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Location perception: The *X-Files* parable

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Three aspects of visual object location were investigated: (1) how the visual system integrates information for locating objects, (2) how attention operates to affect location perception, and (3) how the visual system deals with locating an object when multiple objects are present. The theories were described in terms of a parable (the *X-Files parable*). Then, computer simulations were developed. Finally, predictions derived from the simulations were tested. In the scenario described in the parable, we ask how a system of detectors might locate an alien spaceship, how attention might be implemented in such a spaceship detection system, and how the presence of one spaceship might influence the location perception of another alien spaceship. Experiment 1 demonstrated that location information is integrated with a spatial average rule. In Experiment 2, this rule was applied to a more-samples theory of attention. Experiment 3 demonstrated how the integration rule could account for various visual illusions.

The perception of an object's location is probably one of the most fundamental acts of perception, influencing everything from object recognition to action. For example, in object processing, the perception of "what" and "where" may be somewhat anatomically separate (see, e.g., Mishkin, Ungerleider, & Macko, 2001). Yet, the identification of an object (i.e., "what") is almost always accompanied by some idea of the object's location (i.e., "where"; see, e.g., Hazeltine, Prinzmetal, & Elliot, 1997; Johnston & Pashler, 1990). The perception of location may have a special status in attention (see, e.g., Tsal & Lavie, 1993). Finally, although there might be instances in which perception and action can be dissociated (Goodale & Milner, 1992), there is generally a close correspondence between where we perceive an object to be and our actions toward that object.

The objectives of this study were threefold. The first was to understand the basic mechanisms of location perception. Two contrasting models of the mechanisms of object location were developed and tested. Both models specified how information from a population of "detectors," each with a different "receptive field," is combined to yield the perception of a unique location of an object. The second goal was to understand how attention affects location perception. Two classes of models of attention and location perception were compared. One of the models invoked smaller receptive fields and tighter tuning functions as the mechanism of attention (Desimone & Duncan, 1995). The other model postulated "more samples" as the mechanism whereby attention affects location perception (see, e.g., Luce, 1977). The final goal was to understand how the presence of one object affects the perceived location of another object. Again, two classes of models were compared. The consequence of one of these classes of models is that the perceived location of one object will be repelled from a landmark object. The other class of models predicts that the perceived location of an object will be attracted to a landmark object.

For each of these three issues, the analysis proceeded in four steps. The first step consisted of a verbal description of two alternative theories. For example, two descriptions of the mechanisms of attention were developed. The theories were described in terms of a story called the *X-Files parable*. The parable was inspired by the television show of the same name. Different versions of the parable will be described in detail later, but in general it goes as follows: An alien spacecraft lands somewhere in the United States. The task of the FBI is to decide where the spacecraft landed so that they can send their agents out to investigate. Simply put, each competing model was first described in a metaphor.

The inspiration for describing a formal model in terms of a loose parable is Pandemonium, which was one of the first cognitive theories simulated by computer (Selfridge, 1959). The reason for using a parable is that, although the models involve neurally plausible mechanisms, I do not want to equate a particular theoretical mechanism with a particular neural structure. Thus, although the theory uses constructs such as detectors and receptive fields, these detectors are not necessarily the detectors in physiological areas V1 or V2. Indeed, the aim is to outline and test general design features of the visual system that enable us to locate objects in space.

The second step in the analysis was to perform computer simulations of each parable. The output of the sim-

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ulations was the distribution of locations that the FBI would investigate in a given situation. Hundreds of simulations were performed for each *X-Files* story. The results in some cases were rather surprising: The models performed in ways that were at odds with claims in the literature about how these models behave.

As the third step, a mathematical description of the consequences of each theory was inferred from the simulations. The mathematical description involved statistics not often used in psychological research. For example, in deciding between models of the integration of information from different detectors, the kurtosis (peakedness) of the response distributions was critical. The final step, of course, was to conduct an experiment to determine which theory provided the best fit of the data on human location perception.

In each experiment, data were collected using the method of reproduction (see Huttenlocher, Hedges, & Duncan, 1991; Newby & Rock, 2001; Prinzmetal, Amiri, Allen, & Edwards, 1998; Prinzmetal, Nwachuku, Bodanski, Blumenfeld, & Shimizu, 1997; Prinzmetal & Wilson, 1997; Tsal, Meiran, & Lamy, 1995). On each trial, observers were briefly presented with a target dot. The observers responded by moving a cursor (with a mouse) to the location where they thought the target had appeared. The method provides very rich data. In the present study, the variance and shape of the distribution of location responses was used to test different theories of location perception, attention, and the effect of landmarks.

The method of reproduction was used by Prinzmetal and colleagues in an extensive series of experiments to study attention (Prinzmetal et al., 1998; Prinzmetal et al., 1997; Prinzmetal & Wilson, 1997). For example, Prinzmetal et al. (1997, Experiments 5 and 6) examined the effect of attention on the perception of location. On each trial, observers were briefly presented with a small gray target dot. The task was to locate the dot by moving a mouse cursor to the position of the dot. Figure 1 illustrates the distribution of a typical observer's responses around the target location (plotted at the origin) in two conditions. The goal of Experiment 1 of the present study was to develop a theory that accounted for the statistical properties of distributions such at those shown in Figure 1. A model that successfully accounts for the shape of the distribution of responses was a critical first step in understanding other aspects of location perception.

In the experiments by Prinzmetal et al. (1997), attention affected the variance of location responses. The left panel of Figure 1 illustrates the distribution of responses under divided attention, and the right panel shows the distribution of responses under focused attention. Prinzmetal and colleagues also tested the effect of attention with a number of other stimulus attributes, including color (hue), line orientation, line length, spatial frequency, brightness, and contrast (Prinzmetal et al., 1998; Prinzmetal et al., 1997; Prinzmetal & Wilson, 1997). In every case, attention affected the variance of responses but not the mean response (however, also see Festinger, Coren, & Rivers, 1970; Tsal, Shalev, Zakay, & Lubow, 1994). The second goal of the present study was to develop and test an account of how attention affected the variance of location responses.

In the final section of this study, I investigated how the presence of a distractor object (a landmark) affected the location perception of a target object. The model that best described the results may have application in describing illusions as diverse as the Müller-Lyer illusion and the gravity lens illusion (Greene, 1998; Naito & Cole,

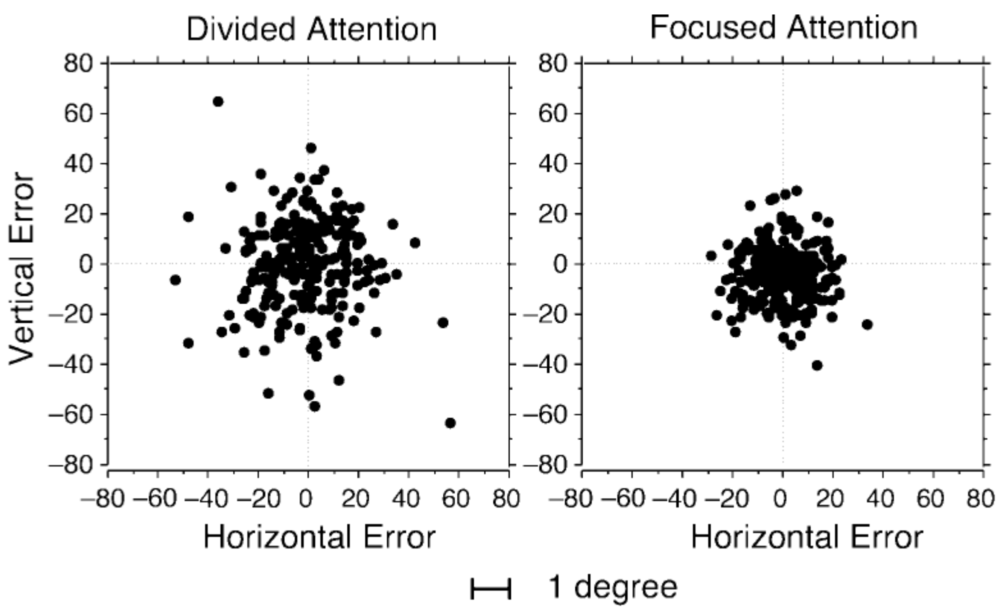

Figure 1. Location errors, in screen pixels, from Prinzmetal, Nwachuku, Bodansky, Blumenfeld, and Shimizu (1997). Although the stimuli were in randomly chosen locations, the responses are plotted as if the target were at the origin.

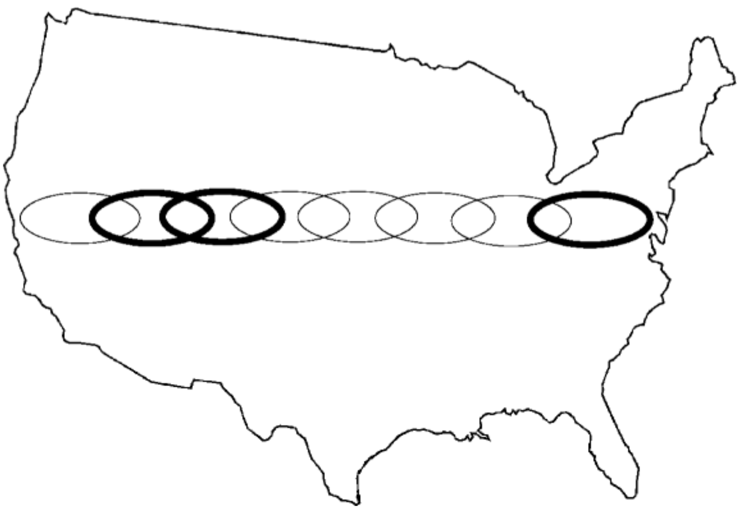

Figure 2. A series of alien detectors in a horizontal array across the United States. Each detector has a radar field (RF) represented by an oval. The bold RFs have registered a paranormal event.

1994). Tests of these models critically depended on an accurate description of the distribution of location responses, the topic of Experiment 1.

EXPERIMENT 1
The Integration Rule

The goal of the first experiment was to determine how the visual system integrates information from many location detectors to yield the perception of a target dot's location. The integration rule turns out to determine the distribution of location responses. The *X-Files* parable will be used to introduce the integration problem and two classes of solutions to this problem.

A typical theme in the television show *The X-Files* is that a strange anomaly occurs somewhere in the United States. The event is often the arrival of an alien spacecraft. Two FBI agents are sent to the site to investigate. The question that arises is, how do they know where to investigate? The question, phrased in terms of visual perception, is, of course, how does a human observer locate a briefly presented stimulus?

To simplify the situation, imagine that the alien could have landed on a 1-D line from approximately Los Angeles to New York (e.g., Figure 2). One might imagine that the FBI (or the National Security Agency) has set up a series of alien-detecting radars from Los Angeles to New York. Each detector has a radar field (RF), shown as an oval. If an alien lands in the RF of a detector, the detector issues an alert (i.e., a hit). However, it is possible that a detector can miss an alien landing in its RF (i.e., a miss). Similarly, there is some probability that a detector will issue a false alarm (FA)—that is, report a nonexistent alien spacecraft.

On a particular day, three detectors issued an alert, as is shown with the bold ovals in Figure 2. The question is, where should the FBI agents go to find the alien? Where has the alien landed? Central headquarters might tabulate the alerts, as is shown in Figure 3A and 3B. In the figure, one location received alerts from two detectors, so that location has received a total of two alerts. The other locations have received either one or zero alert. A decision must be made on the basis of the alerts. There are many ways in which the information could be integrated and a location determined. Two integration rules are the *winner-take-all rule* and the *spatial average rule*.

In the winner-take-all rule, the location that has the highest activation will be chosen as the alien landing location (shown in Figure 3A). This location is indicated with an arrow, and its activation is 2. In this particular episode, the FBI agents will go to the position indicated by the arrow. In a different episode, given the same alien landing location, the agents might go to a different location. That is because the detectors operate in a probabilistic manner (hit rate < 1.0; FA rate > 0.0). Hence, the pattern of activation (Figure 3) will be different from episode to episode, even if the alien lands in the same location.

Figure 4A shows the result of a simulation of a typical distribution of chosen locations over 10,000 "TV episodes" by the winner-take-all rule. The winner-take-all—or highest activation—rule has the attribute of maximizing the number of episodes in which the FBI agents arrive at the exact location of the alien. Note that with this rule, remote detectors have no influence on performance. As is shown in Figure 4A, the shape of the resulting distribution is leptokurtic.

