (log L) of Σ. Recall that $\theta' = (\theta_1, \theta_2, \ldots, \theta_s)$ is a vector of the unknown elements of the matrices Λ_y, Λ_x, B, Γ, Φ, Ψ, Θ_ϵ, and θ_δ. Then F may be regarded as a function $F(\theta)$ of $\theta_1, \theta_2, \ldots, \theta_s$ (Jöreskog, 1973). The procedure in effect minimizes a scalar function of the difference between S and Σ. If the distribution of $z = (y', x')'$ is multivariate normal, then maximum-likelihood estimates result, which are efficient in large samples. The minimization of F with respect to the independent parameters is accomplished through a modification of the Fletcher and Powell (1963) iterative procedure, as described by Gruvaeus and Jöreskog (Note 2). The minimization procedure uses the first-order derivatives and approximations to the second-order derivatives of F and rapidly converges from some arbitrary starting point to a local minimum of F. If several minima of F exist, the procedure does not necessarily converge to the absolute minimum.

The LISREL computer program (Jöreskog & Sörbom, 1978) checks on the identification problem for each of the s distinct and independent parameters by computing the information matrix at the starting point of the iterative procedure. If the matrix is positive definite (i.e., invertible), the model is identified. If the information matrix is singular (i.e., not invertible), the model is not identified. The matrix is evaluated at each iteration by building up information about the function F. Once the minimum of F is found, the information matrix is again computed and yields standard errors for each of the independent parameters. By starting with an identified model and allowing a fixed parameter with a nonzero derivative to become free in a subsequent model, one guarantees an identified model. At the minimum of F, all derivatives of the independent parameters are zero; the derivatives are also zero for any fixed parameter that would not be identified if it were allowed to become free. The constrained parameters sum to zero. Thus, when allowing a fixed parameter to become free, the parameter is identified (and thus estimable) only if its associated derivative is nonzero.

Model Testing

Once the maximum-likelihood estimates of the independent parameters have been computed, the goodness of fit of the LISREL model may be tested in large samples by the likelihood ratio procedure. Let H_0 be the null hypothesis of the LISREL model under the specification of fixed, constrained, and free parameters. Consider first the situation in which H_1, the alternative hypothesis, is that Σ is any positive definite matrix. Then minus twice the logarithm of the likelihood ratio is $(N/2)F_0$, where F_0 is the minimum of F. If such a model holds, it is distributed in large samples as chi square, with degrees of freedom.

$$d = \tfrac{1}{2}(p + q)(p + q + 1) - s. \tag{8}$$

Next, consider the situation in which H_0 is a specific hypothesis of a LISREL model that is more restrictive than an alternative H_1. In large samples, H_0 can be tested against H_1. If F_0 is the minimum of F under H_0, and F_1 under H_1, then $F_1 < F_0$, and minus twice the logarithm of the likelihood ratio becomes $(N/2)(F_0 - F_1)$. Under H_0, this function is approximately distributed as chi square with $s_1 - s_0$ degrees of freedom, which is the difference in the number of independent parameters estimated under H_1 and H_0. Such a set-up allows one to sequentially test a series of hierarchical hypotheses.

In research that is more exploratory in nature, each chi-square value may be utilized as a goodness-of-fit test. If the value of chi square obtained is large relative to its degrees of freedom, the fit may be further evaluated through an inspection of (1) the magnitude of the first-order derivatives of F for the fixed parameters and (2) the difference between S and Σ, the residual resulting from $S - \Sigma$. Usually, such an examination suggests ways in which the LISREL model may be relaxed by allowing additional fixed parameters to become free, in order to arrive at a better fitting model. For the revised model, if the drop in the value of chi square is large relative to the difference in the degrees of freedom, it indicates that the changes resulted in a "significant" improvement in the fit of the model. However, if the drop in the value of chi square is approximately equal to the difference in the degrees of freedom, then the changes resulted in a "minimal" improvement in the fit of the model due to a chance occurrence. A limitation in using the chi-square index as a goodness-of-fit test for an individual model is that the chi-square value is a direct function of sample size. For a large sample, a "good fit" cannot usually be found using this index, even though the residuals may be essentially zero. For a small sample, many competing models may yield a "good fit." Other indexes that are unrelated to sample size are described by Bentler and Bonett (1980).

