# Locational representation in imagery: The third dimension 

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#### Abstract

Six experiments were conducted to test the relative processing characteristics of picture-plane and three-dimensional imagery as indexed by tasks that required subjects to keep track of successive locations in multiunit visual displays. Subjects were shown symmetrical displays either drawn on cardboard or constructed with three-dimensional blocks. They then were required to imagine these matrices and follow pathways through a series of adjacent squares (blocks) within the matrices. The pathways were described by a series of verbal terms that indicated the direction of the next square (block) in the pathway. Subjects experienced difficulty in performing the task with picture-plane displays composed of as few as 16 squares ( $4 \times 4$ ), but they rarely made errors with a three-dimensional matrix of 27 blocks $(3 \times 3 \times 3$ ). Performance with the threedimensional task dropped dramatically when the matrix size was increased to $4 \times 4 \times 4$. The results replicated previous findings that the image processing capacity for location in two-dimensional imagery is about three units in each direction, and they indicate that adding the depth dimension increases the capacity for representation of spatial location in imagery.


There recently has been much interest in the question of the representational capacity of mental imagery (e.g., Attneave \& Curlee, 1983; Kosslyn, 1980; Weber \& Malmstrom, 1979). Research findings consistently have suggested limitations in the amount of information that can accurately be processed imaginally, although interpretations of these data vary greatly. However, despite welldocumented evidence that accurate mental processing is possible for three-dimensional as well as two-dimensional information (e.g., Pinker, 1980; Shepard \& Metzler, 1971), studies of imagery capacity heretofore have focused only on imagery of stimuli depicted on the twodimensional picture plane.
The research reported here explores the relative capabilities and limitations of two- and three-dimensional imaginal processing in a series of experiments similar to those reported by Attneave and Curlee (1983). Attneave and Curlee were specifically interested in the capacity of the imagery system for the representation of distinct locations in the two-dimensional picture plane. They tested subjects' ability to imagine a "spot" as it moved through a matrix in a pathway that was dictated by the spoken directions: up, down, left, right. Each new direction named the next square in the pathway that the spot was to follow. The size of the matrices varied from $3 \times 3$ to $8 \times 8$, and half of the subjects received organizational instructions that encouraged them to divide mentally the larger matrices into smaller component matrices. Attneave and Curlee found that performance on the task dropped considerably as matrices increased in size, with the lar-

[^0]gest single-step performance difference between $3 \times 3$ and $4 \times 4$ matrices. Organizational strategies produced better performance on all but the $3 \times 3$ matrix. The authors concluded that matrices larger than $3 \times 3$ exceed the capacity of the normal imagery processing system.
The subjects in the research reported here also performed a task that required that they mentally follow a pathway through a matrix, but in some cases the matrix was a three-dimensional display built of wooden blocks, and in others it was a two-dimensional picture drawn on cardboard (see Figure 1). Whether the display was twoor three-dimensional, the task was essentially the same. The subjects were told that for each trial the experimenter would indicate a starting square (block), which they were to consider the first step in a pathway through the matrix of squares (blocks). The subsequent squares (blocks) in the pathway were indicated by tape-recorded statements of direction (up, down, left, right, and [for blocks only] forward and back). The physical matrix was not visible while the subject listened to the seven statements of direction, but at the end of each such series, the matrix again was shown to the subject, whose task it was to point to the location of the final square (block) in each pathway. The purpose of Experiment 1 was to compare the subjects' ability to follow mentally the pathway when imagery processing included two versus three dimensions. To compare performance in two- and three-dimensional imagery, matrices were selected to match as closely as possible the absolute number of distinct spatial locations in matrices built of three-dimensional blocks and those drawn on the picture plane. The four matrices selected were $3 \times 3(9$ squares), $2 \times 2 \times 2$ ( 8 blocks), $5 \times 5$ ( 25 squares), and $3 \times 3 \times 3$ ( 27 blocks).
Attneave and Curlee (1983) showed that matrices larger than $3 \times 3$ exceeded the imagery system's processing ca-


Figure 1. Figures built of blocks or drawn on the picture plane.
pacity for information contained in the picture plane. The design of Experiment 1 provided a test of whether capacity limit is restricted by the absolute number of distinct spatial units (9), or by the number of units in each dimension ( 3 horizontal $\times 3$ vertical). If the total number of units is the limiting factor for the imagery processing system, then the smaller matrices ( $3 \times 3$ and $2 \times 2 \times 2$ ) should produce significantly better performance than the larger ones ( $5 \times 5$ and $3 \times 3 \times 3$ ). However, if the limit is based on the number of units in each direction, and if the imagery processing system includes the depth dimension, then performance with the $3 \times 3 \times 3$ matrix should be equivalent to that with the $3 \times 3$ matrix and should be significantly better than that with the $5 \times 5$ matrix.

## EXPERIMENT 1

## Method

Subjects. Sixteen ( 3 female, 13 male) Mercer University students received extra credit in a psychology course for their participation.

Materials. Materials in this experiment included two twodimensional matrices ( $3 \times 3$ and $5 \times 5$ ) and two three-dimensional matrices ( $2 \times 2 \times 2$ and $3 \times 3 \times 3$ ). The two-dimensional matrices were drawn in black ink on a white cardboard background. Each individual square in a matrix was $4 \times 4 \mathrm{~cm}$. Thus the total size of the $5 \times 5$ matrix was $400 \mathrm{~cm}^{2}$ and that of the $3 \times 3$ matrix was $144 \mathrm{~cm}^{2}$. The three-dimensional matrices were constructed from unpainted
wooden cubes that measured 4 cm in each direction. Thus the $3 \times 3 \times 3$ and $2 \times 2 \times 2$ block matrices were 1,728 and $512 \mathrm{~cm}^{3}$, respectively.
Eight tape recordings were prepared, each of which included one practice and eight experimental trials for each of the four matrixstimuli. A trial consisted of the introductory statement, "begin with the starting square (block)," followed by a series of seven statements of direction, read at a $2-\mathrm{sec}$ rate and indicating a pathway through a matrix. Each series of statements described a pathway that began in a corner square or block, was contained within the matrix for that trial, never passed through the same square or block more than once, and never moved more than two consecutive steps in the same direction. The number of statements per trial was selected to accommodate the limited number of steps available for the smallest matrix ( $2 \times 2 \times 2$ ).