The simulated data shown in Figure 4A were produced in the following manner. A number of discrete (integer) locations were defined (for example, from -150 to $+150$). Associated with each location was an activation counter. The counters were initially set to 0.0. The midmost location was the target location. Next, detector RFs were

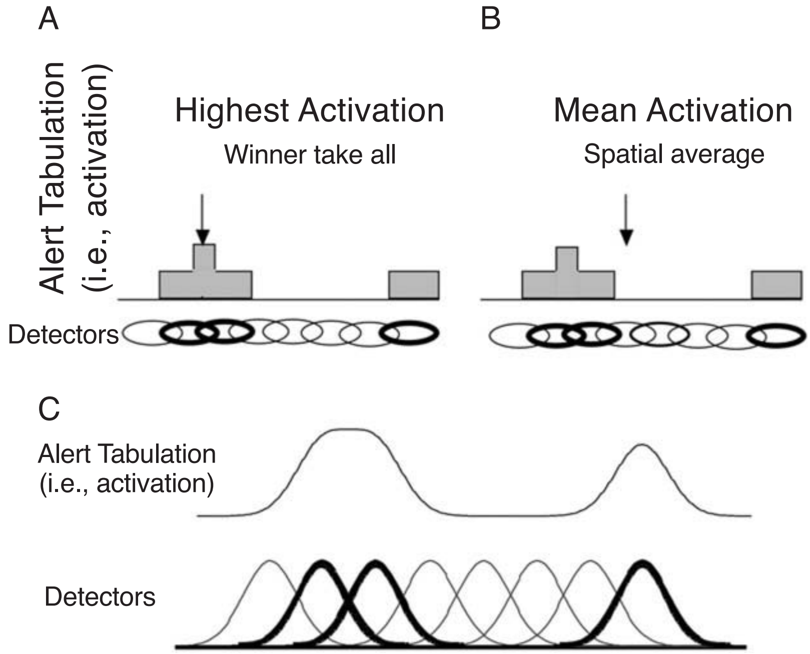

Figure 3. (A) The winner-take-all integration rule. (B) The spatial average rule. (C) Gaussian activation of locations.

defined. The size of the RF, n, was one of the inputs to the simulations. In this example, $n = 31$, the first RF covered Locations -150 to -119, the second covered Locations -149 to -118, the third, Locations -148 to -117, and so forth. (No locations were skipped.) Next, each detector whose RF included Location 0 (the target location) fired with probability p(hit), and the others fired with probability p(FA). In the simulations illustrated in Figure 4, p(hit) = .40 and p(FA) = .20. In this example, when a detector fired it incremented all of its locations by a constant amount. This is called a *rectangular activation profile* (see Figure 3A); other profiles are discussed later. With the winner-take-all rule, the location whose activation counter was the highest was chosen for the response. On trials in which there was a tie for the highest activation, two different tie-breaking procedures were used. In the simulation shown in Figure 4, when there was a tie the trial was rerun. In other simulations, one location was randomly selected from the tied locations. As will be illustrated below, the method of breaking ties did not affect the general shape of the distribution. To generate the distributions shown in Figure 4, this simulation was run 10,000 times. Figure 4 is the frequency histogram of 10,000 location trials.

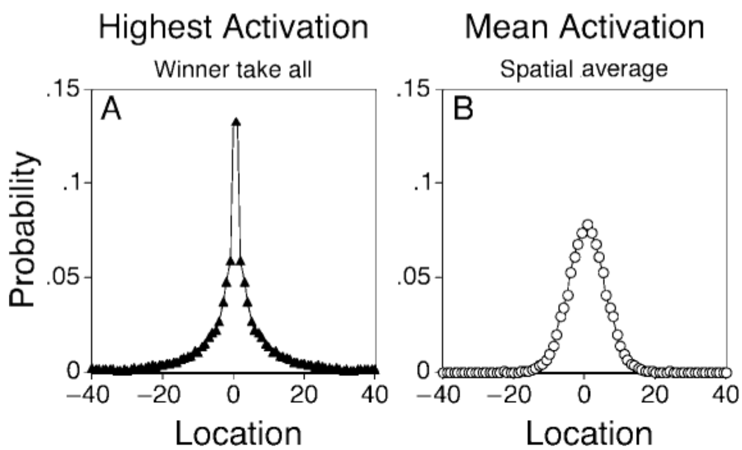

Figure 4. The output of a simulation of (A) the winner-take-all rule and (B) the spatial average rule.

Simulations for the spatial average rule were similar to those for the winner-take-all rule, except that in each trial, rather than select the location with the highest activation, the spatial average was calculated. The "average" location is simply the sum of location number times its activation, divided by the sum of all the activation. The average location in the figure is the location where the activations would balance, if the horizontal axis were a see-saw (see Figure 3B). The mean activation rule has the property that it minimizes errors (or, to be more precise, the square of the summed errors). That is, when the agents arrive on scene, they are likely to be not far from the actual location, but the probability of being at the exact location is less than it would be with the winner-take-all rule. The chosen location need not be the location with the highest activation. A simulation of the mean activation rule over 10,000 episodes is shown in Figure 4B.

As can be seen in Figure 4B, the mean activation rule yields a normal distribution of chosen locations. This is because the distribution of sample means is a normal distribution (according to the central limit theorem). As is discussed in the Appendix, the leptokurtic distribution in Figure 4A (winner–take all) is the extreme value distribution (Johnson, Kotz, & Balakrishnan, 1995, chap. 1). (This distribution is called the *extreme value distribution* because it results from the distribution of maximum—or minimum—values from n independent random samples.) The distribution can be related to the normal distribution in the following manner.[1] Plotted in Figure 5 are the cumulative distributions of the chosen locations with the two integration rules using the simulated data shown in Figure 4. The top abscissa indicates locations across the country from, say, Los Angeles (−40) to New York (40). (Assume the correct location is at ground zero.) If the abscissa is stretched by a power, p, the highest activation distribution can be fit by a normal distribution (see Kontsevich & Tyler, 1999; Pelli, 1985). (Negative values on the abscissa are first multiplied by -1, raised to p, and then multiplied again by -1.) In Figure 5, if $p = .56$ then the two distributions match exactly. Hence, if the exponent, p, is 1, the cumulative distribution by the winner-take-all rule becomes the same as the distribution by the spatial average rule. Thus the exponent, p, is an indication of the shape of the distribution. When $p = 1$, the distribution of location responses is normal (spatial average rule). When $p < 1$, the distribution is leptokurtic (winner-take-all rule).

Several hundred simulations were run, and the relation between the spatial average rule and the winner-take-all rule was quite regular. The inputs to the simulations were p(hit), p(FA), RF size, and activation profile. In the simulation in Figure 4, the profile was rectangular. That is, when a detector "fired" all of its locations were activated equally. Other profiles were tested. Figure 3C shows the results of activation with Gaussian weights so that when a detector fires, it activates the location at its center more than others. Other activation

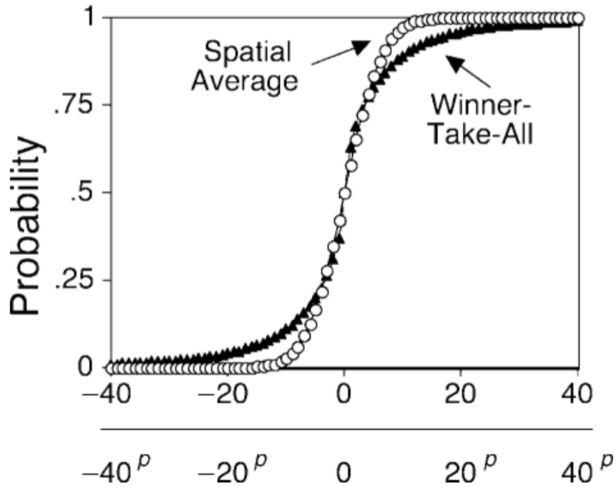

Figure 5. The cumulative distributions, moving from west to east, of the two integration rules.

profiles (e.g., the Mexican hat and triangular profiles) were tested.

The results of a few of the simulations are illustrated in Table 1. The fits of the spatial average model are easy to describe: They always yielded a normal distribution, with the exponents close to 1.0. The exponents with the winner-take-all model were always considerably less than 1.0. There are a few things to note about the winner-take-all simulations. First, as the RF gets smaller, the distribution becomes more leptokurtic (compare Simulations 2 and 3). Hence, the distribution of location responses where RFs are smaller (central vision) should be more leptokurtic (i.e., should have a smaller exponent) than the distribution of location responses with larger RFs. Second, as the signal-to-noise ratio decreased, the distribution of the winner-take-all rule became more leptokurtic (e.g., compare Simulations 3 and 4). This is because locations far from the target are more likely to be selected. Third, the method of breaking ties did not change the shape of the distributions markedly. Finally, the winner-take-all model did not handle some nonrectangular activation filters very well. For example, with a Mexican hat filter and the parameters in Simulation 9, the distribution of location responses was bimodal with a dip at the actual target location. Hence, the winner-take-all model might not be a good mechanism for integration in parts of the visual system that contain units that have an excitatory center and an inhibitory surround. Finally, and most strikingly illustrated in Figure 4, the winner-take-all model is more likely to find the exact location, but when it errs the error will be large. The spatial average model is less likely to "nail" the exact location, but it is usually not too far off.

The winner-take-all rule has been used successfully in a number of theories of luminance detection for stimuli at threshold. For example, Pelli (1985) proposed that there are a number of detectors, some relevant and some irrel-

Table 1
Model Simulations

Simulation	Hit	FA	Width	Filter	Spatial Average	Winner Take All: Resample Ties	Winner Take All: Random Selection
1	.4	.2	31	rect	0.99	0.56	0.59
2	.4	.2	15	rect	1.00	0.47	0.47
3	.4	.2	45	rect	0.99	0.55	0.60
4	.4	.3	45	rect	1.01	0.76	0.77
5	.8	.5	15	rect	1.00	0.42	0.42
6	.8	.6	15	rect	1.00	0.46	0.47
7	.8	.7	15	rect	1.01	0.71	0.70
8	.4	.2	15	Gauss	1.00	0.52	0.52
9	.4	.2	17	hat	1.01	0.61	0.60
10	.4	.2	15	ramp	1.00	0.53	0.52

Note—The right three columns are the exponents of the fits. Two methods of handling ties are illustrated with the winner-take-all model. The Gaussian activation filter (Gauss) was approximated with the following weights: 1, 3, 6, 8, 11, 13, 14, 15, 14, 13, 11, 8, 6, 3, and 1. The Mexican hat filter (hat) was approximated with the following weights: −1, −1, −3, −3, −3, 4, 12, 21, 24, 21, 12, , 4, −3, -3, −3, −1, and −1. The following weights were used for the ramp filter: 15, 14, 13, 12, 11, 10, 9, 8, 7, 6, 5, 4, 3, 2, and 1. FA, false alarm rate; rect, rectangular distribution.

evant. The value of the detector with the highest activation is passed to a decision mechanism. If the value associated with this detector exceeds a threshold, the observer responds that a signal was present (also see Kontsevich & Tyler, 1999). Note that although this theory is highly successful in accounting for detection at threshold, it may not be appropriate for localizing stimuli that are clearly above threshold. In terms of the *X-Files parable*, a detection task is similar to asking whether an alien landed anywhere in the United States, not to asking where in the United States the alien has landed. In their theory of guided search, Cave and Wolfe (1990) used a winner-take-all rule. Tsal et al. (1995) used a winner-take-all integration rule in accounting for the effects of attention.

The spatial average rule has also been used successfully. For example, Ashby, Prinzmetal, Ivry, and Maddox (1996), in accounting for the phenomenon of illusory conjunctions, assumed that the distribution of perceived locations would be a normal distribution. The spatial average rule has the nice property that it is robust. Even with low signal-to-noise ratios, the selected location will not be too far from the actual location.

The goal of Experiment 1 was to characterize the integration rule used for location simply by obtaining enough data from each observer to precisely characterize the distributions of errors, as in Figures 4 and 5. The question is simply whether the cumulative distribution of location responses can be fit by a normal distribution (with only mean and variance parameters) or whether the additional power parameter, p, is necessary to fit the data.

The task was very simple. On 90% of the trials, a small dot was briefly presented. The observer's task was to move the screen cursor with a mouse to the location where the dot had appeared and click the mouse. On 10% of the trials, there was no target dot. On these trials, a correct response was to press a control button labeled "absent" that was always present in the bottom right corner of the screen (see Prinzmetal et al., 1998). These catch trials were included to ensure that the dots were clearly visible, above threshold. The goal of this research was to understand the perception of location of stimuli that were above threshold, not the perception of location of stimuli at or below threshold.

There were two independent variables: target eccentricity and exposure duration. The target dots appeared at a randomly chosen location on an imaginary circle that subtended either 2.29º (near) or 4.58º (far) of visual angle. It seemed desirable to include a variable that should affect perceptual performance. To the extent that the task measures perceptual processes as opposed to memory processes, performance should vary with a variable related to perception. (See Werner & Diedrichsen, 2002, for an example of the effect of a memory-related variable.) Hence, responses should be more variable with a larger stimulus eccentricity—that is, precision should decrease with eccentricity. Furthermore, the winner-take-all rule predicts that the exponent should be small with stimuli that are closer to the fixation point, as was explained above.