It is not the statistical estimation procedure that "creates" a causal model. A theory or structure of relationships among a set of latent variables is first hypothesized and then tested via statistical techniques. The primary reason for modifying a LISREL model is not to obtain a "significantly" better fit to the data. Model modifications should not be made when they are inconsistent with the underlying theory, but rather, only when the changes are meaningful to the researcher. Thus, the major goal of LISREL-type structural equation modeling is confirmatory, in the sense of substantiating some theory, and exploratory, in the sense of making finer theoretical distinctions than were initially hypothesized.

A general problem associated with causal modeling is that of model misspecification. Specification errors arise when some of the relevant and important variables are not included in the model, and they may result in biased estimates of the structure coefficients and/or statistical rejection of the model. Procedures for detecting misspecification in structural equation models have been devised by Saris, dePijper, and Zegwaart (1980).

The Program

The LISREL system of structural equation modeling is presently implemented in the LISREL IV (Jöreskog & Sörbom, 1978) computer program.[1] The LISREL computer program, and thus the LISREL model, is extremely flexible, in that it can accommodate virtually any type of causal model. The LISREL model/program makes allowances for equation errors (i.e., disturbances), measurement errors, correlated errors of measurement, and models with reciprocal causation, as well as confirmatory factor analysis, longitudinal analysis, simultaneous analysis in several groups, and covariance structure analysis.

The LISREL program yields numerous types of results. Of particular interest to the present discussion are the following: the matrix to be analyzed, parameter specifications and starting values (as selected by the researcher), the LISREL maximum-likelihood estimates and their standard errors, the results of the goodness-of-fit test (the chi-square value, degrees of freedom, and associated probability level), computed z values (LISREL estimates divided by their standard errors) for each of the independent parameters estimated, the estimated variance-covariance matrix $\hat{\Sigma}$, a matrix of the $S - \hat{\Sigma}$ residuals, first-order derivatives of the function F, and the estimates for the standardized solution. Other optional output features include tables describing the behavior of the iterative procedure (which may be useful for models that do not converge on or reach a final solution), correlations among the parameters estimated, and factor score regressions (i.e., regressions among the latent variables).

A GUIDE TO LISREL-TYPE CAUSAL MODELING

This guide represents an initial attempt to construct a sequential series of steps whereby the prospective modeler can proceed from an initial hypothesized "gut-level" model to a final best-fitting model based on theory and statistical estimation. For each step, examples are drawn from the reading comprehension model described by Lomax (1980, Note 1). The procedure is as follows.

Step 1—Construct the hypothetical causal or structural model based upon one's training and experience in the substantive area. Define the latent dependent and independent variables (i.e., η and ξ, respectively). Define the causal relationships among the latent dependent variables (i.e., B) and between the latent dependent and independent variables (i.e., Γ). In the reading studies, Lomax (1980, Note 1) considered skills believed to be necessary for the development of reading comprehension and assembled them into the causal model, as shown in Figure 1. The model consisted of phonological, word recognition, reading rate, and reading comprehension components. The causal relationships were formulated through classroom observation of reading behavior and familiarity with reading curricula.

Step 2—Substantiate the structural model by reviewing the relevant literature and by attending to possible alternative models as well. Although in the reading content area, researchers have been content to examine the relationships among pairs of components rather than causal networks, some evidence was found for each of the relationships in the hypothesized model. However,

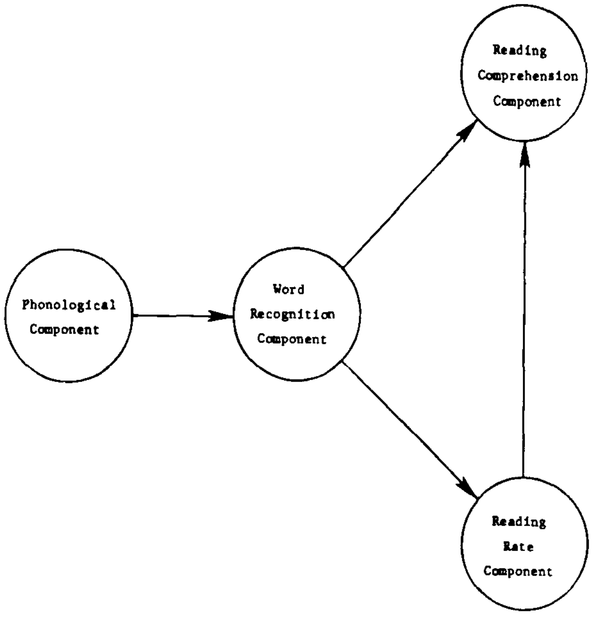

Figure 1. A causal model of the component processes of reading comprehension.

evidence for the influence of reading rate on reading comprehension was minimal, indicating that such a relationship may not exist.

