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Variance of d' for the *same–different* method

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Variance estimates of d' are derived toward the *same–different* method based on Taylor-series expansion with one and two variables. The variance estimates can be used for statistical comparison of d's obtained from various discrimination paradigms. Formulas and tables for estimating variance of d' for the method are provided. One S-PLUS program, which can produce both d' and variance of d', is also provided.

The *same–different* method is one of the discrimination methods commonly used in sensory analysis and psychophysical research. The regular two-interval same–different method involves four sample pairs, of which two are concordant (AA or BB) and two are discordant (AB or BA). The observer's task is to report whether a sample pair that he or she receives on a trial is the same or different. The same–different method is different from the one-interval yes–no task (i.e., the A–not-A method), in which one of two samples A and B (not-A) is presented on each trial. The observer's task is to report whether a sample is A or B (not-A). Macmillan, Kaplan, and Creelman (1977) developed a Thurstonian model for the two-interval same–different method. Kaplan, Macmillan, and Creelman (1978) provided tables of d' for the method.

In Thurstonian models or the theory of signal detection (TSD), d' is a measure of sensory difference or discriminative sensitivity. It is theoretically unaffected by the criterion that subjects adopt or the method that is used to test them. Strictly speaking, δ, which is often used in Thurstonian models, should denote a true value of the measure, and d' is an estimate of δ, just as μ denotes a true mean and $\overline{X}$ denotes an estimate of μ. As a statistic, d' is also a variable and has its variance estimate. In order to compare statistically d's from the same paradigm or different paradigms—for example, to compare the two d's obtained from the A–not-A method and the same–different method, respectively—variances of the d's are needed. Gourevitch and Galanter (1967) gave estimates of the variance of d' for the yes–no task (i.e., the A–not-A method), and Miller (1996) obtained similar results using a different approach. Bi, Ennis, and O'Mahony (1997) provided estimates and tables for the variance of d' from four forced-choice methods: two-alternative forced choice (2-AFC), 3-AFC, triangular, and duo-trio. The purpose of this paper is to provide variance estimates of d' for the same–different method.

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Estimating the Variance of d'

Macmillan et al. (1977) derived the Thurstonian model for the same–different method shown in Equations 1 and 2:

$$P_{ss} = 2\Phi\left(\frac{k}{\sqrt{2}}\right) - 1, \tag{1}$$

and

$$P_{sd} = \Phi\left(\frac{k - d'}{\sqrt{2}}\right) - \Phi\left(\frac{-k - d'}{\sqrt{2}}\right), \tag{2}$$

where P_{ss} is the proportion of the "same" responses for the concordant sample pairs <AA> or <BB>; P_{sd} is the proportion of the "same" responses for the discordant sample pairs <AB> or <BA>; k is a criterion; d' is a measure of sensory difference or sensitivity; and $\Phi(.)$ is the standard normal distribution function. The estimate of d' can be numerically obtained from Equations 1 and 2. The model is based on a monadic design, in that it assumes that the responses in an experiment are independent of each other.

The estimate of variance of d' can also be derived from Equations 1 and 2. The main theoretical basis for obtaining the variance of d' is Taylor-series expansion with one and two variables. According to the Taylor-series expansion for a univariable, Equation 1 can be expressed as follows:

$$P_{ss} \approx p_{ss} + P'_{ss}\Big|_{k_0}\left(k - k_0\right), \tag{3}$$

where p_{ss} is an observed value of P_{ss}; $P'_{ss}|_{k_0}$ denotes the first derivative with respect to k evaluated at k_0;

$$k_0 = \sqrt{2}\Phi^{-1}\left(\frac{p_{ss}+1}{2}\right)$$

from Equation 1; and

$$\Phi^{-1}\left(\frac{p_{ss}+1}{2}\right)$$

denotes the

$$\frac{p_{ss}+1}{2}$$

quantile of the standard normal distribution. On the basis of Equation 1,

$$P'_{ss}\Big|_{k_0} = \sqrt{2}\phi\left(\frac{k_0}{\sqrt{2}}\right) \equiv u,$$

where

$$\phi\left(\frac{k_0}{\sqrt{2}}\right).$$

is the ordinate of the standard normal curve at

$$\left(\frac{k_0}{\sqrt{2}}\right).$$

From Equation 3, the variance of k can be obtained by the following:

$$V(k) \approx \frac{V(P_{ss})}{P'^2_{ss}} = \frac{p_{ss}(1-p_{ss})}{N_s u^2}, \tag{4}$$

where N_s is the sample size for the concordant sample pairs.