Two exposure durations were used: 66.7 (short) and 500 (long) msec. The long exposure duration was sufficient for observers to fixate the stimulus dot and clearly perceive its location. Thus, the variance in the long exposure conditions should reflect processes that are not related to a brief nonfoveal presentation. These would include memory and motor processes, collectively called "other" processes. The variance in responses in the short exposure duration condition would be affected by these "other" processes, as well as limits in performance due to a brief nonfoveal presentation (i.e., *perceptual processes*). By examining the shapes of both distributions of responses, it is possible to infer the shape of the distribution of perceived location, independent of these "other" processes.

Method

Observers. Three observers, 20 to 23 years of age, were paid $50 to participate. They were naive as to the purpose of the experiment.

Procedure. The 3 observers were individually tested in eight sessions, each session lasting about 1 h. The first session began with 2 blocks of 100 practice trials followed by 10 blocks of data collection. Each block consisted of 100 trials. On subsequent days, the observers had approximately 20 warm-up trials and then 10 blocks of data collection trials. Thus, the data set consisted of 8,000 trials per observer.

In each block of 100 trials, 90 were target-present trials (a target dot appeared) and 10 were target-absent trials (no dot appeared). The task was to move the cursor (which was in the form of a plus sign) to the location where the stimulus appeared and then click the mouse. On target-absent trials, the observers were to click a control button labeled "absent" at the bottom right corner of the screen. When the observer made a miss (i.e., pressed the "absent" button when a dot was present) or a FA (i.e., did not press the "absent" button when a dot was not present), the computer emitted a loud sound like that of a foghorn. Such errors were rare.

Stimulus eccentricity was varied within blocks. On half of the target-present trials, the near eccentricity was used, and on half the far eccentricity was used. The order of trials within a block was randomly determined. The exposure duration alternated between blocks. Two of the observers always began with the long exposure duration, and 1 began with the short exposure duration.

Stimuli. Stimuli were presented on an Apple Computer 15-inch color monitor with a screen resolution of 640 × 480 pixels. The viewing distance was 66 cm, and the observers' heads were restrained with a chinrest. From this distance, 32.7 pixels subtended 1º. Normal office ambient lighting was used.

The target stimulus consisted of a filled circle subtending approximately 0.30º of visual angle. The stimulus was gray (approximately 96 cd/m^2) and was presented on a white background (approximately 122 cd/m^2). A black fixation dot and the "absent" button were present throughout the block of trials. The screen cursor appeared 250 msec after the stimulus dot disappeared, and it remained in view until the observer responded. The cursor would reappear at the location from which it had disappeared on the previous trial. The target circles appeared on a randomly chosen location on an imaginary circle with a radius that subtended either 2.29º or 4.58º of visual angle. The observers were not told of this constraint in stimulus location. The observers were questioned at the end of the experiment, and none had noticed this constraint on the target location.

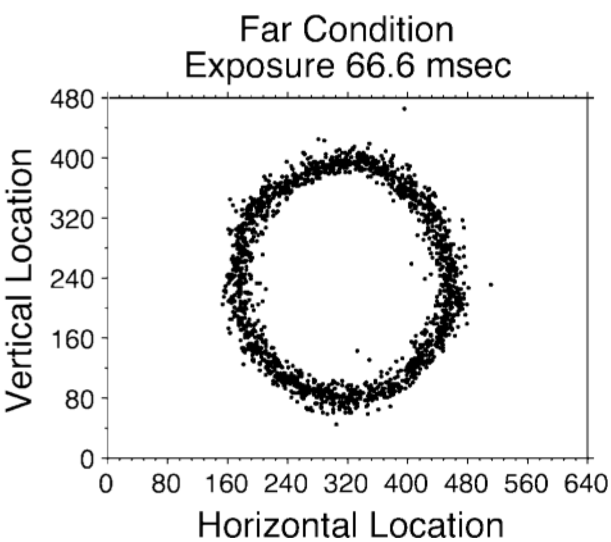

Figure 6. All of the responses of 1 observer with the short exposure duration and far eccentricity. The axes are in screen pixels; 32.7 pixels subtended 1º of visual angle.

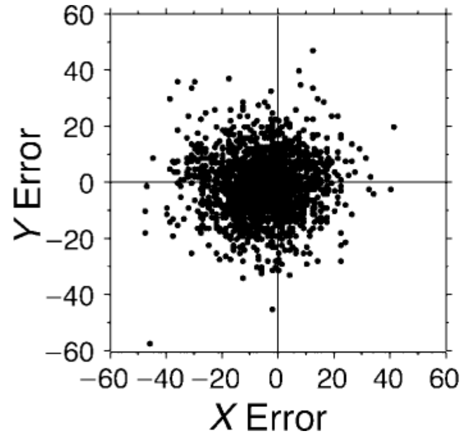

Figure 7. The responses in Figure 6 displayed as errors from the target location.

There were 500 msec between the response and the next trial. Each trial began with the computer emitting a click sound. On target-present trials, the click coincided with the onset of the target circle. On target-absent trials, the click coincided with the instant the target would have appeared if there had been a target.

Results

Raw data. The stimuli were clearly above threshold, as indicated by the presence/absence judgments. The hit rate was over .99 for each observer, and the FA rate averaged .010 (the FA rate never exceeded .025). The distance from the target to the response was calculated for every trial. Trials in which this distance exceeded 7 *SD*s were excluded from the analysis (<1% of the trials).

Figure 6 shows all of the responses for 1 observer in one of the four conditions (short exposure, far eccentricity). The actual stimulus locations were always on the circle. For this observer, there was a tendency for response locations along the vertical median to be farther from fixation than the stimulus locations and for those along the horizontal median to be closer than the actual stimulus locations. Two of the observers exhibited this pattern, but the third exhibited the opposite pattern. The responses in Figure 6 are replotted in Figure 7 in terms of error, with the target location at the center of the plot. This observer had a slight tendency to respond to the left of the actual target location. The constant errors did not systematically vary across the four conditions.

Variability (or dispersion) was measured in terms of precision.[2] Precision was the average squared distance, in screen pixels, from the stimulus to the response. This measure will be referred to as *pixels^2*. The precision for each observer and condition is given in Table 2. For each observer, the responses are more variable in the far condition than in the near condition, and the responses are

Table 2
Precision in Experiment 1

	Short				Long			
	Near		Far		Near		Far	
Subject	*M*	*SE*	*M*	*SE*	*M*	*SE*	*M*	*SE*
1	178.7	5.9	608.7	20.1	54.0	1.8	65.4	2.2
2	161.1	5.8	524.1	18.5	76.0	2.7	157.4	5.6
3	208.9	6.8	297.4	10.0	35.0	1.2	49.8	1.6

Note—Precision measured in pixels^2 (32.7 pixels are 1º of visual angle).

also more variable with the short than the long exposure duration.

Model fits. For each observer and condition, the data were analyzed in three separate ways: horizontal error, vertical error, and what will be termed *radial error*. The distribution of horizontal errors for 1 observer in one condition is illustrated in Figure 8. For all observers and conditions, the correlation between errors in the horizontal and vertical directions was small (averaging r = .10) so that the horizontal and vertical errors could be treated separately. The radial error distributions were the distributions of errors from the imaginary circle on which the stimuli fell along a line radiating from the fixation point.

Figure 9 shows the cumulative distribution of the data plotted in Figure 8, along with the best-fitting normal distribution. The normal distribution, of course, has two parameters: the mean and the variance. This observer's data in this condition (short exposure, far condition) is captured quite well by a normal distribution. The good fit of this observer and condition was not unique, as is discussed below.

Each observer's results in each condition were fitted with a normal distribution (two parameters) and the distribution given by the winner-take-all rule (three parameters). The third parameter in the winner-take-all rule was the exponent, p (see Figure 5). All of the data were extremely well fit with the normal distribution. The fits are shown in Table 3. In all cases, the normal distribution accounted for over 99.99% of the variance (R^2). The results were also fit with the exponent. For the winner-take-all rule, the exponent should be less than 1.0. For a normal distribution, the exponent should equal 1.0. The exponent hovered around 1.0 and averaged 0.992.

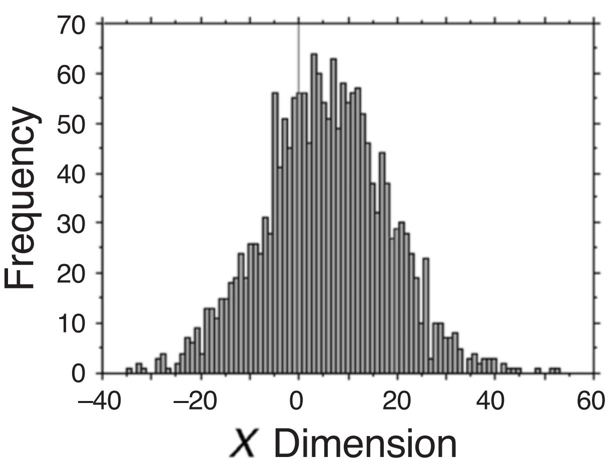

Figure 8. A frequency histogram of the errors along the horizontal axis.

To evaluate statistically the adequacy of the normal distribution, the fit of the normal distribution was compared to the fit of the winner-take-all distribution with the following F statistic (Ashby & Lee, 1991):

$$F = \frac{\left(SSE_{\mathrm{w}} - SSE_{\mathrm{n}}\right) / \left(P_{\mathrm{w}} - P_{\mathrm{n}}\right)}{SSE_{\mathrm{w}} / P_{\mathrm{w}}},$$

$$df = \left(P_{\mathrm{w}} - P_{\mathrm{n}}\right), P_{\mathrm{w}}, \qquad (1)$$

where P_{w} is the number of parameters of the winner-take-all model (i.e., the model with the larger number of parameters), P_{n} is the number of parameters of the normal distribution model (i.e., the model with fewer parameters), and SSE_{w} and SSE_{n} are the sum of squares for the two models. Significant F values of this statistic would mean that the (two-parameter) spatial average model should be rejected in favor of the (three parameter) winner-take-all model. Table 3 shows the results of this analysis. The additional parameter of the winner-take-all model did not significantly improve any of the fits. Furthermore, a prediction of the winner-take-all model is that the smaller the RFs (i.e., the closer to fixation), the smaller the exponent will be. There was no hint of that tendency in the data. The average for the short exposure duration for near and far conditions (averaged over X, Y, and Rad) were 1.02 and 1.00, respectively.

Discussion

The data from each observer and condition are fit very well by the cumulative normal distribution. This result implies that the visual system can be characterized as integrating above-threshold location information by a spatial average rule. The spatial average rule has the consequence of minimizing errors rather than being precisely correct in location judgment. The winner-take-all model has the consequence of maximizing the probability of being precisely correct, but at the expense of larger errors. For the visual system, minimizing error might be more important than being precisely correct. When one is locating a predator, it is perhaps better to be approximately correct than to risk a large error (and death) by running toward the predator.

One might wonder about the generality of these finding with other observers and other conditions. Experiments 2 and 3, both of which involved above-threshold stimuli and slightly different tasks, provide additional support for the spatial average model. In Experiment 2, there were 9 observers in 24 different conditions, and in Experiment 3 there were 5 observers. The average exponent of all 17 observers in all conditions in the three experiments was 1.04, and there was never a case in which

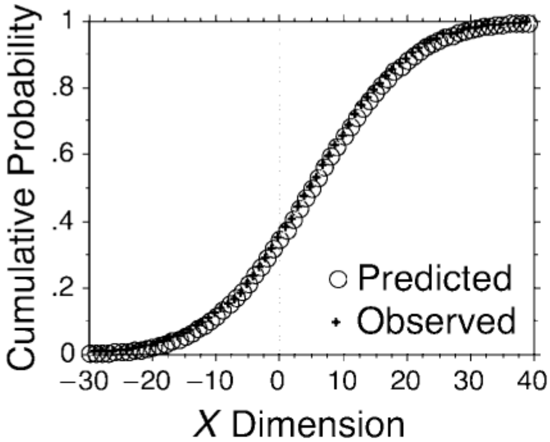

Figure 9. The predicted and observed cumulative response distributions for a typical observer and condition. The predicted distribution is a normal distribution.

the exponent was significantly less than 1. (Note that in each of the winner-take-all simulations in Table 1, the spatial average model would have been rejected by the statistical test in Equation 1 and $p < .05$.)

The present results contrast with results with stimuli at threshold, in which the winner-take-all (i.e., nonlinear) combination rule seems to apply (see, e.g., Kontsevich & Tyler, 1999; Pelli, 1985). The hypothesis behind these theories is that the activation of the location with the highest activation is compared to a threshold. If the activation of the location with the highest activation is above the threshold, the observer responds that the stimulus was present. It may not be surprising that the winner-take-all rule applies in a detection experiment with locations at threshold. If the highest activated location is below the threshold, a weakly activated location will also be below the threshold and hence provide little additional information. In a location experiment with stimuli above threshold, such as the present experiments, all active locations potentially provide useful information.