Step 3—Select an appropriate population to be sampled. In the reading study, the learning-disabled population was selected because the group was very heterogeneous with respect to their level of reading development.

Step 4—Define a set of indicator variables (preferably three or more) for each of the latent dependent and independent variables (i.e., y and x, respectively). From a comprehensive battery of reading tests, 17 relevant indicators were chosen for the analysis. Multiple indicators per latent construct are useful for evaluating the relative merits of each measure, as shall be seen in later steps.

Step 5—Collect the data.

Step 6—Decide whether to utilize the correlation or variance-covariance matrix for analysis, depending upon the relative metrics of the indicator variables. When the unit of measurement for the indicator variables is of no particular importance to the researcher (i.e., arbitrary or irrelevant), then only an analysis of the correlation matrix is typically of interest. Since the correlation matrix involves a rescaling of the observed indicator variables, the parameters estimated for the measurement model (in particular, the factor loadings Λ_y and Λ_x) will be of the same order of magnitude (i.e., along the same metric). When the same indicator variables are measured over time (i.e., longitudinal analysis) or for multiple groups (i.e., simultaneous analysis of several groups), an analysis of the variance-covariance matrix is useful so as to capitalize upon the metric similarities of the variables. Because of the large and arbitrary metric differences among the indicators (e.g., reading rate vs. word recognition), the correlation matrix was analyzed for the reading study.

Step 7—Construct a detailed figure of the proposed causal model that allows derivation of the matrix equations for both the measurement and structural models. A solution to the identification problem may be useful to obtain, although the LISREL program does check on identification. In the measurement model, one indicator of each latent variable in Λ should be fixed to a value of one, which sets the scale for the remaining indicators and simplifies solution of the identification problem. The reference indicator to be selected is incidental, since the importance of the measurement model is the magnitude of the loadings relative to the reference indicator. The relative magnitudes of the loadings are maintained regardless of the reference indicator selected. If there is only a single indicator for a given latent variable (that is, the indicator and latent variables are equivalent, with no measurement error), then the error variance for that indicator variable must be fixed at zero. In other words, assume perfect measurement of the latent variable. It is often desirable in an initial model to specify Ψ, Φ, Θ_ϵ, and Θ_δ as diagonal matrices for ease of identification, where only the diagonal elements are allowed to be free. When Ψ is a diagonal matrix, each diagonal value estimated is equivalent to the amount of unexplained variance for a particular dependent latent variable. Note that when these variance-covariance matrices are full rather than diagonal, starting values for the diagonal elements must be larger than those for the off-diagonal elements (otherwise, the model will not reach a final solution). The free elements in B need to be given negative starting values so as to allow convergence on a final solution, although the sign should be reversed in the reporting of results. Formulate the LISREL deck set-up. A detail of the reading comprehension model and the set of matrix equations are shown in Figure 2 and Table 1, respectively.

Step 8—Test the initial hypothesized model. If standardization of the latent variables is desirable, consider use of the standardized solution rather than the LISREL maximum-likelihood solution. The initial reading comprehension model resulted in a poor fit [$\chi^2(115) = 561.411$] and necessitated some additional model-fitting procedures.

Step 9—Examine the measurement portion of the LISREL model by following Steps 9 and 10. Why investigate aspects of the measurement model prior to those of the structural model? Since the latent variables are defined by the indicator variables, the optimal measurement model should be established in the initial stages of the model-fitting process. Conduct a residual analysis by (a) tallying the number of large residuals (e.g., $|r| \geqslant .100$) for each indicator variable, (b) investigating those indicators with a large number of these residuals