Possible order effects were controlled by both between- and within-subjects counterbalancing. Four of the tape recordings described a series of four trials with matrices in the order $3 \times 3$, $2 \times 2 \times 2,5 \times 5,3 \times 3 \times 3,3 \times 3 \times 3,5 \times 5,2 \times 2 \times 2,3 \times 3$. The other four employed the order $2 \times 2 \times 2,3 \times 3,3 \times 3 \times 3,5 \times 5,5 \times 5$, $3 \times 3 \times 3,3 \times 3,2 \times 2 \times 2$. Thus subjects always encountered smaller matrices before larger matrices, but trials were otherwise fully counterbalanced. Each series of four trials began with a constant starting position. Starting squares were in the lower right or upper left corner; starting blocks were in the lower front right or upper back left corner for both sizes of matrix (again with order counterbalanced across subjects). Each tape was played for two experimental sessions.
Procedure. The subjects were tested individually, seated at a distance of about 60 cm from the stimulus displays. The task was described as one that required the subjects to follow mentally a pathway that moved sequentially through a series of adjacent squares (blocks) in a figure. The instructions did not refer to a "spot" that moved through the figure (cf. Attneave \& Curlee, 1983), because spots are two-dimensional entities that traditionally appear on surfaces, and pilot testing indicated that reference to a spot confused subjects in the three-dimensional task because it inappropriately focused their attention on a particular side of the cube instead of on the cube as a whole. Therefore, the subjects were instructed to think of each individual square (block) as a "step" in a pathway that moved from square to square (block to block) through the figure.
The instructions for the $3 \times 3 \times 3$ task emphasized that all blocks in the figure, including those blocks that were hidden from direct view, could serve as steps in the pathway. The experimenter moved the front "layer" of blocks to point out the middle blocks that could not be viewed directly. Some subjects adopted this strategy of removing obstructing blocks when they wanted to point to a final block that was not on the surface. Others responded by pointing to a surface block and describing its relationship to the final block (e.g., "It's the block behind this one"). When subjects used this mode of reporting, the experimenter double-checked the location by probing about the other two spatial coordinates for the final block (e.g., "Do you mean the block below this one, and to the left of this one?').
The subjects were given instructions for each matrix prior to their first four trials with that matrix. Following the instructions for each matrix, the subjects were given two practice trials, one in which the experimenter read the directional statements aloud while the matrix was still visible, and a second with tape-recorded directions and the matrix hidden from view. The subjects who made errors on either practice trial were shown the correct pathway on the matrix as the directions were repeated. The starting locations for practice trials were corner squares or blocks different from those used on experimental trials. The subjects' answers were recorded on an answer sheet that allowed the coding of both two- and threedimensional information. Except for interruptions for instructions about a new matrix, trials proceeded consecutively at a rate comfortable for the subject. The session generally lasted about 35 min .

## Results

The mean percentage of correct trials for each of the four matrices is indicated in Table 1. Performance was literally errorless for the small matrix of squares and practically so for both of the matrices of blocks. The data, based on the number of correct trials per figure with a possible range of $0-8$, were analyzed in a $2 \times 2$ analysis of variance (ANOVA). All comparisons were significant at $p<.001$. For the complexity factor (large vs. small matrices), $F(1,15)=61.73, M S e=.77$; for the dimensionality factor (three- vs. two-dimensional matrices), $F(1,15)=30.04, M S e=1.15$; and for the interaction, $F(1,15)=44.31, M S e=.99$. All significant effects are attributable to lower performance with the 25 -square matrix than with any other.

## Discussion

The results of Experiment 1 clearly indicate that adding a third dimension to imagery facilitates the imagery process. The subjects in this experiment were able to keep track of 27 units in three-dimensional space with nearly perfect accuracy, although performance on a twodimensional task with slightly fewer units ( 25 squares) was substantially impaired.

These results may be surprising to some readers. The subjects themselves often literally gasped when they first saw the $3 \times 3 \times 3$ matrix, and protested that they would never be able to keep track of that many blocks. Most subjects reported, however, that although they had expected the task with the $3 \times 3 \times 3$ matrix to be difficult, in actual experience it was relatively easy. This is not to say that their performance was effortless. Subjects reported that the task with the $3 \times 3 \times 3$ matrix required great concentration, but, due to such concentration, they felt confident that their answers were correct.

The pattern of the observed results was predictable from the findings typically observed in absolute judgment tasks (see Attneave, 1959, for a review). In general, absolute judgment tasks require subjects to discriminate among a number of alternative stimuli that vary along one or more dimensions. "Channel capacity" is determined in terms of the number of stimuli beyond which subjects begin to make errors in discrimination. The relevant research finding is that subjects usually can discriminate only about 2 or 3 bits of information on a single dimension, but that as the number of dimensions increases, so too does the total amount of information that subjects can process ac-