According to Taylor-series expansion with two variables, Equation 2 can be expressed as follows:

$$P_{sd} \approx p_{sd} + \frac{\partial P_{sd}}{\partial k}\Big|_{k_0,d'_0}\left(k-k_0\right) + \frac{\partial P_{sd}}{\partial d'}\Big|_{k_0,d'_0}\left(d'-d'_0\right), \tag{5A}$$

or,

$$\frac{\partial P_{sd}}{\partial d'}\Big|_{k_0,d'_0}\left(d'-d'_0\right) \approx \left(P_{sd}-p_{sd}\right) - \frac{\partial P_{sd}}{\partial k}\Big|_{k_0,d'_0}\left(k-k_0\right), \tag{5B}$$

where p_{sd} is an observed value of P_{sd};

$$\frac{\partial P_{sd}}{\partial k}\Big|_{k_0,d'_0}$$

and

$$\frac{\partial P_{sd}}{\partial d'}\Big|_{k_0,d'_0}$$

are partial derivatives with respect to k and d' evaluated at k_0 and d'_0. d'_0 is an estimate of d'.

On the basis of Equation 2,

$$\frac{\partial P_{sd}}{\partial k}\Big|_{k_0,d'_0} = \phi\left(\frac{k_0-d'_0}{\sqrt{2}}\right)\frac{1}{\sqrt{2}} + \phi\left(\frac{-k_0-d'_0}{\sqrt{2}}\right)\frac{1}{\sqrt{2}} \equiv v$$

and

$$\frac{\partial P_{sd}}{\partial d'}\Big|_{k_0,d'_0} = -\phi\left(\frac{k_0-d'_0}{\sqrt{2}}\right)\frac{1}{\sqrt{2}} + \phi\left(\frac{-k_0-d'_0}{\sqrt{2}}\right)\frac{1}{\sqrt{2}} \equiv w.$$

Under the assumption that P_{sd} is independent of P_{ss}, P_{sd} is also independent of k. From Equation 5B, the variance of d' can be obtained by the following:

$$V(d') = \frac{V(P_{sd}) + \left(\frac{\partial P_{sd}}{\partial k}\Big|_{k_0,d'_0}\right)^2 V(k)}{\left(\frac{\partial P_{sd}}{\partial d'}\Big|_{k_0,d'_0}\right)^2}. \tag{6}$$

Since

$$V(P_{sd}) = \frac{p_{sd}\left(1-p_{sd}\right)}{N_d},$$

where N_d is the sample size for the discordant sample pairs, the variance of d' in Equation 6 can be estimated as follows:

$$V\left(d'\right) = \frac{B_d}{N_d} + \frac{B_s}{N_s}, \tag{7}$$

where

$$B_d = \frac{p_{sd}(1-p_{sd})}{w^2}$$

and

$$B_s = \frac{v^2 p_{ss}(1-p_{ss})}{w^2 u^2}.$$

B_s reflects the variation from the concordant sample pairs, whereas B_d reflects the variation from the discordant sample pairs.

The d' can be estimated from Equations 1 and 2 by using the two observed proportions p_{ss} and p_{sd}. The variances of d' can be estimated from Equation 7 by using p_{ss}, p_{sd}, N_s, and N_d. It is assumed that $P_{ss} > P_{sd}$. If the observed proportion of P_{ss} is smaller than the observed proportion of P_{sd}, it is assumed that the true $d' = 0$.

One S-PLUS program is given in the Appendix for estimating both d' and the variance of d'. There are four arguments for the program: x_{ss}, N_s, x_{sd} and N_d, where x_{ss} is the number of "same" responses for the concordant sample pairs and x_{sd} is the number of "same" responses for the discordant sample pairs. Here

$$\frac{x_{ss}}{N_s} = p_{ss}$$

and

$$\frac{x_{sd}}{N_d} = p_{sd}.$$

Tables for the Variance of d'

Tables 1A–1B provide variance estimates of d' for the same–different method. The tables can be used to get a variance estimate of d' quickly, without one's having to compute them. The tables give B_s and B_d values, the main components of variance of d'. The variance of d' can be obtained easily by dividing the B_s and B_d values with N_s and N_d. Tables 1A–1B give the B_s and B_d values corresponding to different ranges of p_{ss} and p_{sd}.