One might question whether the method of reproduction (i.e., the observers directly indicated the stimulus location with a mouse) captures perceptual processes. The response distributions contain variability due to nonperceptual sources. For example, observers have to remember the stimulus location, if only for a short time. There may be variability due to positioning of the cursor. These other sources of variability can be assessed by examining the long (500-msec) exposure condition. In the long exposure condition, the observers had time to move their eyes to the stimulus location so that the exposure was foveal, and there was probably little variability due to visual processes. Hence, the short exposure condition contained variability due to perceptual and nonperceptual sources, whereas the long exposure condition represented variability due only to these other sources. The distributions for the long and short exposures could be treated as normally distributed random variables. Since the sum of any two normally distributed random variables is also a normally distributed random variable, one can infer that the component of the short exposure duration due only to perceptual processes is also a normally distributed variable.

It is possible that these "other" sources of variance are normally distributed but that the perceptual component is not. There may be circumstances in which these other sources of variance mask a leptokurtic perceptual component. Although possible, this result seems unlikely in the present circumstance because the variance for the short exposure condition (perception + other) is much

Table 3
Model Parameters for Experiment 1

	Observer 1			Observer 2			Observer 3		
Condition	*SD*	Exp.	*F*(1,3)	*SD*	Exp.	*F*(1,3)	*SD*	Exp.	*F*(1,3)
Near short X	9.17	1	0	8.28	1	0.04	9.41	1.08	0.27
Near short Y	7.98	0.99	0.02	7.58	0.92	0.44	10.67	1.08	0.32
Far short X	**12.58**	**0.98**	**0.23**	**16.48**	**1.02**	**0.03**	**11.9**	**0.94**	**0.58**
Far short Y	13.97	1.08	4.92	15.96	1.07	2.18	10.25	0.91	1.29
Near long X	3.95	0.97	0.51	5.77	0.96	0.1	3.35	1	0
Near long Y	3.72	0.92	0.43	5.29	0.94	0.64	3.29	0.89	0.55
Far long X	4.52	1	0	8.1	0.98	0.48	4.47	0.96	0.35
Far long Y	4.67	0.91	0.17	8.27	1	0	3.78	0.93	0.53
Near short Rad	9.49	1.05	0.77	8.59	1	0	10.74	1.08	1.05
Far short Rad	16.83	1	0	14.95	1.03	0.07	11.63	1	0
Near long Rad	5.21	1.06	0.73	5.71	0.95	0.83	3.48	1	0
Far long Rad	5.24	0.97	0.88	7.84	0.99	0.03	4.47	1	0
Means	8.11	0.99		9.40	0.99		7.29	0.99	

Note—*Near* and *far* refer to the eccentricity; *short* and *long* refer to the exposure duration. *SD*, standard deviation used to fit the normal distribution; Exp., the exponent used to fit the winner-take-all model; X and Y, horizontal and vertical dimensions, respectively; Rad, distribution along a radial axis. The critical value for the *F* statistic ($p < .05$) is 10.27, so that any value above this would violate the spatial average model. The spatial average model was not rejected with any observer or condition. The condition in bold is illustrated in Figure 9 and discussed in the text.

larger than that for the long exposure condition (other). The average precision in the short exposure condition is 330 pixels, in comparison with 50 pixels for the long exposure condition (see Table 2).

It was mentioned in the introduction that several theories have used a winner-take-all integration rule (see, e.g., Cave & Wolfe, 1990; Tsal et al., 1995). The particular integration rule that was used was probably not central to these theories, and they could have used a spatial average rule as well as a winner-take-all rule. However, interesting questions are raised by these theories. For example, Cave and Wolfe hypothesized that attention in visual search is guided by the most active location. On the one hand, it may be the case that the mechanism for guiding visual search is different from the mechanism for the conscious perception of location. Thus, the winner-take-all rule might be appropriate for the Cave and Wolfe guided search model. On the other hand, the spatial average mechanism, which on average will minimize errors (or, to be more precise, error squared), might be a good mechanism for guiding attention as well as for the conscious location of objects.

It can be concluded that the spatial average model provides a good characterization of the integration of location information for the visual system *as a whole*, and it is not necessary to add a nonlinear parameter. Note, however, that there is a fundamental difference between the model (*X-Files*) and the visual system: The model has only a single layer of detectors and the visual system contains many layers of analysis. In fact, the visual system may contain many nonlinear components (e.g., winner-take-all model), but the combination of these can be described by the spatial average (i.e., linear) system. The linear combination of nonlinear processes would yield a normal distribution by the central limit theorem. The best we can do in a behavioral experiment such as the present one is to describe the output of the system as a whole, and not just, for example, area V1.

Even though we can characterize the integration rule of the system only as a whole, the finding that the resulting distribution of location responses is normal turns out to be important in studying attention and the effect of landmarks. The assumption of a spatial average model leads to a potentially powerful measurement of attention. Furthermore, to test the models of the effect of a landmark in Experiment 3, the distribution of location responses was critical. There maybe circumstances in which the winner-take-all (or other) integration rule is more appropriate (e.g., near-threshold exposure). However, as a first step in any model of attention (Experiment 2) or in determining how the presence of one object affects the location of another object (Experiment 3), we must make assumptions about the integration of perceptual information. In the experiments reported in this article, the spatial average model is the most consistent with the data.

EXPERIMENT 2
Attention

In several previous experiments, it has been found that attention reduces the variance of location responses, as is shown in Figure 1 (Newby & Rock, 2001; Prinzmetal et al., 1998; Tsal & Meiran, 1993; Tsal et al., 1995). Two mechanisms for this reduction in variance have been proposed: reduction in receptive field size and more-samples theory.

The first proposed mechanism is based on the idea that attention reduces the size of receptive fields or, more generally, sharpens tuning functions. Figure 10 shows a simplified situation with nine possible target locations and nine detectors. (The locations are the dots, and the detector RFs are the ovals and circles.) In Figure 10A, each RF covers three locations; in Figure 10B, each RF covers one location. It may be that attention reduces RF size. In the visual system, Moran and Desimone (1985) described cells in area V4 whose receptive fields appear to shrink around an attended location (also see Spitzer, Desimone, & Moran, 1988). However, it is not clear whether the behavior of these cells represents the mechanism of attention or is the result of another attentional mechanism. For example, an attentional mechanism (e.g.,

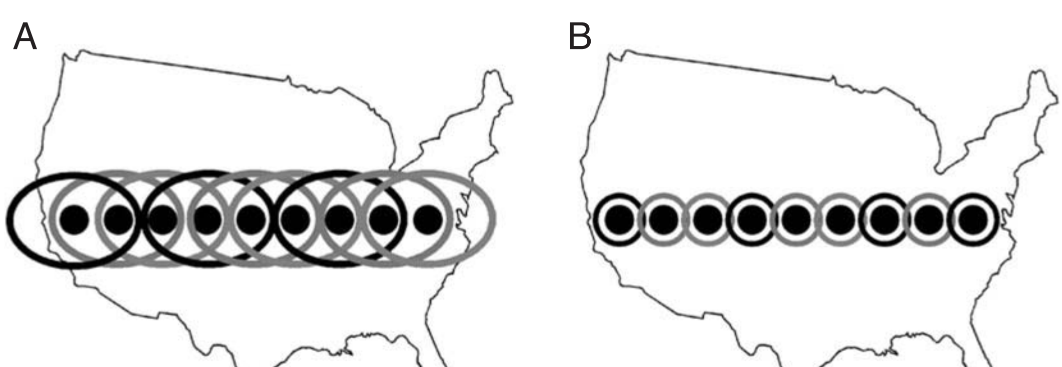

Figure 10. Possible target locations and detectors. In both panel A and panel B, there are nine locations and nine detectors. In panel A, most detectors guard three locations, and in panel B each detector guards one location.

more samples) operating in an area prior to V4 may be the cause of the smaller RF in V4. Tsal et al. (1995) proposed a "resolution theory of attention," which is mediated by smaller receptive fields.

The main problem with the smaller-receptive-fields account of attention is that it has never been shown to work. In fact, it generally makes the wrong prediction. In every simulation that I have run, as the size of the RF gets smaller the variability of location responses increases, provided that the following conditions are met: (1) The number of detectors remains constant, (2) there are enough detectors to locate the stimulus, (3) the hit rate is less than 1.0 and the FA rate is greater than 0, and (4) the hit and FA rates do not vary with RF field size. The reason for this result is quite clear. Consider one location in Figure 10 (e.g., Salt Lake City). In Figure 10B, only one detector is guarding each location. There is a probability $[1 - p(\text{hit})]$ that an alien attack will be missed, possibly with unfortunate consequences for the citizens of Salt Lake City. In Figure 10A, however, with three detectors guarding each location the probability of missing an alien attack on Salt Lake City is only $[1 - p(\text{hit})]^3$. Of course, with multiple detectors guarding a location, there is a greater probability of a FA at that location, but if hits and FAs both increase exponentially and $p(\text{hit}) > p(\text{FA})$, then more detectors guarding a location must lead to better performance. The observation that smaller RFs can lead to worse performance than larger RFs has been made by other investigators (e.g., Heiligengerg, 1987; Hinton, McClelland, & Rumelhart, 1988; O'Reilly, Kosslyn, Marsolek, & Chabris, 1990). Thus, the shrinking of receptive fields with attention observed by Moran and Desimone (1985) could be a consequence of another attentional mechanism.

The second type of theory to account for the effect of attention proposes that attention is like obtaining more samples (see, e.g., Bonnel & Miller, 1994; Luce, 1977). In the *X-Files* parable, obtaining more samples could be similar to the following scenario: Suppose that instead of reporting to the FBI at the end of each day whether or not there was a hit on that day (sample size = 1), the agents waited 3 days and reported whether there were zero, one, two, or three hits (sample size = 3). (Of course, this analysis assumes that the alien stays in the same location for 3 days.) In the latter case, the spatial average is calculated after 3 days (three samples). If a decision about the alien's location is made after 3 days (three samples) instead of after 1 day (one sample), the variance of distributions over episodes will be reduced. According to the spatial average rule, the variance of an observer's responses, Var_{resp}, is simply the square of the standard error of the mean:

$$\text{Var}_{\text{resp}} = \frac{\text{Var}}{n} \tag{2}$$

where n is the sample size and *Var* is the variance of each sample. The variance of responses will naturally be less after three samples ($n = 3$) than after one sample ($n = 1$). The concept of discrete samples is a bit obscure in terms of the visual system because it is impossible to quantify the exact number of samples an observer might take. However, consider the ratio of the variance of more attention (more samples) to responses with less attention (fewer samples):

$$\frac{\text{Var}_{\text{more attn}}}{\text{Var}_{\text{less attn}}} = \frac{\text{Var}/n_{\text{more}}}{\text{Var}/n_{\text{less}}} = \frac{1/n_{\text{more}}}{1/n_{\text{less}}} = \frac{n_{\text{less}}}{n_{\text{more}}} \tag{3}$$

Here, the ratio of the observed variances is directly related to the ratio of samples. The ratio of observed variances provides a theoretically motivated metric of attention assuming integration by spatial average and assuming that attention is mediated by more samples. The variances are obtained directly from the distribution of the observers' responses. Thus, more-samples theory, together with the spatial average integration rule, makes possible a simple and theoretically motivated metric of attention. This measure of attention is more tightly coupled to the mechanisms of attention than are other measures (e.g., slopes of the display size function). However, as will be described below, a more general approach that does not assume spatial averaging will be taken in the present experiment.

More samples might be implemented in physiological terms in many different ways. For example, the refractive period might be reduced so that cells are allowed to fire at a higher rate (McAdams & Maunsell, 1999; Wurtz, Goldberg, & Robinson, 1980). Perhaps a higher firing rate would be reflected in cerebral blood flow as well as in single-cell activity (Gandhi, Heeger, & Boynton, 1999). McAdams and Maunsell have found that the change in behavior of V4 cells is better described by a change in firing rate than by a change in the cell-tuning function. A second way in which more samples might be implemented is that detectors could be recruited from an unattended region of space by shifting their receptive fields to include the attended region of space (Connor, Preddie, Gallant, & Van Essen, 1997). Finally, attention might change the tuning function of a cell so that it includes stimulus values that normally do not activate it. Indeed, attention has many physiological effects (Colby, 1991) and many of these might be related to more samples.