Table 1
Mean Percentage of Correct Trials as a Function of Matrix Size and Dimensionality in Experiment 1

|  |  |  |
| :--- | :---: | :---: |
| Size | Picture-Plane | Dimensionality |
| Small | $(3 \times 3)$ | $(2 \times 2 \times 2)$ |
|  | $100 \%$ | $98 \%$ |
| Large | $(5 \times 5)$ | $(3 \times 3 \times 3)$ |
|  | $58 \%$ | $97 \%$ |

curately. For example, Hake and Garner (1951) found that the channel capacity for visual discrimination of distinct positions along a line (a single-dimensional task) was about 3.25 bits, whereas Klemmer and Frick (1953) found that the capacity for judging the location of a dot within a square (a two-dimensional task) was 4.4 bits. Thus, presuming that the visual processing and imagery systems share capacity characteristics, it is entirely predictable that adding a third dimension will increase imagery processing efficiency.
Although the most parsimonious explanation of the results of Experiment 1 focuses on the differences in the dimensionality of the materials, it also is possible to attribute the differences to the fact that three-dimensional matrices (e.g., $3 \times 3 \times 3$ ) can be separated more easily into units of a manageable size (e.g., into three $3 \times 3$ "layers") than can the $5 \times 5$ matrix. The implication is that if the two-dimensional matrices also could be easily segmented into component parts, pathways through them also could be imagined with high accuracy. This possibility is consistent with Attneave and Curlee's (1983) finding that subjects who were instructed to divide mentally larger twodimensional matrices into smaller segments performed better than those who were not.
To test the effects of a segmentation strategy on performance under conditions maximally similar to those of Experiment 1, an additional 14 subjects were tested. These subjects were instructed to use a segmenting organizational strategy for the $5 \times 5$ matrix, and then were tested with both the $3 \times 3$ and $5 \times 5$ matrices. The organizational strategy was illustrated by shading in red the center row and column of the $5 \times 5$ matrix so that the matrix was divided into a center "cross" of 5 squares that separated four 4 -square segments. The cross pattern was selected because it was the most frequently reported $5 \times 5$ strategy of the subjects in Experiment 1. (Although independently selected, this pattern also was the design used by Attneave and Curlee [1983] as the organization for their $5 \times 5$ matrix.) The subjects instructed in this strategy were correct on $62 \%$ of the trials with the $5 \times 5$ matrix and on $96 \%$ of the trials with the $3 \times 3$ matrix. Thus, performance on the $5 \times 5$ matrix with an organizational strategy was slightly better than without it, but still far below the nearly perfect performance found with the $3 \times 3 \times 3$ matrix in Experiment 1 . These results suggest that the superior performance with 27 blocks in Experiment 1 is attributable to the inclusion of the depth dimension and not simply to the ease of segmentation. They further indicate that performance is dependent upon the number of units in each spatial dimension and not the total number of units in a figure. This finding is important because it eliminates the possibility that performance is purely a function of the proportional number of units in a figure that are included in the eight-step pathway.

Experiment 2 was designed as an initial test of the relative performance on the two- and three-dimensional tasks when the figures were larger than three units in each direction. In this experiment, performances on matrices only
slightly larger than those used in Experiment 1 were compared. The matrices differed in dimensionality but were equivalent in number and size of component matrices that might serve as useful segments. The two critical matrices were $6 \times 6$ squares and $3 \times 3 \times 4$ blocks, with the added layer of blocks in the depth dimension. Thus the $6 \times 6$ matrix contained four $3 \times 3$ component matrices situated side by side on the picture plane and the $3 \times 3 \times 4$ contained four $3 \times 3$ matrices stacked behind one another in the depth plane. The total number of units in each figure was 36 .

## EXPERIMENT 2

## Method

Subjects. Sixteen ( 14 female, 2 male) Emory University summer school students participated for extra course credit.
Materials. A $6 \times 6$ matrix and a $3 \times 3$ matrix were drawn on the same scale as those of Experiment 1 . A $2 \times 2 \times 2$ matrix of blocks was identical to that of Experiment 1. A $3 \times 3 \times 4$ matrix was built of the same blocks and was identical in height and width to the earlier experiment's $3 \times 3 \times 3$ matrix, but it extended an extra unit in the depth direction.
The eight tapes from Experiment 1 were again employed, with 2 subjects assigned to each tape. Thus, although the subjects believed that they must keep track of information in a larger area of space for the larger matrices, the pathways never moved outside the bounds of a $3 \times 3 \times 3$ or $5 \times 5$ matrix.
Procedure. The procedure was identical to that of Experiment 1 except in the description of the two larger matrices. The nonsymmetricality of the $3 \times 3 \times 4$ matrix was emphasized to ensure that subjects attended to the "extra" layer of depth, and the relationship of component $3 \times 3$ matrices to the larger matrix was pointed out for both matrices.

## Results

The results of Experiment 2 are presented in Table 2. Data analysis was based on the number of correct trials per figure with a possible range of $0-8$. A $2 \times 2$ ANOVA revealed significant effects for all comparisons. For the complexity factor (large vs. small matrices), $F(1,15)=$ $46.15, \mathrm{MSe}=2.17, p<.001$; for the dimensionality factor (two- vs. three-dimensional matrices), $F(1,15)=$ $8.26, M S e=1.48, p<.05$; and for the interaction, $F(1,15)=10.07, M S e=1.22, p<.01$.

## Discussion

The significant effects for both dimensionality and the interaction clearly indicate that the subjects had more difficulty with the two- than the three-dimensional task, even when the matrices were equated for absolute number of units and the organizational potential of $3 \times 3$ subcomponents.

A comparison of performance on the $3 \times 3 \times 4$ matrix of Experiment 2 with that of the $3 \times 3 \times 3$ matrix of Experiment 1 showed performance on the latter to be significantly better $(t=3.59, p<.01)$. This result suggests that increasing the size of a three-dimensional figure from 27 to 36 blocks exceeds the processing capacity of threedimensional imagery. Just as a $3 \times 3$ matrix defines the number of distinct locations that can easily be represented