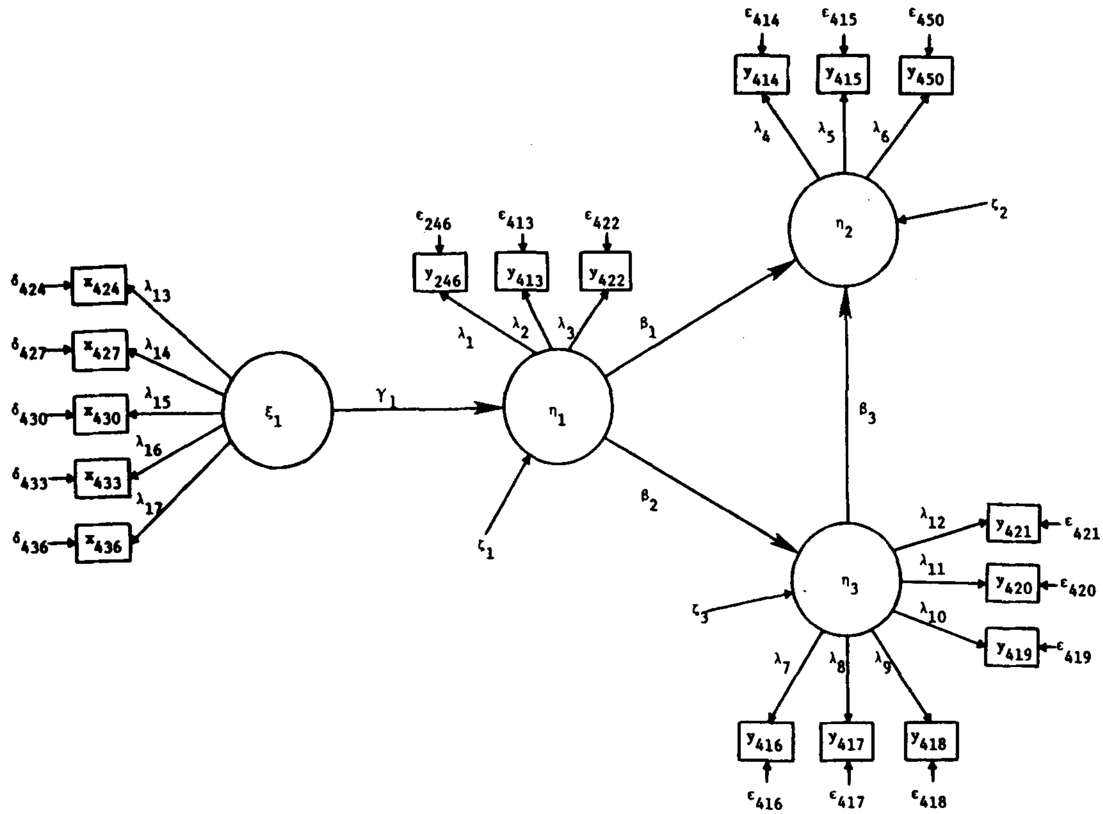

Figure 2. Detail of the proposed causal model.

to determine if there are measurement problems (e.g., ceiling effects, test administration problems, etc.), and (c) testing a new model with the problematic indicators not included, seeking to find a significant improvement in the fit of the model (for a similar approach, see Costner & Schoenberg, 1973). As shown in Table 2, three such measurement problems were detected in the residual analysis for the reading components model and resulted in a better fitting causal model. These problems were due to a lack of convergent validity evidence (i.e., construct related) and a severe ceiling effect that resulted in minimal variability. This is a benefit of obtaining multiple measurements for each latent variable.

Step 10—Perform a derivative analysis by (a) devising a list for those pairs of indicator variables for which correlated measurement error seems reasonable theoretically, (b) selecting from these pairs the largest absolute first-order derivative of Θ_ϵ and Θ_δ and allowing that parameter to become free in the next model, and (c) determining whether the difference in chi-square values is significant for the two models. If the difference is significant, return to Step 10b; otherwise, go on to Step 11.

Step 11—Examine the structural portion of the LISREL model by following Steps 11-13. Inspect the resulting z statistic for each of the structure coefficients (i.e., B and Γ) to see if they are significantly different from zero. Then, fix the nonsignificant parameters to be zero and test a subsequent model for which the difference in chi-square values should be nonsignificant (due to essentially zero structure coefficients). For the reading comprehension model, β_3 (i.e., the influence of rate on comprehension) was initially estimated to be nonsignificant and was set equal to zero for the subsequent model. No significant difference was found between the models, and thus the notion that β_3 was indeed equal to zero was supported. Recall that there was minimal empirical support for the influence of rate on reading comprehension.