In Experiment 2, I looked for evidence consistent with the more-samples theory of attention. The number of samples is a theoretical construct that cannot be directly manipulated. The claim is that attention affects the number of samples. To test this idea, attention was varied using the dual task paradigm described below. Another variable, related to the number of samples, was also manipulated. This variable was exposure duration: The longer the exposure duration is, the greater the number of samples there will be. The question addressed was whether exposure duration would affect performance in a manner that is quantitatively similar to attention. The method is similar to one used by Loftus and his colleagues, who asked how much additional exposure duration was equivalent to the duration of an iconic image (Loftus, Johnson,

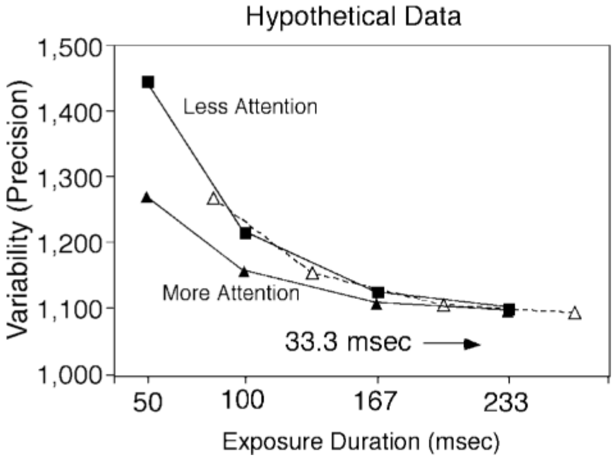

Figure 11. A theoretical distribution based on the idea that attention can be considered similar to increasing the exposure duration and therefore the number of samples. In this example, attention is "worth" 33.3 msec of additional exposure duration.

& Shimamura, 1985). Here, I am asking how much exposure duration attention is worth.

Exposure duration and number of samples are of course not identical. However, to the extent that exposure duration is equivalent to number of samples, the variance of responses should decrease in a similar manner. Consider the hypothetical data in Figure 11. The solid symbols are the results of simulations (spatial average model). In the simulations, exposure duration is simulated with the number of samples taken before a location decision is made. The location decision is based on the sum of activation over *n* samples. The more samples (or the longer the exposure duration), the less the variance will be. If the data were generated from an experiment with conditions in which attention was varied, in the hypothetical data (Figure 11) attention would have been worth exactly 33.3 msec, because more attention is worth the equivalent of 33.3 msec more time for more samples.

The hypothetical data in Figure 11 could be fit by Equation 2 because they were generated by a spatial average integration rule. However, simulation from the winner-take-all rule could not be fit by Equation 2. I found that the decrease in variance with increased number of samples could always be fit with the following expression, regardless of the integration rule:

$$\mathrm{Var} = d\left(e^{-bx}\right) + a. \tag{4}$$

The parameters are *b*, which determines the shape of the function; *d,* a scale factor; and *a,* the asymptote. The quantity *x* refers to the number of samples (in a simulation) or the exposure duration (in an experiment). Hence, Equation 4 was a flexible tool for characterizing the decrease in variance that one would expect with increasing exposure duration and/or samples. The two solid line functions in Figure 11 were created with the same parameters, except that having more attention is like having more samples by parameter *c*:

$$\mathrm{Var} = d\left[e^{-b(x+c)}\right] + a. \tag{5}$$

Thus, *c* is equivalent to 33.3 msec. The dotted line is the function labeled "more attention" moved over by exactly 33.3 msec. The empirical question is, to what extent can attention and exposure duration be considered different methods of influencing the same underlying theoretical variable (i.e., number of samples)? Notice that in Figure 11 the two functions asymptote at the same value. In this experiment, the exposure durations were not such as to obtain asymptotic performance (except perhaps for 1 observer). To the extent that attention and exposure duration reflect the same underlying variable, it should be possible to model the variances of the response distributions with high and low attentional loads by the addition of a parameter, *c*.

A second version of the more-samples model was also tested. In this version, rather than having an additive effect, attention changed the sampling rate. In the rate version, instead of adding a constant (*c*) a rate parameter (*d*) was estimated. In this parameterization, $x + c$ became $x * d$ (see Equation 5). [3] This version provided fits that were similar to those of Equation 5.

Using the methods presented here, whereby observers directly locate a target, as in Experiment 1, attention has been manipulated in three ways. Tsal and Meiran (1993) used a variant of a spatial cuing method developed by Posner (1980). Three regions were marked by large circles. Just before the presentation of a target dot, one of the circles got brighter, indicating that the target was most likely within the indicated circle (also see Tsal et al., 1995). The observers then located the target in a manner similar to that of Experiment 1. Prinzmetal et al. (1998) used a variety of dual task manipulations to manipulate attention. In one method, used here, the dual task was letter identification. Letter identification was either easy or difficult. When letter identification was easy, presumably it took little attention and the observers could devote their attention to the dot localization task. When letter identification was difficult, the observers could not devote their attention to the dot localization task. Finally, Newby and Rock (2001) used the inattention paradigm, in which the target to locate was either not expected (inattention) or expected (divided attention; see Mack & Rock, 1998).

With all three methods, the result of the attentional manipulation was the same: greater variance in location responses with less attention (e.g., Figure 1). In principle, the analysis above could be applied to any method of varying attention, but the procedure of Prinzmetal et al. (1998) was chosen for the following reasons. The cuing paradigm used by Tsal and Meiran (1993) differs from the original Posner (1980) paradigm in that the cue not only affects attention but also provides direct information relevant for the response. Thus, the attention effect was confounded with information on location provided directly by the cue. In the method used by Newby and Rock (2001), only one inattention trial can be obtained per observer, and many thousands of trials per observer are required for the present analysis.

Method

Procedure. The experiment was similar to Experiment 1 except that a second task was introduced to manipulate attention. On each trial, not only did a small dot appear in the periphery, as in Experiment 1, but at the same time a 3 × 3 matrix of letters was presented in the center of the screen. The letter matrix always contained either the target letter F or the target letter T. The observer's tasks were to indicate which target letter had been presented and to indicate the location of the dot. After the presentation of the dot and letter matrix, the observer moved the screen cursor over the location where the dot had been presented. If the letter matrix contained the letter F, the observer would press the left button on the mouse. If the letter matrix contained the letter T, the observer would press the right button on the mouse. On 8.88% of the trials, no dot (only the letter matrix) was presented. On these trials, the observer was to move the cursor to the "absent" button in the bottom right corner of the screen and press the appropriate button to indicate the target letter. Thus, by moving the cursor and pressing one button, the observer indicated the identity of the target letter, whether a target dot was present, and, if the target dot was present, its location.

Attention to the dot task was manipulated by making the letter task easy or difficult. On the easy trials, all of the nontarget letters in the matrix were Os. On difficult trials, the nontarget letters were heterogeneous and randomly drawn from all the letters of the alphabet. Prinzmetal and colleagues found that this manipulation of attention affected performance in a wide variety of tasks (Prinzmetal et al., 1998; Prinzmetal et al., 1997, Prinzmetal & Wilson, 1997). With the easy letter condition, there was less variance on the dot localization task, presumably because there was more attention "left over" to perform the localization task. In each block, half of the trials were in the easy condition and half were in the difficult condition, and they were presented in a random order.

In addition to the attention manipulation, there were four different exposure durations. For the first 3 observers, these were of 50, 100, 166.7, and 233.3 msec. For the remaining 6 observers, they were of 40, 106.7, 173.3, and 240 msec. The reason for the difference was that the first 3 observers were run with a 60-Hz monitor and the remaining observers with a 75-Hz monitor. In each block, there were 96 target dot-present trials and 12 target dot-absent trials. Each combination of exposure duration and attention condition was used equally often for target-present and target-absent trials within a block.

Each trial began with a fixation dot in the center of the screen. The target dot and the matrix of letters came on at the same time. The letters were presented for 250 msec (first 3 observers) or 240 msec (remaining 6 observers). The target dot was presented for one of the four exposure durations described above.

On each trial, the observers were given feedback on the correctness of their present/absent judgments, as in Experiment 1. In addition, whenever the observer identified the target letter incorrectly, the computer sounded a brief beep. There was no feedback on the accuracy of the location judgment.

Each of 9 observers, selected as in Experiment 1, participated in 10 1-h sessions. During each session, data were collected on eight blocks of 108 trials per block, yielding 8,640 trials per observer. There were practice and warm-up trials, as in Experiment 1. The observers' ages ranged from 22 to 55 years. All the observers were naive as to the purpose of the experiment, except W.P. (the author).

Stimuli. The stimulus displays were similar to those of Experiment 1 except that the monitor had a screen resolution of 832 × 624 pixels and the viewing distance was 50.8 cm. At this viewing distance, 26.6 pixels subtended 1º of visual angle (in comparison with Experiment 1, in which 32.7 pixels subtended 1º). The brightness values of the background (white) and the gray target dot were the same as in Experiment 1.

The target dot, when present, subtended 0.34º of visual angle and was located on an imaginary circle whose radius subtending 3.5º. The 3 × 3 letter matrix was created with Helvetica uppercase 12-point type. The letters were black, and the entire matrix of letters subtended approximately 1.05º in height and 1.40º in width.

Results

The stimuli were clearly above threshold as indicated by the presence/absence judgments. The hit rate was over .999 for each observer, and the FA rate was under .010 for each observer. The average percents correct on the letter task were 98.8% versus 93.8% for the easy and difficult conditions, respectively [$t(8) = 3.87$, $p < .01$, one-tailed].

The data were fit to the spatial average (normal) distribution and the winner-take-all distribution, as in Experiment 1. There were 24 fits per observer (difficult vs. easy letter condition × 4 exposure durations × horizontal vs. vertical vs. radial error). The normal distribution provided excellent fits of the data. When the exponent parameter was added, none of the fits were significantly improved. If this parameter equals 1.0, the distribution is the normal distribution. The exponent averaged 1.06.

As in Experiment 1, the precision of responses was used as a measure of variance (i.e., the averaged squared distance from the stimulus to the response; see note 3). The average precision on the location task in the easy and difficult letter conditions are given in Table 4. Thus, the difficulty of the letter task had a substantial influence on the precision of location accuracy. The letter condition affected location precision for each observer except P.E.

The precision results in Table 4 were fit with Equations 4 and 5. The value of parameter c is given in Table 4. For every observer except P.E., adding c to the easy letter condition improved the fit. The F ratios used to compare the augmented model to the model without c ranged from $F(1,4) = 115.06$ (Observer K.R.) to $F(1,4) = 6.68$ (Observer T.A., $p = .054$). There was no improvement for Observer P.E. [$F(1,4) = 0$]. Because Observer P.E. didn't show an effect of attention, his data is not informative as to how attention affects location perception. (It is not at all clear why P.E. did not show an attention effect. The difficulty of the letter task did not affect his performance on the localization task.) Hence, the remaining analysis concerns the 8 observers who did show an attention effect.

For 7 of these 8 observers, the model presented in Figure 11 and Equations 4 and 5 provides a good qualitative and quantitative account of the data. The effect of attention is similar to that of increased exposure duration. Only Observer E.R. did not follow this pattern. I will first discuss the 7 observers who conform to the pattern and then contrast their performance with that of Observer E.R.

Table 4
Precision in Experiment 2

Observer	Exposure Duration (msec)										
	50	100	166.7	233.3	*M*	50	100	166.7	233.3	*M*	*c*
	Easy Letter Condition					Difficult Letter Condition					
E.R.	666	532	442	403	510	733	591	503	465	573	28.6
J.E.	1,422	1,196	1,058	1,123	1,200	1,590	1,303	1,104	1,149	1,287	21.8
K.R.	781	680	658	584	676	1,041	941	864	709	889	158.1
	Exposure Duration (msec)										
	42	109	175	242	142	42	109	175	242	142	*c*
	Easy Letter Condition					Difficult Letter Condition					
W.P.	2,121	1,542	1,290	1,000	1,488	2,947	2,443	2,182	2,039	2,403	184.5
J.O.	2,024	1,875	2,000	1,668	1,892	2,808	2,229	2,337	1,931	2,326	179.1
S.H.	816	735	628	560	685	936	770	701	574	745	69.1
T.A.	568	512	436	386	476	756	605	560	458	595	67.8
Z.E.	842	729	723	698	748	878	868	819	819	846	197.0
P.E.	992	711	572	500	694	992	711	572	500	694	−12.6

Note—Precision of location responses as a function of attention condition and exposure duration in milliseconds. The parameter *c* is the number of milliseconds that one would add to the easy letter condition to make it match the difficult letter condition.

To obtain a qualitative sense of whether exposure duration and attention may both be characterized, in part, by the same underlying variable (i.e., number of samples), the results are plotted in Figure 12 in the same format as in Figure 11. The solid lines represent the actual data. The dashed line represents the easy letter condition displaced by parameter *c*. The error bars represent 2 standard errors of the variance.[4] Each point represents approximately 960 trials (depending on the number of misses).