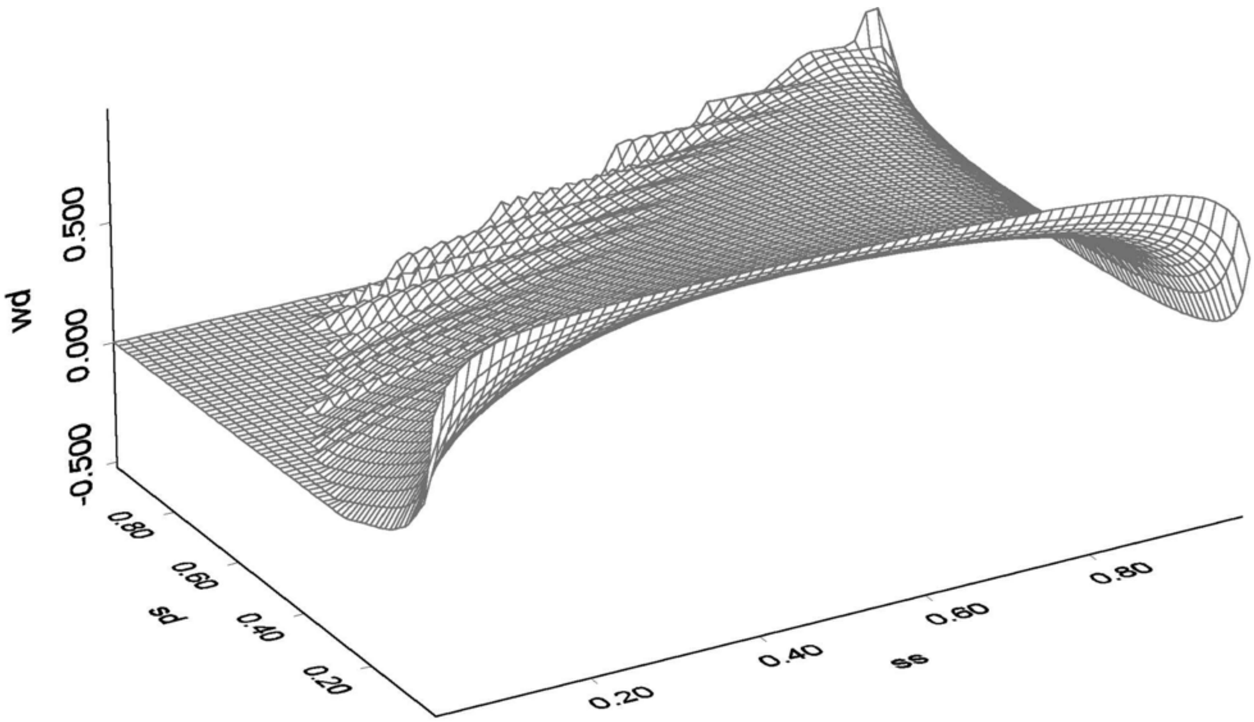

Figure 1. The weight of variation of d' from discordant sample pairs. The weight is defined by $w_d = B_d / (B_s + B_d)$. B_s and B_d are components of variance of d'. B_s reflects the variation from the concordant sample pairs, whereas B_d reflects the variation from the discordant sample pairs.

Table 1A corresponds to p_{sd} from .01 to .11 with a step of .01 and from .11 to .19 with a step of .02, as well as corresponds to p_{ss} from .02 to .2 with a step of .02, from .25 to .8 with a step of .05, from .81 to .89 with a step of .02, and from .9 to .99 with a step of .01. Table 1B corresponds to p_{sd} from .2 to .80 with a step of .05 and from .8 to .98 with a step of .02, as well as corresponds to p_{ss} from .25 to .8 with a step of .05, from .81 to.89 with a step of .02, and from .9 to .99 with a step of .01. For values p_{ss} and p_{sd} that cannot be found in the tables, linear interpolation may be used to compute the B values. However, a good approximate can be obtained from the tables.

From the tables we can find the trend that in most cases, especially when p_{sd} is smaller, B_d is larger than B_s. This relationship means that the variance of d' in the same–different test is mainly determined by performance on the discordant sample pairs. Hence, in order to reduce variance of d' in the test, sample size for the discordant sample pairs should generally be larger than that for the concordant sample pairs. However, when p_{ss} is close to 1, the situation is different. Here B_s is larger than B_d. In that situation, sample size for the concordant pairs should be more important than that of the discordant pairs in order to reduce variance of d'. Figure 1 shows the weight of variation from discordant sample pairs in the total variation. The weight, defined by

$$w_d = \frac{B_d}{B_s + B_d},$$

measures the relative importance of B_d in the total B values. The weight (the *wd* axis in Figure 1) lies between zero and one, $0 < w_d \leq 1$. It is large when P_{sd} (the *sd* axis in Figure 1) is close to zero, whereas it is small when P_{ss} (the *ss* axis in Figure 1) is close to one.