Table 2
Mean Percentage of Correct Trials as a Function of Matrix Size and Dimensionality in Experiment 2

|  | Dimensionality |  |
| :--- | :---: | :---: |
| Size | Picture-Plane | Three-Dimensional |
| Small | $(3 \times 3)$ | $(2 \times 2 \times 2)$ |
|  | $99 \%$ | $99 \%$ |
| Large | $(6 \times 6)$ | $(3 \times 3 \times 4)$ |
|  | $57 \%$ | $79 \%$ |

in images of the picture plane, a $3 \times 3 \times 3$ matrix apparently defines the limits on the processing capacity of threedimensional imagery. The difference between the $3 \times 3 \times 3$ and $3 \times 3 \times 4$ matrices cannot, of course, be attributed to the particular pathways employed since they were identical; only the imagined context of these pathways was varied systematically.
The $3 \times 3 \times 4$ matrix of Experiment 2 was selected in order to equate the absolute number of units and subunits in the two-dimensional and three-dimensional displays without drastically changing the sizes of the figures in Experiment 1 . However, the asymmetrical $3 \times 3 \times 4$ matrix is not strictly comparable to the $6 \times 6$ matrix since the former exceeds the hypothetical three-unit limit by only one additional layer in the depth dimension, whereas the $6 \times 6$ matrix exceeds the three-unit limit in both pictureplane dimensions. A test of performance with threedimensional imagery similar to Attneave and Curlee's (1983) test of two-dimensional imagery requires fully symmetrical displays that extend an equal number of units in each dimension.
Experiment 3 was designed to test performance on the three-dimensional imagery task with the next-larger-sized symmetrical matrix, $4 \times 4 \times 4$. It also included test trials with $3 \times 3 \times 3$ and $3 \times 3$ matrices to replicate Experiment 1, and trials with an $8 \times 8$ matrix for comparison with the $4 \times 4 \times 4$ matrix ( 64 blocks vs. 64 squares).

## EXPERIMENT 3

## Method

Subjects. Sixteen summer-school students (6 female, 10 male) at Emory and Oglethorpe Universities volunteered to serve as subjects.
Materials. The matrices of squares and blocks were constructed from materials identical to those used in the previous experiments.
Four tape recordings were made, each of a different set of 32 randomly generated pathways. To reduce the predictability of the pathways, the stipulation that pathways could never pass through the same square or block more than once was not used, and each pathway began in a randomly selected square or block in the figure rather than in a corner square. Rules for generating the pathways were otherwise the same as in previous experiments. Because subjects had reported no difficulty with the 2 -sec presentation rate in the previous experiments, and because Attneave and Curlee's (1983) subjects apparently had little difficulty with their task at much faster rates, the presentation rate was increased to 1.5 sec . Trials were completely counterbalanced within subjects in blocks of four trials with each matrix. Half of the subjects received experimental trials in the order $3 \times 3,3 \times 3 \times 3,8 \times 8,4 \times 4 \times 4,4 \times 4 \times 4,8 \times 8,3 \times 3 \times 3$,
$3 \times 3$; and the other half in the order $3 \times 3 \times 3,3 \times 3,4 \times 4 \times 4,8 \times 8$, $8 \times 8,4 \times 4 \times 4,3 \times 3,3 \times 3 \times 3$. Each tape was played for four experimental sessions.

Procedure. The subjects were given instructions for both the picture-plane and three-dimensional tasks before any experimental trials were attempted. Practice trials were given with the $3 \times 3$ and $3 \times 3 \times 3$ matrices. Thereafter, the subjects were allowed as much time as they wanted to inspect each new matrix, but no additional practice trials were given. The experimenter had a coded list that indicated the starting blocks so that she could point to the starting block prior to each trial. Hidden blocks were indicated as in previous experiments.

## Results

The results again showed nearly perfect performance for both the $3 \times 3(97 \%)$ and $3 \times 3 \times 3(91 \%)$ matrices. The subjects performed the task with accuracy of $48 \%$ for the $4 \times 4 \times 4$ matrix and $30 \%$ for the $8 \times 8$ matrix. Data anal$y$ yis was based on the number of correct trials per figure with a possible range of $0-8$. The ANOVA was significant $[F(3,45)=62.22, M S e=1.74, p<.001]$, and Newman-Keuls comparisons indicated that only the $3 \times 3$ and $3 \times 3 \times 3$ matrices were not significantly different from each other at $p<.05$.

## Discussion

The results of this experiment replicate those of Experiments 1 and 2 in the comparison of performance with the $3 \times 3$ and $3 \times 3 \times 3$ matrices. Performance with 27 blocks again was nearly equivalent to that with 9 squares. The results of this experiment also are consistent with the findings of Experiment 2 in suggesting that imagery processing is more efficient with three-dimensional than with picture-plane displays even when the displays exceed the normal imagery processing capacity. The subjects performed significantly better with 64 blocks arranged in a $4 \times 4 \times 4$ display than they did with 64 squares in an $8 \times 8$ design. However, this argument is susceptible to the same segmenting qualification discussed earlier: Perhaps a $4 \times 4 \times 4$ matrix is more easily divided into component layers of $4 \times 4$ blocks each than an $8 \times 8$ matrix is divided into its $4 \times 4$ quadrants.

Experiment 4 was designed to test this segmenting hypothesis by providing structural emphasis on the relation of $4 \times 4$ subcomponents to the $8 \times 8$ matrix. Performance was again tested with the $3 \times 3$ and $3 \times 3 \times 3$ matrices for purposes of replication and comparison. A $4 \times 4$ matrix was added for comparison with the larger matrices to which it was structurally related.

## EXPERIMENT 4

## Method

Subjects. Twelve ( 7 female, 5 male) young adults responded to notices on bulletin boards on the Emory University campus and were paid for their participation.
Materials. The matrices of squares and blocks were constructed from materials identical to those used in previous experiments. The two center lines of the $8 \times 8$ matrix were thickened in black ink to emphasize the structural relationship of the four $4 \times 4$ quadrants to the larger figure.

Four tape recordings similar to those used in Experiment 3 were prepared. Trials with the $4 \times 4$ matrix were included in the serial position that immediately preceded the first trials with a 64 -unit figure ( $8 \times 8$ or $4 \times 4 \times 4$ ).
Procedure. The procedure was identical to that of Experiment 3, except that in the instructions for the $8 \times 8$ and $4 \times 4 \times 4$ matrices, the structural relationship of the smaller $4 \times 4$ component matrices to the larger figure was pointed out, and the subjects were told it might be helpful to use this structural scheme as an organizational strategy.