Step 12—Review the first-order derivatives for the parameters of B and Γ previously fixed at zero. Select the parameter of largest absolute magnitude (unless the structure coefficient runs counter to empirical findings) and allow that parameter to become free in a subsequent model. Determine if the chi-square difference is significant relative to the difference in degrees of freedom. Continue as necessary. As shown in Table 3 by the value −.276, this process leads to the discovery of β_4, that is, the direct influence of reading rate upon word recognition skill, which appears to be theoretically plausible.

Table 1
Matrix Equations for the Initial Model

Structural Model:

$$\begin{bmatrix} 1 & 0 & 0 \\ -\beta_1 & 1 & -\beta_3 \\ -\beta_2 & 0 & 1 \end{bmatrix} \begin{bmatrix} \eta_1 \\ \eta_2 \\ \eta_3 \end{bmatrix} = \begin{bmatrix} \gamma_1 \\ 0 \\ 0 \end{bmatrix} \xi_1 + \begin{bmatrix} \zeta_1 \\ \zeta_2 \\ \zeta_3 \end{bmatrix}$$

Φ = diagonal Ψ = diagonal

Measurement Model:

$$\begin{bmatrix} y_{246} \\ y_{413} \\ y_{422} \\ y_{414} \\ y_{415} \\ y_{450} \\ y_{416} \\ y_{417} \\ y_{418} \\ y_{419} \\ y_{420} \\ y_{421} \end{bmatrix} = \begin{bmatrix} \lambda_1 & 0 & 0 \\ \lambda_2 & 0 & 0 \\ \lambda_3 & 0 & 0 \\ 0 & \lambda_4 & 0 \\ 0 & \lambda_5 & 0 \\ 0 & \lambda_6 & 0 \\ 0 & 0 & \lambda_7 \\ 0 & 0 & \lambda_8 \\ 0 & 0 & \lambda_9 \\ 0 & 0 & \lambda_{10} \\ 0 & 0 & \lambda_{11} \\ 0 & 0 & \lambda_{12} \end{bmatrix} \begin{bmatrix} \eta_1 \\ \eta_2 \\ \eta_3 \end{bmatrix} + \begin{bmatrix} \epsilon_{246} \\ \epsilon_{413} \\ \epsilon_{422} \\ \epsilon_{414} \\ \epsilon_{415} \\ \epsilon_{450} \\ \epsilon_{416} \\ \epsilon_{417} \\ \epsilon_{418} \\ \epsilon_{419} \\ \epsilon_{420} \\ \epsilon_{421} \end{bmatrix}$$

$$\begin{bmatrix} x_{424} \\ x_{427} \\ x_{430} \\ x_{433} \\ x_{436} \end{bmatrix} = \begin{bmatrix} \lambda_{13} \\ \lambda_{14} \\ \lambda_{15} \\ \lambda_{16} \\ \lambda_{17} \end{bmatrix} \xi_1 + \begin{bmatrix} \delta_{424} \\ \delta_{427} \\ \delta_{430} \\ \delta_{433} \\ \delta_{436} \end{bmatrix}$$

Step 13—Be sure that (a) the final structural model is best fitting in a statistical sense and, more important, (b) that the model is consistent with what is known theoretically (i.e., that each path either has some empirical base or that the causal relationship is unknown or uninvestigated). Evidence for the influence of rate on comprehension (i.e., β_3) was minimal, whereas the influence of rate on word recognition (i.e., β_4) was unknown. The standardized solution for the final reading comprehension model is shown in Figure 3.

Step 14—Interpret and discuss the implications of the final model.

SUMMARY

The LISREL model is very useful in a causal model-building context. LISREL includes the following features to investigate causal models: (1) A goodness-of-fit test allows the testing of individual models, as well as testing for improvement across models; (2) the use of multiple indicator variables allows an assessment of measurement error; (3) any possible recursive or non-recursive (reciprocal) causal relationship can be hypothesized and evaluated; (4) the standardized solution is preferable to the LISREL maximum-likelihood solution if standardization of the latent variables is desirable; (5) a "residual" analysis can be conducted to detect measurement problems in the indicator variables; (6) a "derivative" analysis can be performed to determine those indicators for which there is correlated measurement error; and (7) the amount of unexplained variability is estimable for each latent variable. In theory, LISREL is extremely flexible in allowing for virtually any possible causal relationship. In practice, LISREL