Examples

In a same–different test, $N_s = 100$ concordant sample pairs (50 AA and 50 BB) and $N_d = 200$ discordant sample pairs (100 AB and 100 BA) are presented. $x_{ss} = 17$ and $x_{sd} = 18$ are observed. Hence $p_{ss} = .17$ and $p_{sd} = .09$. From the tables in Kaplan et al. (1978) or from Equations 1 and 2, $d' = 1.61$ can be obtained. From Equation 7, the exact estimate of variance of d' at 1.61 is $V(d') = 0.162$. The variance can also be obtained from Table 1A. We can find from Table 1A that for $p_{ss} = .16$ and $p_{sd} = .09$, the B values are $B_s = 9.54$ and $B_d = 17.81$. The variance of d' is

$$V(d') = \frac{9.54}{100} + \frac{17.81}{200} = 0.184.$$

Table 1A
***B* Values (B_s and B_d) for Variance of d' From the Same–Different Method**

p_{ss}	p_{sd} .01	.02	.03	.04	.05	.06	.07	.08	.09	.10	.11	.13	.15	.17	.19
.02	70.75														
	142.86														
.04	17.41	34.73	83.58												
	71.47	70.75	112.49												
.06	8.88	14.41	22.76	38.83	86.21										
	55.36	44.69	46.74	59.30	104.41										
.08	5.71	8.48	11.92	16.80	24.71	40.27	86.57								
	47.77	35.46	33.08	34.74	40.56	54.64	99.83								
.10	4.12	5.81	7.71	10.07	13.24	17.90	25.56	40.74	86.06						
	43.22	30.61	27.00	26.33	27.56	30.83	37.44	51.81	96.47						
.12	3.20	4.35	5.56	6.95	8.67	10.89	13.94	18.45	25.91	40.76	85.15				
	40.14	27.55	23.50	22.01	21.87	22.77	24.84	28.58	35.41	49.74	93.70				
.14	2.62	3.45	4.29	5.22	6.29	7.59	9.22	11.35	14.31	18.71	26.00	84.02			
	37.90	25.44	21.21	19.36	18.64	18.68	19.37	20.76	23.12	27.03	33.90	91.25			
.16	2.22	2.86	3.48	4.14	4.87	5.72	6.73	7.97	9.54	11.62	14.50	25.92	82.75		
	36.18	23.88	19.58	17.55	16.56	16.19	16.29	16.82	17.81	19.41	21.89	32.67	89.01		
.18	1.94	2.44	2.92	3.41	3.95	4.55	5.24	6.04	7.02	8.21	9.74	14.57	25.75	81.43	
	34.83	22.68	18.35	16.23	15.09	14.51	14.31	14.43	14.84	15.58	16.71	20.92	31.62	86.92	
.20	1.74	2.14	2.52	2.90	3.31	3.76	4.26	4.82	5.48	6.26	7.20	9.85	14.56	25.50	80.03
	33.74	21.73	17.40	15.23	14.00	13.29	12.93	12.82	12.93	13.26	13.82	15.87	20.12	30.69	84.91
.25	1.43	1.69	1.92	2.14	2.38	2.62	2.88	3.17	3.48	3.83	4.22	5.19	6.52	8.49	11.75
	31.75	20.04	15.75	13.53	12.20	11.35	10.79	10.43	10.23	10.16	10.19	10.58	11.47	13.09	16.07
.30	1.28	1.45	1.61	1.76	1.91	2.06	2.22	2.39	2.57	2.76	2.97	3.46	4.06	4.84	5.89
	30.45	18.94	14.71	12.48	11.12	10.21	9.57	9.13	8.81	8.60	8.46	8.40	8.58	9.03	9.80
.35	1.22	1.34	1.45	1.55	1.65	1.76	1.86	1.97	2.08	2.21	2.33	2.62	2.96	3.36	3.86
	29.57	18.21	14.01	11.79	10.41	9.47	8.81	8.31	7.95	7.67	7.47	7.22	7.15	7.23	7.46
.40	1.22	1.31	1.38	1.45	1.52	1.60	1.67	1.74	1.82	1.90	1.98	2.17	2.38	2.62	2.90
	28.96	17.70	13.53	11.31	9.92	8.98	8.29	7.78	7.38	7.07	6.83	6.49	6.31	6.23	6.26
.45	1.26	1.32	1.37	1.42	1.47	1.52	1.57	1.63	1.68	1.74	1.79	1.92	2.06	2.21	2.39
	28.55	17.34	13.19	10.98	9.59	8.63	7.93	7.40	6.99	6.67	6.40	6.02	5.76	5.61	5.54
.50	1.33	1.37	1.41	1.44	1.48	1.52	1.55	1.59	1.63	1.67	1.71	1.79	1.89	1.99	2.11
	28.28	17.11	12.96	10.75	9.35	8.39	7.68	7.15	6.72	6.38	6.11	5.69	5.40	5.20	5.07
.55	1.43	1.46	1.48	1.51	1.53	1.56	1.58	1.61	1.63	1.66	1.69	1.75	1.82	1.89	1.97
	28.11	16.95	12.81	10.59	9.19	8.22	7.51	6.97	6.54	6.19	5.90	5.46	5.15	4.92	4.76
.60	1.56	1.58	1.60	1.61	1.63	1.64	1.66	1.68	1.70	1.72	1.74	1.78	1.82	1.87	1.93
	28.00	16.85	12.70	10.48	9.08	8.11	7.39	6.84	6.41	6.05	5.76	5.31	4.98	4.73	4.54