## Results

The mean percentage of correct trials was $99 \%$ for the $3 \times 3,92 \%$ for the $3 \times 3 \times 3,81 \%$ for the $4 \times 4,59 \%$ for the $4 \times 4 \times 4$, and $53 \%$ for the $8 \times 8$ figure. Data analysis was based on the number of correct trials per figure with a possible range of 0-8. A one-way ANOVA was significant $[F(4,44)=16.16, M S e=1.90, p<.001]$. Only three comparisons failed to reach significance at the .05 level in a Newman-Keuls analysis: Performance on the $3 \times 3 \times 3$ did not differ significantly from that on either the $3 \times 3$ or the $4 \times 4$ matrix, and performance on the $4 \times 4 \times 4$ did not differ significantly from that on the $8 \times 8$.

## Discussion

When structural boundaries for its $4 \times 4$ quadrants were added to the $8 \times 8$ matrix, the subjects' performance improved to a level comparable to that with the $4 \times 4 \times 4$ matrix. Thus, the better performance with the $4 \times 4 \times 4$ matrix in Experiment 3 apparently was due to better structural organization of the figure rather than to an inherently greater capacity for locational representation in three dimensions.

It could be argued, however, that the "structured" $8 \times 8$ matrix in fact had an advantage over the $4 \times 4 \times 4$ matrix whose four segments were not physically differentiated. Perhaps if the four segments of the $4 \times 4 \times 4$ matrix were each distinctly marked, performance with that figure would improve. Experiment 5 was designed to test that possibility and similar segmenting hypotheses regarding figures used in previous experiments.
Seven figures were tested in Experiment 5. The four picture-plane figures were $3 \times 3,4 \times 4,6 \times 6$, and $8 \times 8$. The four quadrants of each of the two larger figures were each colored a different hue. The three-dimensional figures were $3 \times 3 \times 3,3 \times 3 \times 4$, and $4 \times 4 \times 4$. Each layer in depth was colored a different hue. Thus the component units of all larger figures were each equally distinctively marked by contrasting colors so that subunits were clearly visible. The uncolored $3 \times 3$ and $4 \times 4$ matrices were included for purposes of comparison with larger figures.

## EXPERIMENT 5

[^1]$4 \times 4 \times 4$ matrices, the picture-plane layer was composed of red blocks, the second layer in depth of green blocks, the third layer of yellow blocks, and the final layer of blue blocks. The blue layer was removed to create the $3 \times 3 \times 3$ matrix.

Picture-plane matrices were composed of $2.5-\mathrm{cm}^{2}$ squares and were drawn in black ink on white cardboard. On the $6 \times 6$ and $8 \times 8$ matrices, the upper left quadrant was shaded red; and upper right, green; the lower right, yellow; and the lower left, blue.

Four tape recordings similar to those used in Experiment 4 were prepared. Two tapes presented blocks of four trials each in the order $3 \times 3,3 \times 3 \times 3,6 \times 6,3 \times 3 \times 4,4 \times 4,4 \times 4 \times 4,8 \times 8,8 \times 8,4 \times 4 \times 4$, $4 \times 4,3 \times 3 \times 4,6 \times 6,3 \times 3 \times 3,3 \times 3$; and on the other two tapes, the positions of the $3 \times 3 \times 3$ trials were exchanged with those of the $3 \times 3$ trials, the positions of the $3 \times 3 \times 4$ trials were exchanged with those of the $6 \times 6$ trials, and the positions of the $8 \times 8$ trials were exchanged with those of the $4 \times 4 \times 4$ trials.

Procedure. The procedure was similar to that of Experiment 4 except in the description of the purpose of the colored segments. For each colored figure, the structural relationship of the different colored component segments was described, and subjects were told that it might be helpful to use this structural scheme as an organizational strategy. Subjects were given practice trials only with the $3 \times 3$ and $3 \times 3 \times 3$ matrices. Thereafter, they were allowed as much time as necessary to inspect each new figure, but additional practice was not given.

## Results

The mean percentage of correct trials was $100 \%$ for the $3 \times 3,95 \%$ for the $3 \times 3 \times 3,79 \%$ for the $6 \times 6,75 \%$ for the $4 \times 4,75 \%$ for the $3 \times 3 \times 4,62 \%$ for the $8 \times 8$, and $59 \%$ for the $4 \times 4 \times 4$. Data analysis was based on the number of correct trials per figure with a possible range of $0-8$. A one-way ANOVA was significant $[F(6,78)=$ 13.18, $M S e=1.60, p<.001]$. Newman-Keuls comparisons showed three distinct groups of scores, the members of which were not significantly different from each other but were significantly different from all other scores. They were $3 \times 3$ and $3 \times 3 \times 3 ; 4 \times 4,6 \times 6$, and $3 \times 3 \times 4$; and $8 \times 8$ and $4 \times 4 \times 4$.

## Discussion

The results confirm the findings of Experiment 4 under conditions designed to equate the number, size, and distinctiveness of segments in picture-plane versus threedimensional figures. Subjects in Experiment 5 performed equally well with the $3 \times 3 \times 4$ as with the $6 \times 6$ matrix and with the $4 \times 4 \times 4$ as with the $8 \times 8$ matrix. The superior performance with the $3 \times 3 \times 4$ matrix in Experiment 2 and with the $4 \times 4 \times 4$ matrix in Experiment 3 thus is not attributable to the three-dimensional qualities of the figures alone, but to the fact that they are more easily divided into manageable segments. The apparent discrepancy between these results and the findings in Experiment 1 will be considered in the General Discussion.

The results of Experiment 5 also replicated the consistent finding that performance with the $3 \times 3 \times 3$ matrix of blocks is similar to that with the $3 \times 3$ matrix of squares, and is significantly better than performance with any other figure, including the $4 \times 4$ matrix of squares.

The fact that performance with the $4 \times 4$ matrix was not superior to performance with the $6 \times 6$ matrix is proba-
bly due to the fact that the $6 \times 6$ was divided into subunits, but the $4 \times 4$ was not. Attneave and Curlee (1983) have established that even a $4 \times 4$ matrix is easier to keep in mind when it is divided into smaller components. The $4 \times 4$ matrix in Experiment 5 was not subdivided because its main point of comparison was with the $4 \times 4 \times 4$ and $8 \times 8$ figures in which the undivided $4 \times 4$ served as the subcomponent.