Consider Observer K.R. (Figure 12), who showed a large effect of attention. Attention was "worth" 158.0 msec for this observer (i.e., $c = 158$ msec). One can easily imag-

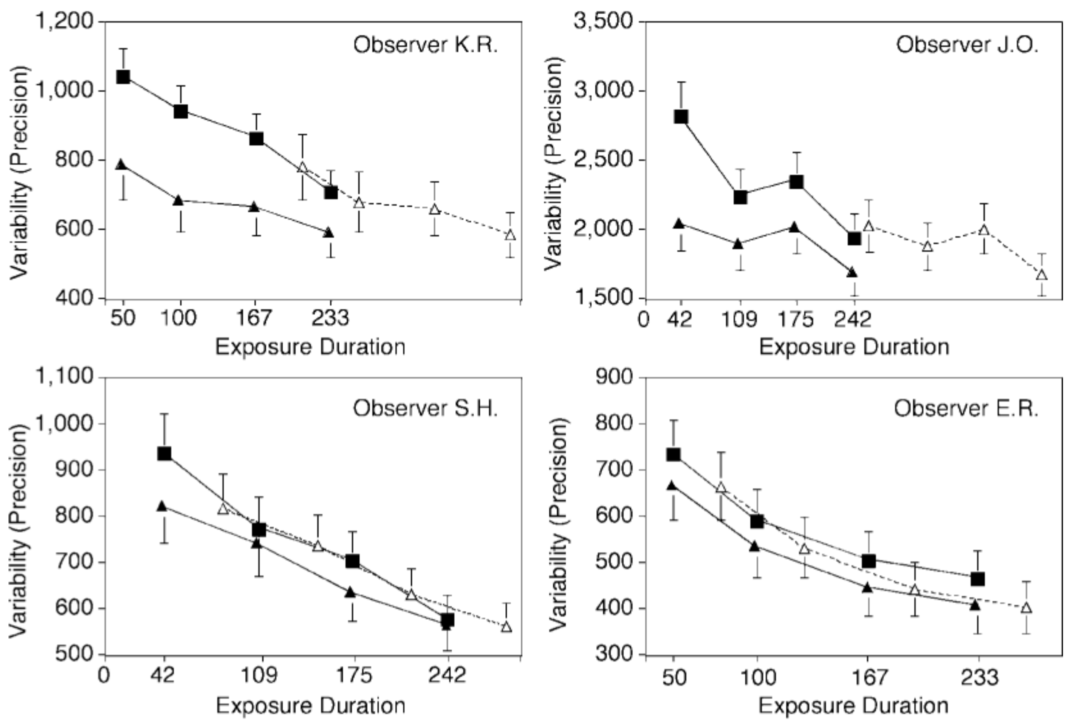

Figure 12. The results of 4 observers. Solid triangles represent results from the easy letter condition, and solid squares represent those from the difficult letter condition. Unfilled triangles represent results from the easy letter condition moved laterally by *c* msec. Each point is based on a minimum of 1,385 trials. Error bars represent 2 standard errors of the variance.

ine that if the easy letter condition was shifted 158 msec (the dotted line), a single function could fit both the easy and the difficult noise conditions. Observer J.O. also had a large attention effect "worth" 216 msec (i.e., $c = 216$). Observer S.H. had a much smaller effect of attention, worth only 42 msec. The reader is invited to plot the data from the other observers from Table 4 and/or test alternative models.

For the 7 observers who followed the predicted pattern, several additional quantitative models were tested. One might question whether anything is happening with these observers other than the addition of the parameter c. Against the model with four parameters (a, b, s, and c), we tested three different five-parameter models. These five-parameter models had different a, b, and s parameters for easy and for difficult letter conditions. These models did not significantly improve the fits for any of these 7 observers. The F values used to compare these five-parameter models and the four-parameter model ranged from $F(1,4) = 0$ to $F(1,4) = 2.40$ (the critical value is 7.71).

E.R. did not follow the predicted pattern (see Figure 12). For this observer, attention could be thought of as 28.6 msec of additional exposure duration. However, the visual fit of this observer's data does not seem very good in that the two attention conditions do not appear to asymptote at the same value. For this observer, each of the five-parameter models provides a significantly better fit than does the four-parameter model. Note that this was not the case for any of the other observers. The problem with this observer's data may be that the exposure durations were not long enough to yield a good indication of asymptotic performance. Alternatively, it may be that for this observer exposure duration (or attention) may have had an additional effect on the asymptote.

For each observer, we also analyzed the data with the rate version of Equation 5. In this version, we estimate a rate parameter, d, and $x * d$ replaces $c + x$. Of the 8 subjects who showed an attention effect, 3 were fit better by the $x * d$ formulation and 5 were fit better by the $x + c$ formulation. The differences were not great, probably because the fits were extremely good to begin with. I think that in order to make the experiment sensitive enough to show the difference between these versions of the more-samples model, one would have to run an experiment with many more than four exposure durations. However, one needs a minimum of approximately 1,000 trials per condition. Adding more exposure durations would make the experiment extremely large.

Discussion

The goal of this experiment was to determine whether or not increased attention could be considered analogous to increased exposure duration. The rationale was that both attention and exposure duration might be related to the same underlying theoretical construct: more samples. If this account is correct, then a single parameter should account for the change in variance with changes in exposure duration and attention.

The results were encouraging for the more-samples model. Of the 8 observers who were affected by the attention manipulation, the performance of 7 was consistent with the more-samples model in that the effect of attention on precision could be accounted for by a single parameter related to exposure duration. For the 8th observer, this was not the case. Although we cannot account for the results of 1 observer, it is encouraging to know that our quantitative methods were powerful enough to detect this exception.

The relation of attention and exposure duration to the same underlying construct does not mean that attention functions exactly as increased exposure duration. There are probably numerous ways in which increasing exposure duration and increasing attention have different effects on the visual system. For example, with long exposure duration, detectors early in visual processing, including retinal receptors, may fatigue. On the other hand, a faster firing rate of cells with increased attention is more likely to fatigue cells later in visual processing.

Nevertheless, a model that assumed that the effect of attention was like the effect of increased exposure duration fit the results of 7 of 8 observers fairly well. The exception was Observer E.R., for whom the function with attention does not seem to asymptote to the same level without attention. As was pointed out above, Prinzmetal et al. (1998), with exposure durations up to 0.5 sec, found performance with and without attention to asymptote to the same value. It may be that the exposure durations used in this experiment were not long enough to yield a good estimate of the asymptotic value for Observer E.R. Alternatively, for this observer exposure duration may have had an influence in addition to that which it shares with attention. For example, although the target letter may have "popped out" of the easy letter arrays, this observer may have tried to remember the arrays during the difficult letter condition. Memory for the letter array may have interfered with memory for the location of the target dot. Thus, it is possible, at least for 1 observer, that the attention manipulation affected nonperceptual processes and that these processes were reflected in the asymptote.

This experiment does not prove that the more-samples notion of attention is correct. However, the results were reasonably consistent with the more-samples theory of attention, and therefore it remains a viable candidate theory. Furthermore, the alternative—smaller receptive fields—makes the wrong prediction. In the simulations, smaller receptive fields resulted in larger variability of location responses. Several colleagues have suggested alternatives that would repair the smaller-receptive-fields theory. One possibility is that with smaller receptive fields, perhaps there are more detectors and hence better performance. There is no question that adding detectors increases performance, but such improvement is the result of more detectors (providing more samples), not of decreasing RF size. The second suggestion is that perhaps with smaller RFs, each detector becomes more accurate (i.e., higher hit rate, lower false alarm rate). There is no question that increasing the sensitivity of detectors improves performance, but it would be the sensitivity of the detectors, not the RF size, that results in better performance.

Two final notes of caution are in order. First, the smaller-receptive-fields theory and the more-samples theory are intended here to account for what Lu and Dosher (1998) would term *signal enhancement* (or internal noise reduction). The stimulus displays were uncluttered. There are other situations in which it might not be clear which item is the target item (cases of "noise exclusion"; see, e.g., Awh, Matsukura, & Serences, 2003; Dosher & Lu, 2000). It is unclear how to apply either of the models discussed here to cases of noise exclusion.

Finally, computer simulations, just like experiments, suffer from the inductive fallacy. That is, it is difficult to prove that there is no version of the smaller-receptive-fields theory that could make the correct prediction (i.e., less variance with more attention). However, despite numerous attempts, I could not get such a model to work. Furthermore, other investigators have independently come to the same conclusion with very different simulations (see, e.g., Heiligengerg, 1987; Hinton et al., 1988; O'Reilly et al., 1990). It is difficult to imagine what version of the smaller-receptive-fields theory would work when actually implemented in a computer simulation. It behooves those who subscribe to the smaller-receptive-fields theory to demonstrate that it can lead to the appropriate behavior. Furthermore, it would be important to understand the necessary conditions for such a model to work. At this point in time, the more-samples theory is the most viable computational account for the effect of attention on location perception.

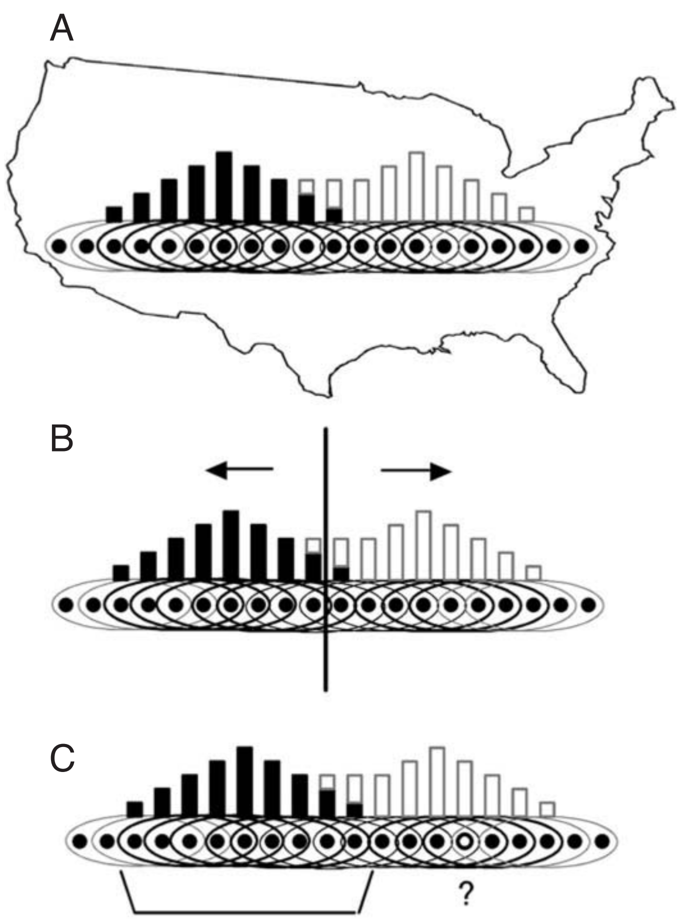

Figure 13. (A) The pattern of activation when two aliens land. The black bars represent activation due to the alien to the west, and the white bars represent activation due to the alien to the east. (B) Separating the activation by the point of minimum activation. (C) Separating the activation by a statistical dispersion method. Only the information within a "neighborhood" (bracket) is considered as arising from one spacecraft.

EXPERIMENT 3
Multiple Objects

The final experiment was conducted to address the question of what happens if two aliens land. The situation is illustrated in Figure 13. How might the perceived location of one alien spaceship be affected by the presence of a second alien spaceship? There are two computational problems. The first is to determine whether there are one, two, or more aliens. This problem is very difficult and will not be addressed here. The second problem is addressed by the following question: Assuming there are two alien spaceships and the FBI knows this, how is the activation from the radar detectors used to determine the location of each spaceship? In Figure 13, for example, RFs in the center get activation from both alien spaceships.

One class of solutions is to look for a point of minimum activation. This solution is illustrated in panel B of Figure 13. All the activation to the west of that minimum are attributed to one alien, and all the activation to the east of that minimum are attributed to the other. Once the activation is assigned, the information attributed to each alien is integrated as in the previous simulations.

Logan's (1996) CODE theory of attention uses this type of parsing rule. According to this theory, objects are defined in terms of an activation pattern, as in Figure 13. The minimum value that separates the two "mountains" in the terrain defines objects.

The consequence of using the minimum point depends on the integration rule. In the computer simulations, if the spatial average rule were applied separately to the activation east and west of the minimum point, the consequence would be to shift the locations apart. On average, the alien to the west of the minimum point would be located farther to the west, and the alien to the east of the minimum point would be located farther to the east. The reason for these shifts is clear from Figure 13. Consider the west alien location. Activation to the right of the minimum point was truncated. Note that the distribution of perceived locations will still be a normal distribution because the distribution of sampling means is always normal regardless of the shape of the population distribution.