Table 1A (Continued)

p_{ss}	p_{sd} .01	.02	.03	.04	.05	.06	.07	.08	.09	.10	.11	.13	.15	.17	.19
.65	1.73	1.74	1.75	1.76	1.77	1.78	1.79	1.80	1.81	1.83	1.84	1.87	1.90	1.93	1.97
	27.93	16.79	12.64	10.42	9.01	8.04	7.32	6.76	6.32	5.96	5.67	5.20	4.86	4.60	4.40
.70	1.94	1.94	1.95	1.96	1.96	1.97	1.97	1.98	1.99	2.00	2.00	2.02	2.04	2.06	2.09
	27.90	16.75	12.60	10.38	8.97	7.99	7.27	6.71	6.27	5.91	5.61	5.13	4.78	4.51	4.30
.75	2.22	2.22	2.22	2.22	2.23	2.23	2.23	2.24	2.24	2.25	2.25	2.26	2.27	2.28	2.30
	27.88	16.73	12.58	10.36	8.95	7.97	7.24	6.68	6.24	5.87	5.57	5.09	4.73	4.46	4.24
.80	2.60	2.60	2.60	2.60	2.60	2.60	2.61	2.61	2.61	2.61	2.61	2.62	2.62	2.63	2.64
	27.88	16.73	12.58	10.35	8.94	7.96	7.23	6.67	6.22	5.86	5.55	5.07	4.71	4.43	4.21
.81	2.69	2.70	2.70	2.70	2.70	2.70	2.70	2.70	2.70	2.71	2.71	2.71	2.72	2.72	2.73
	27.88	16.72	12.57	10.35	8.94	7.95	7.23	6.67	6.22	5.85	5.55	5.07	4.71	4.42	4.20
.83	2.91	2.91	2.92	2.92	2.92	2.92	2.92	2.92	2.92	2.92	2.92	2.93	2.93	2.93	2.94
	27.88	16.72	12.57	10.34	8.93	7.95	7.23	6.66	6.22	5.85	5.54	5.06	4.70	4.42	4.19
.85	3.18	3.18	3.18	3.18	3.18	3.18	3.18	3.18	3.19	3.19	3.19	3.19	3.19	3.19	3.20
	27.87	16.72	12.57	10.34	8.93	7.95	7.22	6.66	6.21	5.85	5.54	5.06	4.70	4.41	4.19
.87	3.52	3.52	3.52	3.52	3.52	3.52	3.52	3.52	3.52	3.52	3.52	3.52	3.52	3.53	3.53
	27.87	16.72	12.57	10.34	8.93	7.95	7.22	6.66	6.21	5.85	5.54	5.06	4.69	4.41	4.18
.89	3.96	3.96	3.96	3.96	3.96	3.96	3.96	3.96	3.96	3.96	3.96	3.96	3.96	3.96	3.96
	27.88	16.72	12.57	10.34	8.93	7.95	7.22	6.66	6.21	5.85	5.54	5.06	4.69	4.41	4.18
.90	4.23	4.23	4.23	4.23	4.23	4.23	4.23	4.23	4.23	4.23	4.23	4.23	4.23	4.23	4.23
	27.88	16.72	12.57	10.34	8.93	7.95	7.22	6.66	6.21	5.84	5.54	5.06	4.69	4.41	4.18
.91	4.56	4.56	4.56	4.56	4.56	4.56	4.56	4.56	4.56	4.56	4.56	4.56	4.56	4.56	4.56
	27.87	16.72	12.57	10.34	8.93	7.95	7.22	6.66	6.21	5.84	5.54	5.06	4.69	4.41	4.18
.92	4.96	4.96	4.96	4.96	4.96	4.96	4.96	4.96	4.96	4.96	4.96	4.96	4.96	4.96	4.96
	27.87	16.72	12.57	10.34	8.93	7.95	7.22	6.66	6.21	5.84	5.54	5.05	4.69	4.41	4.18
.93	5.45	5.45	5.45	5.45	5.45	5.45	5.45	5.45	5.45	5.45	5.45	5.45	5.45	5.45	5.45
	27.87	16.72	12.57	10.34	8.93	7.95	7.22	6.66	6.21	5.84	5.54	5.05	4.69	4.41	4.18
.94	6.09	6.09	6.09	6.09	6.09	6.09	6.09	6.09	6.09	6.09	6.09	6.09	6.09	6.09	6.09
	27.87	16.72	12.57	10.34	8.93	7.95	7.22	6.66	6.21	5.84	5.54	5.05	4.69	4.41	4.18
.95	6.95	6.95	6.95	6.95	6.95	6.95	6.95	6.95	6.95	6.95	6.95	6.95	6.95	6.95	6.95
	27.88	16.72	12.57	10.34	8.93	7.95	7.22	6.66	6.21	5.84	5.54	5.05	4.69	4.41	4.18
.96	8.19	8.19	8.19	8.19	8.19	8.19	8.19	8.19	8.19	8.19	8.19	8.19	8.19	8.19	8.19
	27.88	16.72	12.57	10.34	8.93	7.95	7.22	6.66	6.21	5.84	5.54	5.05	4.69	4.41	4.18
.97	10.15	10.15	10.15	10.15	10.15	10.15	10.15	10.15	10.15	10.15	10.15	10.15	10.15	10.15	10.15
	27.87	16.72	12.57	10.34	8.93	7.95	7.22	6.66	6.21	5.84	5.54	5.05	4.69	4.41	4.18
.98	13.80	13.80	13.80	13.80	13.80	13.80	13.80	13.80	13.80	13.80	13.80	13.80	13.80	13.80	13.80
	27.87	16.72	12.57	10.34	8.93	7.95	7.22	6.66	6.21	5.84	5.54	5.05	4.69	4.41	4.18
.99	23.67	23.67	23.67	23.67	23.67	23.67	23.67	23.67	23.67	23.67	23.67	23.67	23.67	23.67	23.67
	27.87	16.72	12.57	10.34	8.93	7.95	7.22	6.66	6.21	5.84	5.54	5.05	4.69	4.41	4.18