The data from Experiment 5 might be used to argue that the results of the first three experiments were due to the fact that segments in three-dimensional matrices are more easily and distinctively labeled than are those in pictureplane matrices. Such an explanation assumes that subjects are performing the task through verbal or propositional strategies, rather than through spatial or imaginal ones. Consider, for example, the possibility that poor performance with the $8 \times 8$ matrix in Experiment 3 was the result of interference between verbal labels for submatrices and those for position within the submatrix (e.g., upper right quadrant, lower left square). Presumably this sort of interference would be less powerful where names for segments were distinctive to the depth dimension while those for position were limited to the picture plane (e.g., back segment, lower left square). Thus the enhanced performance in Experiment 5 might be attributed to the availability of distinctive color labels for different segments of larger figures. Color labels were not available, however, in Experiment 4 , which also showed equivalent performance for 64 squares and 64 blocks. In fact, if spatial labeling strategies were used in Experiment 4, then encouraging subjects to use a segmenting strategy should have produced greater performance benefits for the noninterfering three-dimensional task than for the interference-laden picture-plane task. But it did not do so. And, if performance is based on verbal labels, it is difficult to understand why teaching subjects a segmenting strategy should produce the consistently positive results it does with picture-plane performances (Attneave \& Curlee, 1983). It is even more difficult to understand why performance with the $8 \times 8$ matrix should have improved to the level of that with the $4 \times 4 \times 4$ matrix in Experiment 4 .

Despite the previous arguments, however, there may remain doubt in the minds of some readers that the reliably good performance with the $3 \times 3 \times 3$ matrix of blocks is attributable specifically to a spatial imagery process. This objection is consistent with arguments that performance on imagery tasks in general depends on "propositional"' rather than '"analog'’ processing (e.g., Pylyshyn, 1973) or that the two processes are empirically indistinguishable from each other (e.g., Anderson, 1978). Experiment 6 was designed to lay this doubt to rest by adding a nonspatial dimension to the picture-plane task to test its effect on performance. In this experiment subjects were required to keep track of the "temperature" of a $3 \times 3$ matrix as it changed from one temperature to another (hot, warm, cold) while they also followed a pathway through the squares. If performance on the $3 \times 3 \times 3$ matrix is mediated by a verbal or other nonspatial strategy, then sub-
jects should be equally adept at keeping track of three levels of temperature as they are at keeping track of three layers in depth.

## EXPERIMENT 6

## Method

Subjects. Sixteen ( 9 female, 7 male) students at Oglethorpe University participated to fulfill a requirement for a psychology course.

Materials. The $3 \times 3$ and $3 \times 3 \times 3$ matrices were the same as those used in the first four experiments.
Sixteen randomly generated pathways through the $3 \times 3 \times 3$ matrix were constructed using the same constraints as those in previous experiments. Each pathway was then altered such that moves on the picture plane remained identical but moves in depth were replaced systematically by changes in temperature. Changes in temperature could move back and forth from hot to warm to cold just as changes in depth moved from back to middle to front. The term "hotter" was systematically substituted for the term "back" and "colder" was substituted for "forward." Likewise, a starting block that had been in the front layer of the $3 \times 3 \times 3$ matrix was described as cold in the $3 \times 3$ matrix, one in the back was described as hot, and one in the middle was warm.
Two tape recordings were made. Each tape included eight temperature-change trials and eight depth-change trials in addition to two practice trials of each. The matched pairs of trials described in the previous paragraph appeared on different tapes, and the order of presentation was counterbalanced between subjects.
Procedure. Subjects learned about their first task and received two practice trials followed by eight experimental trials. They were then given instructions for the second task followed by the same number of practice and experimental trials.
The instructions for temperature changes included the information that the temperature would always remain within the threetemperature range from hot to cold so that hot matrices could get no hotter, cold no colder. The instructions also made it clear that a hot matrix could move to cold and vice versa only by moving through the intermediate stage, warm.

## Results

Subjects were correct on $91 \%$ of the trials that included changes in the depth dimension ( $M=7.25, S D=.86$ ) and $72 \%$ of the trials that involved changes in temperature ( $M=5.69, S D=1.54$ ). The difference was significant at $p<.001(t=4.58)$.

## Discussion

Keeping track of changes in temperature while simultaneously keeping track of location in a picture-plane matrix was significantly more difficult than keeping track of changes in depth as well as location in the picture-plane dimension. These results are consistent with the interpretation that subjects were using a single integrated processing system to keep track of the three spatial dimensions but different, and potentially incompatible, processing systems to keep track of both temperature and picture-plane location. The results are inconsistent with a verbal mediation explanation of the processing of depth information, since such an explanation should apply equally well to temperature and to depth changes.

The patterns of errors in the two conditions provide further evidence that different processing strategies underlie the two tasks. Twenty-five out of 32 errors ( $78 \%$ ) in the temperature condition were errors in temperature alone. Only 3 of 8 errors ( $38 \%$ ) in the three-dimensional matrix were errors in depth alone. Keeping track of temperature apparently was more difficult for subjects than keeping track of depth while simultaneously following directional changes in the picture plane.
Following the experiment, subjects were asked which task had been more difficult for them. Four subjects considered the task with three spatial dimensions more difficult, 10 considered the temperature changes more difficult, and 2 were undecided. Whether subjects considered the temperature task easy or difficult seemed to depend on whether they had been able to develop an effective strategy for dealing with it. The most frequently cited strategies for the temperature task involved constant verbal rehearsal. In contrast, most subjects reported that they needed no special strategy for the task with the blocks because it was possible to keep track of spatial location directly. No subject reported a verbal rehearsal strategy for the spatial task.