Table 2
Residual Analysis for Models 2 and 3

	Model #2		Model #3
Indicator Variable	Number of Residuals ≥ .100	Number of Large Residuals After Removing 417, 419, 436	Number of Residuals ≥ .100
246	0	0	0
413	3	1	1
422	2	1	2
414	3	1	1
415	3	0	1
450	3	3	3
416	2	1	1
417	9	---	---
418	2	2	2
419	9	---	---
420	1	0	0
421	6	4	5
424	4	1	2
427	1	1	1
430	2	0	0
433	3	1	1
436	7	---	---
TOTAL	60/2 = 30	16/2 = 8	20/2 = 10

Table 3
First-Order Derivatives of β and Γ for Model 4

BETA

	ETA 1	ETA 2	ETA 3
EQ. 1	-0.000	-0.093	-0.276
EQ. 2	-0.000	-0.000	-0.085
EQ. 3	0.000	-0.020	0.000

GAMMA

	KSI 1
EQ. 1	-0.000
EQ. 2	0.143
EQ. 3	0.101

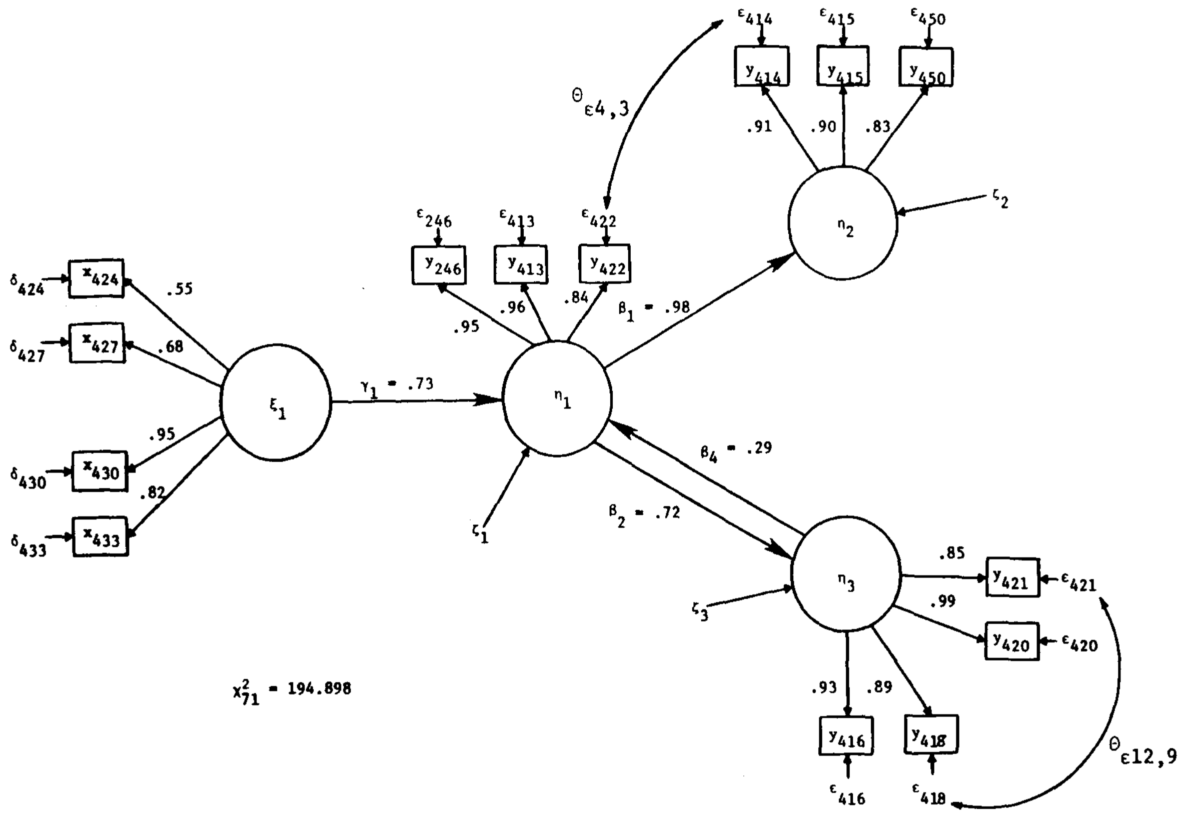

Figure 3. Standardized solution for the final model.

is somewhat complicated for use by the typical researcher. The guide described here should help to bridge the gap between theoretical and practical use and enable researchers in psychology to implement the LISREL causal modeling technique.

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NOTE

1. The LISREL IV computer program is available from International Educational Services, 1525 East 53rd Street, Chicago, Illinois 60615.

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