Note—p_{ss} = The proportion of the "same" responses for the concordant sample pairs (AA or BB). p_{sd} = The proportion of the "same" responses for the discordant sample pairs (AB or BA). There are two B values, B_s and B_d, corresponding to a same pair of p_{ss} and p_{sd}. The variance is then obtained by dividing the B values with corresponding sample sizes,

$$V(d') = \frac{B_s}{N_s} + \frac{B_d}{N_d},$$

where N_s is the number of concordant sample pairs and N_d is the number of discordant sample pairs.

Table 1B
***B* Values (B_s and B_d) for Variance of d' From the Same–Different Method**

	p_{sd}																					
p_{ss}	.20	.25	.30	.35	.40	.45	.50	.55	.60	.65	.70	.75	.80	.82	.84	.86	.88	.90	.92	.94	.96	.98
.25	14.35																					
	18.54																					
.30	6.57	13.96																				
	10.36	17.29																				
.35	4.16	6.51	13.50																			
	7.65	9.54	16.21																			
.40	3.06	4.18	6.40	12.99																		
	6.31	6.98	8.88	15.24																		
.45	2.49	3.12	4.17	6.25	12.46																	
	5.52	5.73	6.47	8.32	14.35																	
.50	2.17	2.57	3.16	4.13	6.08	11.91																
	5.03	5.00	5.29	6.04	7.81	13.51																
.55	2.01	2.27	2.63	3.17	4.08	5.90	11.36															
	4.69	4.54	4.61	4.93	5.67	7.36	12.71															
.60	1.95	2.13	2.36	2.69	3.19	4.03	5.72	10.80														
	4.47	4.23	4.18	4.30	4.64	5.35	6.93	11.93														
.65	1.99	2.10	2.25	2.46	2.76	3.22	3.99	5.55	10.25													
	4.31	4.03	3.90	3.91	4.05	4.39	5.06	6.54	11.18													
.70	2.10	2.17	2.27	2.40	2.59	2.86	3.27	3.97	5.39	9.71												
	4.21	3.89	3.72	3.66	3.69	3.85	4.17	4.80	6.17	10.44												
.75	2.31	2.35	2.41	2.49	2.61	2.76	3.00	3.36	3.99	5.27	9.20											
	4.15	3.80	3.60	3.50	3.47	3.53	3.68	3.99	4.57	5.82	9.72											
.80	2.64	2.67	2.70	2.75	2.81	2.90	3.03	3.23	3.54	4.09	5.22	8.72										
	4.11	3.75	3.53	3.40	3.34	3.34	3.40	3.55	3.84	4.37	5.51	9.02										
.81	2.73	2.75	2.78	2.82	2.88	2.96	3.08	3.25	3.52	3.99	4.92	7.49	36.38									
	4.11	3.75	3.52	3.39	3.32	3.31	3.36	3.49	3.74	4.20	5.14	7.73	36.65									
.83	2.94	2.96	2.98	3.01	3.05	3.11	3.20	3.33	3.54	3.88	4.53	6.05	12.86	34.85								
	4.10	3.74	3.51	3.36	3.29	3.26	3.30	3.39	3.59	3.93	4.59	6.15	13.00	35.02								
.85	3.20	3.21	3.23	3.25	3.28	3.33	3.39	3.49	3.64	3.90	4.35	5.32	8.36	12.49	33.31							
	4.09	3.73	3.50	3.35	3.26	3.23	3.25	3.32	3.47	3.73	4.22	5.23	8.34	12.51	33.36							
.87	3.53	3.54	3.55	3.56	3.59	3.62	3.67	3.74	3.85	4.03	4.35	4.98	6.63	8.27	12.15	31.74						
	4.09	3.72	3.49	3.34	3.24	3.20	3.21	3.26	3.38	3.59	3.95	4.66	6.40	8.08	12.01	31.65						