## GENERAL DISCUSSION

Results of the first five experiments reported here suggest that the capacity for representing locational space in imagery depends less on the absolute number of discriminable units of location than on the number of dimensions in which those units are distributed. The limitation on effective use of imaginal space for the tasks employed here seems to be three distinct locations in each direction: a $3 \times 3$ matrix in the picture plane and a $3 \times 3 \times 3$ matrix with the depth plane included.
Results of Experiments 3, 4, and 5 further indicate that once capacity limitations have been exceeded by one unit in each picture-plane and depth direction, performance drops dramatically. Performance with the $4 \times 4 \times 4$ matrix was markedly poorer than that with the $4 \times 4$ matrix in Experiments 4 and 5 . Adding the depth dimension clearly increases imagery processing efficiency within the limitations of a $3 \times 3 \times 3$ matrix, but when the limits are exceeded, adding the third dimension creates a new confusion by adding another direction in which a subject may "get lost" in a pathway.
The relative effects of structural organization for twoand three-dimensional figures that exceed processing capacity are summarized in Table 3. The results of Experiments 2 and 3 initially suggested that imagery processing in three dimensions was more efficient than picture-plane imagery even when the processing capacity had been exceeded. Performance was better with 36 cubes than 36 squares in Experiment 2 and with 64 cubes than 64 squares in Experiment 3. This finding again illustrates that subjects' performance is not a function simply of the proportionate number of units in a figure that are included

Table 3
Mean Percentage of Correct Trials for Larger Figures as a Function of Type of Matrix and Structural Organization

| Structural Organization | Size of Matrix |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | 36 Units |  | 64 Units |  |
|  | $6 \times 6$ | $3 \times 3 \times 4$ | $8 \times 8$ | $4 \times 4 \times 4$ |
| Unstructured | $57 \%{ }^{2}$ | $79 \%{ }^{2}$ | $30 \%^{3}$ | $48 \%^{3}$ |
| Structured | 79\% ${ }^{5}$ | 75\% ${ }^{5}$ | 62\% ${ }^{\text {s }}$ | 59\% ${ }^{5}$ |

Note-Numerical superscripts indicate the number of the experiment in which the data were obtained.
in a pathway, since proportions would be identical for matched two- and three-dimensional figures. The results of Experiments 4 and 5, however, showed that when distinct structural subunits were clearly indicated for the picture-plane stimuli, performance was equivalent for twoand three-dimensional figures that were matched for number of units and subcomponents. Thus the only advantage of three-dimensional displays larger than $3 \times 3 \times 3$ appears to be in the structural characteristics that make the component subunits more obvious.

The data in Table 3 show an apparent superiority for $4 \times 4 \times 4$ figures with colored segments but not for comparable $3 \times 3 \times 4$ figures. This difference may be due to the fact that subjects in Experiment 2 were shown the relationship of $3 \times 3$ subunits to the larger figures, whereas subjects in Experiment 3 received no segmenting instructions. Subjects in Experiment 4 who received segmenting instructions for the $4 \times 4 \times 4$ figure with uncolored blocks performed as well as those who saw the differently colored sections in Experiment 5, which suggests that any benefit derived from the segmented $4 \times 4 \times 4$ figure is the result of the subject's strategy for imaging the figure rather than the presence of color per se.
The apparent discrepancy between the results reported above and the finding in Experiment 1 that performance with a $3 \times 3 \times 3$ matrix was better than that with a structured $5 \times 5$ matrix may be attributable to performance characteristics for figures that do and do not exceed the imagery processing capacity. If, as claimed earlier, the $3 \times 3 \times 3$ matrix remains within the three-unit limit of imagery processing capacity, then performance with that figure should be superior to performance with any figure that exceeds processing capacity, including any threedimensional figure larger than $3 \times 3 \times 3$ and any twodimensional figure larger than $3 \times 3$. Adding structure to figures that exceed processing capacity may improve performance, but never to the near-perfect levels of performance for the $3 \times 3$ and $3 \times 3 \times 3$ figures. Comparisons of larger figures such as the $4 \times 4 \times 4$ and $8 \times 8$ matrices are essentially comparisons of relative processing deficits, and such comparisons will produce a natural advantage for figures such as the $3 \times 3 \times 3$ matrix, which show minimal if any processing deficits at all. Thus, adding a third dimension to the imagery processing task increases processing capacity from 9 to 27 distinct units, which accounts for superior performance with the $3 \times 3 \times 3$ figure. Once capacity has been exceeded, however, the advan-
tage for three-dimensional figures appears to be one of structural distinctiveness, and when structural distinctiveness is added to two-dimensional figures, the advantage disappears.
Although performance with the $3 \times 3 \times 3$ figure was similar to that with the $3 \times 3$ figure, the consistently high accuracy for both figures suggests the possibility that the failure to find differences was due at least in part to ceiling effects. Further evidence for ceiling effects comes from the finding that for all four experiments that included the comparison, performance with the $3 \times 3 \times 3$ matrix was lower than that for the $3 \times 3$ matrix, albeit nonsignificantly so. Thus, although the $3 \times 3 \times 3$ figure is equal to the $3 \times 3$ figure in processing capacity as defined in the present tasks, it is possible that different tasks or task characteristics could reveal differences. It has been suggested, for example, that imagery processing for three-dimensional stimuli may be slower than for stimuli depicted on the picture plane (e.g., Kerr, 1983). If this is the case, then increasing the speed with which verbal directions are presented in the present task might produce larger performance decrements for the $3 \times 3 \times 3$ than for the $3 \times 3$ matrix. Although this would not necessarily indicate a difference in processing capacity, it would clearly identify differences in processing efficiency.
When subjects were asked what strategy or approach they had used to keep track of pathways, most reported having used a visual-spatial strategy for both the pictureplane and three-dimensional matrices. The most common explanation for either task was, "I simply visualized where the pathway was in the figure." Some subjects reported that they imagined that each square or block "lit up"' or glowed as the pathway passed through it. The task with the blocks was frequently described as more difficult or effortful, but as long as the figure was no larger than $3 \times 3 \times 3$, subjects maintained their accuracy.
In addition to defining the limitations of locational representation in three-dimensional imagery processing, the experiments reported here replicate the findings of related two-dimensional experiments reported by Attneave and Curlee (1983). This replication is noteworthy in light of the many differences in the procedures of the two research projects; for example, (1) Attneave and Curlee's moving spot strategy was not suggested in the current research; (2) Attneave and Curlee used a 12 -step pathway, whereas the current studies used an 8 -step pathway; and (3) directions were read at a $.75-\mathrm{sec}$ rate in Attneave and Curlee's research and at much slower rates here. The replication of Attneave and Curlee's results indicates that their original finding was a robust one: two-dimensional matrices larger than $3 \times 3$ exceed the capacity of imagery processing.
Although Attneave and Curlee (1983) conducted their original experiment with materials the same size as most of those used here, they also conducted a subsequent experiment to ensure that the differences in performance on different-sized matrices were attributable to the number of matrix locations and not to "size" as defined by visual
angle. In their experiment, matrices were drawn at sizes well within the boundaries of visual angle that have been suggested by other researchers (e.g., Kosslyn, 1980; Weber \& Malmstrom, 1979) to define the limits of visual imagery capacity. Results under these conditions were similar to those of their first experiment and provided no evidence to suggest that larger matrices in the previous experiment had "overflowed" the limitations of visual angle. This finding is consistent with the findings of Experiment 5 that performance on the $4 \times 4 \times 4$ matrix remained relatively poor despite the fact that the size of this $4 \times 4 \times 4$ figure was smaller than the size of the $3 \times 3 \times 3$ matrices in previous experiments. Thus the imagery capacity measured by Attneave and Curlee and by the experiments reported here is better defined by the number of distinct locations in each spatial dimension than by absolute size or visual angle.

