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TOES

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Harris and Kaiser (1964) have developed a theoretical algebraic model that defines all possible analytic transformation procedures. Their orthoblique equations describe the computation of both orthogonal and oblique transformation solutions using only orthogonal transformation matrices. They have operationalized two highly specific sets of equations to define two highly specific oblique transformation solutions having several important properties, the independent cluster solution and pattern proportional to factor intercorrelations. The acronym "TOES" refers to a series of subroutines intended to compute and explicate these two Harris and Kaiser (1964) orthoblique solutions. Specifically, TOES refers to "the orthoblique explication subroutines."

The independent cluster solution is fundamental to much multivariate research, and Harris and Kaiser (1964) imply that it should be computed as a matter of formality in all exploratory factor analyses. It is most appropriate when each variable loading is on no more than one factor, while the pattern proportional is most appropriate when some of the variables are factorially complex (some variables loading on more than one factor).

It is well known that factors make direct proportionate contributions, independent contributions, and joint proportionate contributions to the common variance (see Harman, 1967; Hofmann, 1975). The joint contributions, although sometimes negative, represent that proportion of common variance that is explained as a function of the covariation of the factors. More traditionally, we are accustomed to seeing direct contributions alluded to in a components solution when reference is made to "proportion of common variance explained" by an eigenvalue. Contrary to traditional oblique transformation theory, the oblique factors of an independent cluster solution make no joint proportionate contribtuions to the common variance. Alternatively, the maximum possible joint proportionate contributions of the factors to the common variance of the variables is afforded by the pattern-proportional solution. Thus, one solution affords the maximum possible direct proportionate contributions, while the other solution affords the maximum possible joint contributions.

The set of subroutines "TOES" is a highly specific group of subroutines written almost exclusively to explicate the computations of the solutions and the previously mentioned characteristics of the two Harris and Kaiser (1964) operational orthoblique equations.

Computational approach. Let F be any $(n \times r)$ input factor matrix. Form the r eigenvalues, M^2, and eigenvectors, Q, of the major product of F,

$$FF' = QM^2Q'. \tag{1}$$

Then the primary pattern matrix of the independent cluster solution, A_1, and the associated primary factor intercorrelation matrix, L_1, may be defined, respectively, as

$$A_1 = QT_1D_1 \tag{2}$$

and

$$L_1 = D_1^{-1}T_1'M^2T_1D_1^{-1}, \tag{3}$$

where the orthonormal T_1 is determined from Q according to the raw quartimax transformation criterion (Harman, 1967), and D_1^{-1} normalizes the columns of (MT_1). The primary pattern matrix of the pattern proportional, A_2, and the associated primary factor intercorrelation matrix, L_2, may be defined, respectively, as

$$A_2 = QM^{.5}T_2D_2 \tag{4}$$

and

$$L_2 = D_2^{-1}T'MT_2D_2^{-1}. \tag{5}$$

The orthonormal T_2 is computed from $QM^{.5}$ according to the raw quartimax transformation criterion and D_2^{-1} normalized the columns of $(M^{.5}T_2)$.

Let A be a general $(n \times r)$ primary pattern matrix; then, following Hofmann (1976), the complexity, c, of any variable (variable complexity) is determined according to:

$$c_{.} = \frac{\left(\sum_{j=1}^{r} a_{.j}^2\right)^2}{\sum_{j=1}^{r} a_{.j}^4}. \tag{6}$$

The average complexity is just the average complexity for all variables.

Let A and L be a general primary pattern matrix and general primary factor intercorrelation matrix. The joint and direct proportionate contributions of the factors to the total common variance is given by the off-diagonal and diagonal entries, respectively, of the matrix V which is defined (following Hofmann, 1975) by the following equation set:

$$c = 1'(A'A)*L1 \tag{7}$$

and

$$V = 1/c(A'A)*L. \tag{8}$$

The symbol * refers to the element-by-element product of $(A'A)$ and L and 1 is a unit column vector.

Input. The only necessary input for this routine is a factor matrix. The factor matrix may represent either some initial factor matrix or some orthogonally rotated factor matrix.

Output. Program output includes two sets of oblique solutions, a primary pattern matrix and a primary factor intercorrelation matrix, one set for the independent cluster solution and a second set for the pattern proportional. Also output with each primary pattern matrix is the complexity for each variable as well as the average variable complexity for each solution. Finally, the joint and direct proportionate contributions to the total common variance is printed for each solution.

Capacity. The subroutines used may be subdivided into required and optional. Using an IBM 370-168 G-level compiler, the required subroutines utilize 8,202 bytes of core, while the optional subroutines, a matrix printing routine and an eigenvalue-eigenvector routine, utilize an additional 9,750 bytes of core. The routines are programmed in FORTRAN IV, utilizing double precision, single subscripting, and principles of dynamic storage. Thus, there are no practical limits on either the number of variables or the number of factors that may be input into the program.

Availability. Copies of this paper, a source listing, and documented output from an illustrative example can be obtained at no cost by writing to Richard J. Hofmann, Department of Educational Psychology, Miami University, Oxford, Ohio 45056.

REFERENCES

Harman, H. *Modern factor analysis* (Rev. ed.). Chicago: University of Chicago Press, 1967.

Harris, C. W., & Kaiser, H. Oblique factor analytic solutions by orthogonal transformations. *Psychometrika*, 1964, **29**, 347-362.

Hofmann, R. J. Brief report: On the proportionate contributions of transformed factors to common variance. *Multivariate Behavioral Research*, 1975, **10**, 507-508.

Hofmann, R. J. Indices descriptive of factor complexity. *Journal of General Psychology*, 1976, in press.