Table 1B (Continued)

p_{ss}	p_{sd} .20	.25	.30	.35	.40	.45	.50	.55	.60	.65	.70	.75	.80	.82	.84	.86	.88	.90	.92	.94	.96	.98
.89	3.96	3.97	3.97	3.98	4.00	4.02	4.05	4.10	4.18	4.31	4.52	4.94	5.92	6.74	8.25	11.86	30.15					
	4.09	3.72	3.48	3.33	3.23	3.18	3.18	3.22	3.31	3.48	3.76	4.28	5.38	6.26	7.84	11.52	29.89					
.90	4.24	4.24	4.24	4.25	4.26	4.28	4.31	4.35	4.41	4.51	4.69	5.03	5.78	6.39	7.43	9.57	16.13					
	4.08	3.72	3.48	3.32	3.23	3.18	3.17	3.20	3.29	3.44	3.69	4.14	5.04	5.71	6.83	9.06	15.71					
.91	4.56	4.56	4.57	4.57	4.58	4.60	4.62	4.65	4.70	4.78	4.92	5.19	5.78	6.23	6.97	8.35	11.66	28.55				
	4.08	3.72	3.48	3.32	3.22	3.17	3.16	3.19	3.27	3.41	3.64	4.03	4.78	5.31	6.14	7.62	11.05	28.06				
.92	4.96	4.96	4.96	4.97	4.97	4.98	5.00	5.02	5.06	5.13	5.24	5.45	5.90	6.24	6.77	7.70	9.64	15.63				
	4.08	3.71	3.48	3.32	3.22	3.17	3.16	3.18	3.25	3.38	3.59	3.94	4.58	5.01	5.64	6.69	8.76	14.90				
.93	5.45	5.45	5.46	5.46	5.46	5.47	5.48	5.50	5.53	5.58	5.66	5.82	6.17	6.42	6.80	7.45	8.67	11.65	27.00			
	4.08	3.71	3.48	3.32	3.22	3.17	3.15	3.17	3.24	3.36	3.55	3.87	4.42	4.78	5.28	6.06	7.44	10.60	26.17			
.94	6.09	6.09	6.09	6.10	6.10	6.10	6.11	6.12	6.14	6.18	6.24	6.36	6.61	6.80	7.07	7.52	8.32	10.01	15.35			
	4.08	3.71	3.47	3.32	3.22	3.16	3.15	3.17	3.23	3.34	3.52	3.81	4.30	4.60	5.02	5.63	6.61	8.52	14.12			
.95	6.95	6.95	6.95	6.96	6.96	6.96	6.97	6.97	6.99	7.01	7.05	7.14	7.31	7.44	7.63	7.94	8.46	9.49	12.06	25.64		
	4.08	3.71	3.47	3.32	3.22	3.16	3.14	3.16	3.22	3.33	3.50	3.77	4.21	4.48	4.83	5.32	6.06	7.35	10.23	24.20		
.96	8.19	8.19	8.19	8.19	8.19	8.19	8.20	8.20	8.21	8.22	8.25	8.30	8.42	8.50	8.63	8.83	9.16	9.78	11.16	15.70		
	4.08	3.71	3.47	3.32	3.22	3.16	3.14	3.16	3.22	3.32	3.49	3.74	4.15	4.39	4.70	5.11	5.70	6.64	8.41	13.44		
.97	10.15	10.15	10.15	10.15	10.15	10.15	10.15	10.15	10.16	10.16	10.18	10.21	10.27	10.32	10.40	10.51	10.71	11.07	11.81	13.82	25.11	
	4.08	3.71	3.47	3.32	3.22	3.16	3.14	3.16	3.22	3.32	3.48	3.73	4.11	4.33	4.61	4.98	5.47	6.21	7.44	10.08	22.26	
.98	13.80	13.80	13.80	13.80	13.80	13.80	13.80	13.80	13.80	13.80	13.81	13.82	13.85	13.87	13.91	13.97	14.06	14.23	14.59	15.48	18.76	
	4.08	3.71	3.47	3.32	3.22	3.16	3.14	3.16	3.22	3.32	3.48	3.72	4.09	4.30	4.56	4.90	5.34	5.97	6.93	8.65	13.19	
.99	23.67	23.67	23.67	23.67	23.67	23.67	23.67	23.67	23.68	23.68	23.68	23.68	23.69	23.69	23.70	23.72	23.74	23.80	23.90	24.18	25.09	32.12
	4.08	3.71	3.47	3.32	3.22	3.16	3.14	3.16	3.22	3.32	3.47	3.71	4.08	4.29	4.54	4.87	5.29	5.86	6.70	8.07	10.80	21.47