The computer simulations yielded more complex results with the winner-take-all rule. Depending on the parameters of the simulation, the distribution of location responses was shifted farther apart (as above) or closer together. In these simulations, they would generally be shifted closer together when the RF size was small. Using the winner-take-all integration rule, the distribution of locations over episodes was leptokurtic. Furthermore, depending on the parameters of the simulation, it could be asymmetric. That is, the distribution of locations of one spaceship would be skewed toward that of the other. Thus, in making predictions about how the presence of one object affects the perceived location of another object, it was important to characterize the shape of the distribution. Fortunately, the shape of the distributions of the location responses was normal. If the shape of the distribution is normal, then the minimum point always predicts that the aliens would be perceived as being farther apart than they actually are.

A second class of models is one in which a statistical solution based on a measure of dispersion is used. A *t* test would be an example of this sort of solution. A *t* test tests whether a datum is too far from a group of scores to be considered a member of that group. The idea behind the statistical solution is illustrated in Figure 13C. To simplify the simulations, a range statistic was used as the measure of dispersion. For example, all of those locations indicated by the bracket (on the left) were considered a part of one potential object (or spaceship). The location marked "?" was outside the range and therefore not considered part of that object. Thus, all of those locations that were within some critical range were considered together.

The idea behind the statistical dispersion model was implemented in the following manner. A series of "neighborhoods" was defined. A neighborhood is simply a group of contiguous locations that are all within *n* locations of each other. For example, if the neighborhood size was nine locations, the first nine locations from the left would be the first neighborhood. The bracket in Figure 13C is the third neighborhood from the left, and so forth. Many different neighborhood sizes were simulated. In each simulation, the two neighborhoods with the highest total activation were considered possible target locations. The information within each of these two neighborhoods was integrated (separately) according to the spatial average or winner-take-all rule to choose one location on each episode. Regardless of the integration rule, the statistical dispersion model tended to make items appear closer together, although the shape of the distribution was affected by the integration rule, as in the previous simulations. This shift occurs because the winning neighborhood includes activation from the other spacecraft. For example, in the figure the winning neighborhood in brackets includes activation from the spacecraft on the right. As in the previous simulations, the spatial average rule yielded a normal distribution and the winner-take-all rule yielded a leptokurtic distribution. In some circumstances, the latter distribution was asymmetric in the simulations.

There are probably many ways of implementing a statistical dispersion model in neural hardware. The way it was implemented here added a bit of the winner-take-all decision rule because only the two neighborhoods with the highest activations were considered. A better method might be to construct a hierarchical arrangement of detectors, with detectors high in the hierarchy being equivalent to the neighborhoods in the simulation. To the extent that a neighborhood detector is activated, it might send reciprocal activation back down to the members of that neighborhood, creating a "rich-get-richer" effect common in neural networks. Expressed colloquially, the statistical dispersion models clump together items that are close together.

In summary, the effect of a statistical dispersion model, however implemented, is to cause location assimilation: Stimuli will be perceived as closer together than they truly are. The effect of a minimum point model depends on the integration rule. If the distribution of location responses is a normal distribution, then the minimum point model predicts that stimuli will be perceived as farther apart than they actually are.

In this experiment, observers were briefly presented a small target dot and a large "landmark" dot. The primary task was to locate the small target dot, as in the previous experiments. The question is whether, on average, the observers would locate the target dot closer to or farther from the landmark dot than its actual position. The observers also indicated whether the target dot was to the left or to the right of the landmark dot. The reason for this additional task was to ensure that the observers saw two dots on each trial and knew which dot was the target. If the observers mistook the landmark dot for the target dot on some trials, the average location response would be artifactually biased toward the landmark location. Furthermore, to discriminate the models, it was also necessary to determine the shape of the response distribution.

Method

Procedure. The experiment was similar to Experiment 1 except that a pair of dots (a small target dot and a large landmark dot) was briefly presented. The exposure duration was 167.7 msec. The centers of the two dots were always horizontally aligned. The observer's task was to move the screen cursor (with the mouse) to the location of the small target dot. The observer then pressed the left mouse button if the target was to the left of the landmark, or pressed the right mouse button if the target was to the right of the landmark. Thus, with one buttonpress, the observers indicated the position of the target relative to the landmark and also the absolute position of the target dot. On the rare occasions on which the observer indicated the incorrect relative position (e.g., indicated that the target was to the left when it was to the right), the computer emitted a loud sound like that of a foghorn.

Each of 5 observers participated in a single session that lasted about 1.25 h. The observers were first given a practice block of 50 trials, and then data were collected for 10 blocks of 100 trials per block. The observers were volunteers, aged 22 to 30 years, who were naive to the purpose of the experiment.

Stimuli. The experiment was run with the same monitor, computer, and viewing distance as was Experiment 2. However, instead of gray dots, the stimuli were black, on a white background. The target dot was the same size as that used in Experiment 2 and subtended 0.34º of visual angle, with a diameter of 10 screen pixels. The landmark dot was four times larger in diameter than the target dot. The target dot was presented on the diameter of an imaginary circle that subtended 3.5º of visual angle from the fixation point (as in Experiment 2). On half of the trials, the landmark was to the left of the target dot, and on half of the trials it was on the right of the target. (The pair of dots on the left side of Figure 14A illustrates a stimulus with the landmark to the left of the target, and the pair of dots on the right side of Figure 14A illustrates a stimulus with the landmark on the right side of the target.) The distance from the center of the target to the center of the landmark subtended approximately 1.7º of visual angle (50 screen pixels). Note that at the viewing distance used, 26.6 pixels subtended 1º of visual angle.

Results

The observers were quite accurate at knowing whether the target dot was to the left or to the right of the landmark dot. Relative location averaged 99.3% correct (see Table 5). Trials with relative location errors were removed from subsequent analysis.

Because the landmark and target dots were always horizontally aligned, the errors in the horizontal direction

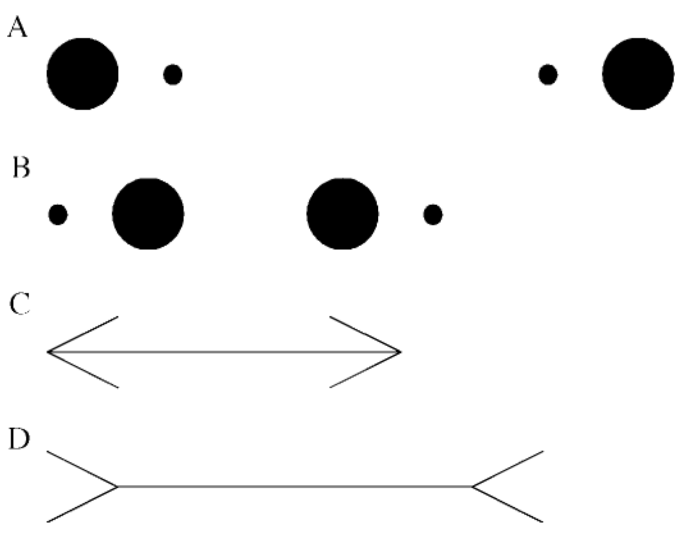

Figure 14. The stimuli used in Experiment 3 consisted of a single small dot and a single large dot that were the same relative size and distance apart as the pairs of adjacent dots in panel A or panel B. The small dots may appear farther apart in panel A than in panel B. Panels C and D suggest the relation to the Müller-Lyer illusion.

Table 5
Results of Experiment 3

Observer	Relative Location (%)	Skew	Landmark Right	Landmark Left	*t* test
1	99.4	−0.401	2.40	−13.98	$t(992) = 17.39$*
2	99.2	0.307	28.20	−5.87	$t(990) = 31.37$*
3	98.3	0.136	2.51	0.65	$t(981) = 1.69$
4	99.8	0.197	7.83	−2.61	$t(996) = 18.70$*
5	99.8	−0.171	−0.79	−4.49	$t(996) = 4.37$*
Means	99.3	0.014	8.03	−5.26	

Note—Skewness, shifts, and *t* values reflect only trials on which the relative location judgment was correct. The Landmark Right and Landmark Left columns are the average shifts in location response in pixels in the horizontal direction (26.6 pixels are 1º of visual angle). Positive numbers indicate a shift to the right, and negative numbers indicate a shift to the left. The *t* test is two-tailed. $^*p < .01$.

were analyzed to answer two questions: First, could the data be fit with a normal distribution, and, second, did responses shift toward or away from the landmark dot?

Before the results were fit to a normal distribution, an "adjusted" horizontal error was calculated in the following manner. When the landmark was on the left of the target, the sign of the horizontal error was reversed. No change was made to errors when the landmark was to the right of the target. Thus, if the landmark caused the responses to be skewed toward the landmark, the two conditions would not cancel each other out. This transformation made it possible to analyze both the landmark-left and the landmark-right conditions together.

Errors in the horizontal direction were fit to the normal (spatial average) and winner-take-all models, as in the previous experiments. The data were fit reasonably well by a normal distribution. For all the observers, more than 99.98% of the variance was accounted for by a normal distribution. The predicted and observed data for Observer 2 are shown in Figure 15. This observer had the worst fit (in terms of *SSE*), yet the normal distribution does a good job of describing the observer's performance with just two parameters. The addition of the third parameter did not significantly improve any of the fits.

Predictions of the winner-take-all integration model are not as simple as those in Experiments 1 and 2. With a landmark near the stimulus, the simulations sometimes produced skewed distributions. The moment coefficient of skew was calculated for each observer and is shown in Table 5. The distributions were very symmetrical, which of course is consistent with the normal distribution and the spatial average integration rule.

The mean of the responses was shifted toward the landmark. On average, the observers responded 8.03 pixels too far to the right when the landmark was to the right of the target and 5.26 pixels too far to the left when the landmark was to the left of the target. Table 5 shows the shifts for each observer. Each observer's average location shifted toward the landmark, and the shift toward the landmark was significant (by a *t* test) for each observer but Observer 3.

Discussion

In this experiment, two dots were briefly presented: a target dot and a landmark dot near the target dot. The observers had to indicate whether the target was to the left or to the right of the landmark, and they did this nearly flawlessly. However, the landmark affected the perceived location of the target. The observers perceived the target as being closer to the landmark than it actually was. This effect might be called *location assimilation*.

Although there was location assimilation, the distribution of perceived locations was still normal. It was not skewed toward (or away from) the landmark. Thus, the integration rule of the system as a whole still appears to be the spatial average rule.

The finding of location assimilation is perhaps not surprising. There have been several experiments in which a dot was presented inside a circle, and observers then reproduced the location of the dot (see, e.g., Huttenlocher et al., 1991; Laeng, Peters, & McCabe, 1998; Nelson & Chaiklin, 1980). The distortion is generally toward the periphery of the circle, which could be considered a landmark. Huttenlocher et al. and Laeng et al. also found distortions toward the center of the quadrant that contained the dot. These results were described as a drift toward the prototypical location of each quadrant. It might be that mental landmarks attract objects just as well as physical landmarks do.

Suzuki and Cavanagh (1997) found spatial repulsion away from an attended object. Because attention was not

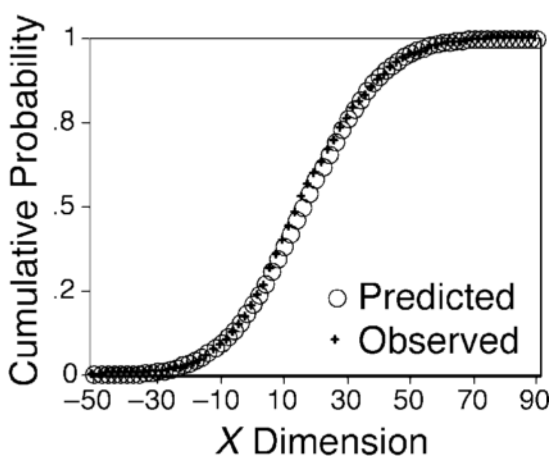

Figure 15. The fit of 1 observer to the normal distribution in Experiment 3.

controlled in Experiment 3, it is difficult to judge the relevance of these findings to the present experiment. Their explanation was that attention recruits more detectors in the attended location expanding mental space. Certainly, this explanation is not inconsistent with the more-samples model. More relevant, Werner and Diedrichsen (2002) found attraction toward landmarks except when the target was very close to the landmark. When the target was close to the landmark, the spatial distortion was away from the landmark. There were many differences between the Werner and Diedrichsen experiment and the present experiment, including the distances that were used and the fact that Werner and Diedrichsen used two landmarks. In general, as is discussed below, the literature suggests that location assimilation is the rule, but there may be exceptions.