The finding that performance is essentially errorless for the $3 \times 3 \times 3$ figure clearly shows that subjects were able to perform the imagery task despite the fact that not all blocks in the three-dimensional figure were visible from the subject's point of view. This finding contradicts theories that limit the information that can be encoded in an image to the surfaces that would be visible to a subject from a particular point of view. Keenan (1983; Keenan \& Moore, 1979), for example, characterized imagery as a system incapable of directly encoding information about objects that are hidden from a subject's view by occlusion or some other form of concealment. By this account, information about concealed objects cannot be directly represented in an image, and if any information about such an object is encoded, it must be incorporated in one of two ways: in a nonimagery form such as a verbal or propositional representation, or as a part of the image that is made "visible" by a shift to a different point of view. Thus, subjects should "lose track" of pathways that pass through "concealed" blocks unless they manage to code the information about depth in propositional form, or to change their imaginal vantage point.

But, as described earlier, the results of Experiment 4 are inconsistent with a verbal-coding hypothesis for depth information, and the results of Experiment 6 directly contradict such a hypothesis. The finding in Experiment 6that keeping track of location in depth was significantly easier than keeping track of changes in temperaturesupports the view that in imagery, the depth dimension has no special status. Instead it is part of a coordinated imagery system that represents the relationships among objects in three-dimensional space.

Although it is possible for a subject to change the vantage point in imagining the $3 \times 3 \times 3$ figure so as to view the side and back blocks from an orientation in which they are directly visible, it seems unlikely that subjects employed this strategy for the present task. Subjects were instructed to imagine the block figures as viewed directly from the front so that the directions forward, back, left, and right would be interpreted with reference to the figure in its standard orientation. Subjects who shifted their van-
tage point to imagine the figure as viewed from the back would need to reinterpret the directions since the term "back" would then mean "front," and "left" would be "right." A view from the side would require an even more complicated set of transformations. Even if subjects could perform the task of transforming the verbal directions to correspond to each new vantage point (and my experience as an experimenter suggests that this is no mean feat), they would still be unable directly to view the blocks in the middle of each figure. Yet performance with three-dimensional figures never suffered significantly compared with performance with the fully visible twodimensional figures of comparable size. The results of the experiments are consistent with subjects' introspective reports that the figure was consistently imagined from a single vantage point facing the front of the figure.
The finding that subjects are able to imagine pathways that move through blocks that often are not simultaneously "visible" from a given point of view is important because it is based on a research paradigm that is resistant to the biasing effects of specific instructions or descriptions of what an image should be (Intons-Peterson, 1983). Previous controversy about whether images may include objects that are hidden from direct view has focused on questions about whether researchers have correctly "defined" for subjects what an image is or should be (Keenan, 1983; Keenan \& Moore, 1979), and whether they have created specific "demand characteristics" that produce the desired results (Kerr \& Neisser, 1983). However, the data here come from a research paradigm that shares the characteristics of Attneave and Curlee's (1983) paradigm, which Intons-Peterson identified as resistant to bias. The central feature recommended by Intons-Peterson is that imagery experiments use tasks that are difficult to perform without imagery. The most parsimonious explanation of the present data is that subjects perform the task by direct imaginal processing of three-dimensional space, and that imagery encodes all spatial relationships including those that produce visual occlusion or concealment.
The main finding-that adding a third dimension in a mental imagery task increases the capacity for locational representation-is consistent with theories proposing that imagery directly encodes information about threedimensional spatial relationships (e.g., Attneave, 1972; Neisser, 1978), and with research that indicates that objects that are visually behind or within some other object can be encoded in a visual image (Neisser \& Kerr, 1973; Kerr, 1983; Kerr \& Neisser, 1983-cf. Keenan, 1983; Keenan \& Moore, 1979). The data reported here support the characterization of imagery processing as similar to perceptual processing, and emphasize the potential of the imagery system for the efficient representation of threedimensional space.

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[^1]:    Method
    Subjects. Fourteen (8 female, 6 male) Oglethorpe University students participated to fulfill a psychology course requirement.
    Materials. A new set of blocks was constructed. Each block was $2.5 \mathrm{~cm}^{3}$ and was spray painted one of four colors: red, blue, green, or yellow. When the blocks were used to create the $3 \times 3 \times 4$ and