Note—p_{ss} = The proportion of the "same" responses for the concordant sample pairs (AA or BB). p_{sd} = The proportion of the "same" responses for the discordant sample pairs (AB or BA). There are two B values, B_s and B_d, corresponding to a same pair of p_{ss} and p_{sd}. The variance is then obtained by dividing the B values with corresponding sample sizes,

$$V(d') = \frac{B_s}{N_s} + \frac{B_d}{N_d},$$

where N_s is the number of concordant sample pairs and N_d is the number of discordant sample pairs.

For p_{ss} = .18 and p_{sd} = .09, the B values are B_s = 7.02 and B_d = 14.84. The variance of d' is

$$V(d') = \frac{7.02}{100} + \frac{14.84}{200} = 0.144.$$

Hence a good approximate of the variance of d' at 1.61 is a value between 0.184 and 0.144. It is about 0.164, which is close to the exact estimate value 0.162.

In another same–different test, N_s = 200 and N_d = 400, x_{ss} = 134, and x_{sd} = 52 are observed. We get p_{ss} = .67 and p_{sd} = .13. From the tables in Kaplan et al. (1978) or from Equations 1 and 2, d' = 2.96. The exact variance estimate of d' is 0.023 from Equation 7. In Table 1A, the closest proportions are p_{ss} = .65 and p_{sd} = .13, the corresponding B values are B_s = 1.87 and B_d = 5.2. The approximate estimate of d' at 2.96 is 0.022. It is very close to the exact estimate value (0.023).

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APPENDIX
S-PLUS Code for Estimating d' and Variance of d' for the Same–Different Method

Function Name
    sddvn

Description
    Returns estimates of d' and variance of d' for the same–different method.

Usage
    sddvn (sn, n1, dn, n2)

Required Arguments
    sn: The number of "same" responses for concordant sample pairs
    n: The number of concordant sample pairs
    dn: The number of "same" responses for discordant sample pairs
    n2: The number of discordant sample pairs

Output
    Values of d' and variance of d'

Examples

```
> sddvn(17,100,18,200)
[1] 1.607 0.162
> sddvn(134,200,52,400)
[1] 2.963 0.023
```

```
S-PLUS Code
sddvn
function(sn, n1, dn, n2)
{
# d' and variance of d' for same-different method.
     s <- sn/n1
     d <- dn/n2
     p1 <- s + (1 - s)/2
     ta <- qnorm(p1) * sqrt(2)
     del <- function(delta, ta, d)
     {
       pnorm((ta - delta)/sqrt(2)) - pnorm(( - ta -
     delta)/sqrt(2)) - d
     }
     delta <- uniroot(del, interval = c(0, 10), ta = ta, d
     = d)
     delta <- delta[[1]]
     fi <- dnorm(qnorm(p1))
     fi <- fi^2
     btau <- (s * (1 - s))/(2 * n1 * fi)
```

APPENDIX (Continued)

```
    bvp2 <- (d * (1 - d))/n2
    a <- (ta - delta)/sqrt(2)
    b <- ( - ta - delta)/sqrt(2)
    dx <- - dnorm(a)/sqrt(2) + dnorm(b)/sqrt(2)
    dy <- dnorm(a)/sqrt(2) + dnorm(b)/sqrt(2)
    bdelta <- bvp2/dx^2 + ((dy^2) * btau)/(dx^2)
    dv <- c(delta, bdelta)
    dv <- round(dv, 3)
    dv
}
```

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