It is interesting to speculate whether the location assimilation observed in this experiment is caused by the same mechanism as other kinds of spatial assimilation. For example, Pressey (1971) extended the idea of spatial assimilation to a number of classical illusions. Pressey's idea can be illustrated with the stimuli used in Experiment 3. Figures 14A and 14B illustrate the stimuli. However, in the figure two pairs of stimuli are presented in a Müller-Lyer-inspired arrangement. For many observers, the small dots in Figure 14A appear farther apart than the small dots in Figure 14B. This illusion might be considered a form of the Baldwin illusion (Coren & Girgus, 1978, pp. 31–33). It is interesting to note that after the experiment, all of the observers who participated in Experiment 3 were debriefed and shown a drawing that was similar to that of Figures 14A and 14B. Observer 3 was the only observer that did not perceive the dots as farther apart in panel A than in panel B. This observer also showed the smallest assimilation effect in the experiment.

Figures 14B and 14D provocatively include the Müller-Lyer illusion in comparison with the stimuli used in the present experiment. One could think of the Müller-Lyer illusion as a case of position assimilation, as is suggested in the figure. In fact, the statistical dispersion model could be considered an instantiation of Müller-Lyer's original theory: "two lines are judged to be of different lengths because it is not only both lines themselves which are taken into account, but, involuntarily, part of the region on either side of them" (Müller-Lyer, 1896, translated in Day & Knuth, 1981, p. 137).

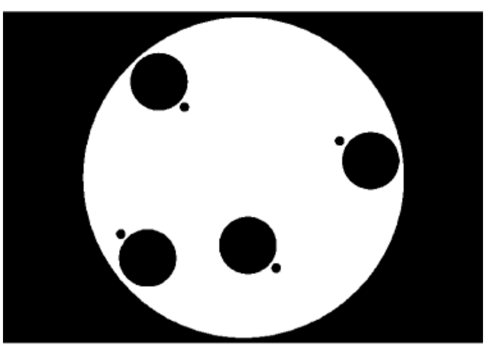

Figure 16. The gravity-lens illusion. Imagine a line connecting the two small dots on the left of the figure and a different line connecting the two small dots on the right of the figure. These two imagined lines do not seem parallel, but they are.

As a final example of location assimilation, consider the gravity lens illusion, shown in Figure 16 (Greene, 1998; Naito & Cole, 1994). Imagine a line connecting the two small dots that are on the left side of the figure and a separate line connecting the two small dots on the right side of the figure. The reader might be surprised to learn that a line drawn between the small dots on the left of the figure is parallel to a line drawn between the two small dots on the right. It is as if the larger dots attract the smaller dots just as gravity attracts planets. The model of assimilation depicted here could be described as a mechanism for implementing this mental gravity.

Position assimilation illusions have also been explained in terms of the smearing that might occur with low-pass filtering (see, e.g., Ginsburg, 1984). A fourier analysis of an image includes all the information in the image. Hence, the issue is not whether a fourier image analysis is an appropriate way to represent the information in an image. The issue is whether these illusions are best conceptualized in the frequency domain or in the phase domain. The mechanism proposed here is in the phase domain as opposed to the frequency domain. That is, the location of objects, and not their spatial frequency, is misperceived. An attractive aspect of the present approach is that it corresponds to the phenomenology of the experiment: The position of the target appears shifted in location; it does not appear blurred.

GENERAL DISCUSSION

In the experiments reported in this article, I examined three issues involving the mechanisms of location perception: (1) how the visual system integrates location information, (2) how attention affects location perception, and (3) how the presence of one object affects the location of other objects. In each case, an attempt was made to contrast two classes of theories. For example, in the first experiment information integration by a winner-take-all rule was compared with integration by a spatial average rule. Of course, these different classes of models could be placed on a continuum. The winner-take-all rule gives a large (infinite) weight to the highest activated detector, whereas the spatial average rule weights all information equally. One could imagine a full range of models between these two. For example, the activation value at each location might be raised to a power. Nevertheless, highlighting the differences between the two basic models served the purpose of making clear, testable predictions possible.

Two ways in which an attentional mechanism could be modeled were also examined. One was based on the idea that attention could be conceptualized as an increase in sampling; the other was based on the idea that attention produced tighter tuning functions. Simulations of the tighter tuning functions theory simply did not work. Operationalizing the "more-samples" idea is admittedly difficult. The approach taken in the present work was to determine whether the effect of attention was quantitatively similar to the effect of increasing exposure duration. The assumption was that both attention and exposure duration are related to the same underlying variable: more samples. For 7 of the 8 observers who showed an affect of attention, attention and exposure duration fit the data with a single parameter by which more attention was considered to be equivalent to a constant (or multiplicative) increase in exposure duration.

In the final experiment, models that divide activation in terms of a minimum value (Logan, 1996) were contrasted with a class of models that use a statistical dispersion approach. Because we could characterize the integration rule, these theories made clear predictions. The statistical dispersion approach predicts location assimilation, and the minimum value model predicts location repulsion. Although there may be cases of repulsion (as discussed above), Experiment 3 was consistent with the statistical dispersion model.

The three issues are interrelated. Each of the models considered had to make assumptions about the integration rule. A characterization of the integration rule was necessary to test the models of the effect of a landmark. Furthermore, the spatial average rule suggested a theoretically motivated theory of attention. The issues could be considered separately, but much of the theoretical power of the models would not have been possible had the issues not been considered together.

In the remaining discussion, I first consider methodological issues raised by the present work and, second, I consider one critical issue for future research. In both sections, I attempt to highlight the strengths and limitations of the present work.

The methodological approach taken here involved four steps: a theory expressed verbally (i.e., the *X-Files* parable), computer simulations, a mathematical description of the output of the simulations, and, finally, an experiment. The alternative theories were first expressed in terms of a fanciful story of the FBI's locating alien spacecraft. In addition to amusing the author, expressing the models in this manner was useful in several ways. Although the models can easily be implemented in neural hardware, it is important to keep in mind that they refer to general design characteristics of the visual system, not to any particular area in the brain. By discussing radar fields instead of receptive fields, it should be clear that the model is not a model of a particular visual area (e.g., V1). Furthermore, by referring to easily comprehended physical mechanisms, the temptation to evolve the models into multiple-layer neural networks was avoided. The issue is not whether such implementations are correct or incorrect; rather, the concern is that such implementations can be difficult to comprehend. A simple parable made it easy to describe the problem and the general structure of different solutions. The story and the models are simply metaphors for the visual system. It is clear that the visual system did not evolve to (only) detect and locate alien spacecraft.

Computer simulations played a vital role in all of the work. Before the simulations were conducted, the consequences of the theories were not always obvious. For example, my colleagues and I had previously predicted that the distribution of perceived locations would be Gaussian (Ashby et al., 1996). The motivation for this prediction was guided as much by mathematical convenience as by anything else. Computer simulations using a winner-take-all integration rule never yield a Gaussian distribution. Similarly, in thinking about the mechanisms of attention, the first guess was that the tighter-tuning-functions theory could account for the improvement of location perception with attention (see, e.g., Prinzmetal et al., 1998). This idea seemed to be in accord with physiological evidence (Moran & Desimone, 1985; but see McAdams & Maunsell, 1999). After all the attempts to simulate attention in this manner failed, it became obvious why such a scheme was doomed: If each detector gets a smaller receptive field, the number of detectors guarding each location decreases. One way to make such a model work would be to add more detectors (i.e., more samples). However, adding more samples in and of itself was sufficient to model the effect of attention. In the final experiment with two objects, the fact that predictions of the minimum point model depended on the integration rule was a complete surprise. In summary, it is well worth going beyond a verbal description of a theory. There may be surprises when a theory is actually simulated.

These experiments differed from many psychological experiments in another way. The typical experiment usually makes predictions about the ordinal relation among means. For example, as experimental psychologists, we are usually satisfied if one mean reaction time is significantly greater than another. Occasionally, we might look for a particular functional relation between means (see, e.g., Sternberg, 1969). In the present experiments, the exact shape of the response distribution was critical in deciding between theories. In addition, precise predictions about variance were also critical in testing other theories. Of course, this study is not unique in examining statistics beyond measures of central tendency. However, it does illustrate that we can make theoretical progress by looking beyond the first moment of the distribution. There is a cost to this approach, however. It was necessary to gather much more data per observer than if only means had been examined.

One of the more interesting challenges for future study arises from the fact that the present work was limited to strictly metric aspects of location perception, a kind of simplistic "dot-ology." The perception of object location in natural environments tends to involve categorical or other types of relations (Huttenlocher et al., 1991; Stevens & Coupe, 1978; Tversky, 1981). In localizing my computer, for example, I know that it is *on* my worktable, to the *left* of the telephone. An important issue is how to model the transition from a simple metric spatial representation to one that involves spatial relations. In the last experiment, for example, the observers indicated not only the absolute location of the target dot but also whether it was to the left or to the right of the landmark. That is, the observers extracted a spatial relation, not just a position.

It is likely straightforward to derive models that extract spatial relation from purely metric relationships. Imagine an *X-Files* problem involving two alien spacecraft, one from Mars and one from Venus. An example of a question involving spatial relations is whether the Martian craft is to the west or to the east of the Venusian craft. This question is precisely the task given to the observers in Experiment 3, who were asked whether the small dot was to the left or to the right of the large dot. One might imagine a model with a 2-D array of detectors, but some detectors are more sensitive to Martian spacecraft and others are more sensitive to Venusian spacecraft. In addition, this problem probably could not be solved without some metric of latitude or an idea of meridians of longitude. In perceptual terms, some frame of reference would be required.

The question, therefore, is not whether the visual system becomes “categorical” in the perception of location; it surely does. Furthermore, modeling such processes will not be particularly difficult when the approach taken here is used. Such a model might provide a good fit of the data. However, as Roberts and Pashler (2000) argued, model fits by themselves are not very informative. An effort was made in the present work to ask fundamental computational questions about perception. For example, in the first experiment I asked whether the visual system maximizes the probability of precisely localizing objects (winner-take-all model) or whether it functions to minimize errors in location judgments (spatial average model). The challenge for future research is to ask this type of fundamental question about the derivation of spatial relations in location perception.

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NOTES

1. I thank and acknowledge Leonid Kontsevich for suggesting this method for fitting the winner-take-all function.

2. Variance is the average squared distance from the response to the centroid of responses for each condition, whereas precision is measured from the stimulus location. Since there was very little constant error, the results in terms of precision were almost identical to the results in terms of variance. In the present experiments, precision gave slightly more systematic results than variance, perhaps because the centroid of responses for a condition is only an estimate of the constant error of that condition. However, in no case were the results with precision systematically different from those with variance.

3. This parameterization of the more-samples model was suggested by Thomas Sanocki.

4. The standard error of the variance is

$$\sigma_{\mathrm{SE}}^2 = \sigma^2 * \sqrt{\frac{2}{n}}.$$

Precision was used for σ. Note that the sampling distribution of the variance is χ^2. With such a large number of observations, however, the sampling distribution can probably be treated as a normal distribution.

APPENDIX

The distribution of location responses for the spatial average rule is easy to characterize, but the distribution for the winner-take-all rule is more complex. To understand this distribution, first consider the spatial average rule. Locations were numbered, starting from the leftmost location. The sum of location x activation divided by the sum of all the activation was the mean location. Each trial (i.e., each *X-Files* episode) can be considered a sample. The distribution of sample means is a normal distribution.

To understand the distribution from the winner-take-all rule, first consider only the target location and all the locations to the left of the target. The "highest value" (or winner) is the location with the highest activation value. The distribution of maximum values will be determined according to the following function:

$$f(x) = c * \exp\left[\frac{-(u-x)^{a}}{s}\right] \text{ for } x < u.$$

Considering the location to the right of the target, the distribution of minimum values will be the mirror of the above:

$$f(x) = c * \exp\left[\frac{-(x-u)^{-a}}{s}\right] \text{ for } x > u.$$

These functions are Type 2 and Type 3 extreme value functions (Johnson et al., 1995, chap. 1). The squares in Figure A1 are the results of the cumulative distribution of a typical winner-take-all simulation, and the diamonds are generated by the extreme value function shown above. The results of the winner-take-all simulation could always be fit by the extreme value functions, just as the results of the spatial average simulation could always be fit by the normal distribution.

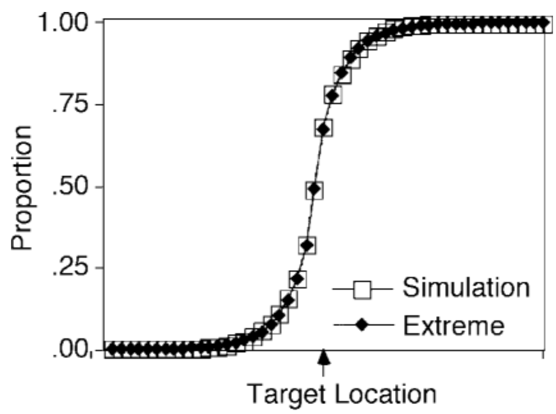

Figure A1. The extreme value function and a winner-take-all simulation.

The extreme value function becomes a normal distribution when $a = 2$. Thus, a is critically related to the kurtosis of the distribution.

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