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Do multiplication and division strategies rely on executive and phonological working memory resources?

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The role of executive and phonological working memory resources in simple arithmetic was investigated in two experiments. Participants had to solve simple multiplication problems (e.g., 4 × 8; Experiment 1) or simple division problems (e.g., 42 ÷ 7; Experiment 2) under no-load, phonological-load, and executive-load conditions. The choice/no-choice method was used to investigate strategy execution and strategy selection independently. Results for strategy execution showed that executive working memory resources were involved in direct memory retrieval of both multiplication and division facts. Executive working memory resources were also involved in the use of nonretrieval strategies. Phonological working memory resources, on the other hand, tended to be involved in nonretrieval strategies only. Results for strategy selection showed no effects of working memory load. Finally, correlation analyses showed that both strategy execution and strategy selection correlated with individual-difference variables, such as gender, math anxiety, associative strength, calculator use, arithmetic skill, and math experience.

Working memory is a system devoted to short-term storage and processing and is used in various cognitive tasks, such as reading, reasoning, and mental arithmetic. Throughout the past decennium, research into the role of working memory in mental arithmetic has flourished (for a review, see DeStefano & LeFevre, 2004) and has shown that solving both simple arithmetic problems (e.g., 8 + 5, 3 × 9) and complex arithmetic problems (e.g., 23 + 98, 12 × 35) relies on working memory resources. The present study further investigates the role of working memory in simple-arithmetic strategies, on the basis of the multicomponent working memory model of Baddeley and Hitch (1974). In this model, there is an attentional system (the central executive) that supervises a phonological subsystem and a visuospatial subsystem. The phonological subsystem guarantees short-term maintenance of phonological information, and the visuospatial subsystem guarantees short-term maintenance of visuospatial information.

The role of executive working memory resources in simple arithmetic has been shown extensively (see, e.g., Ashcraft, 1995; De Rammelaere, Stuyven, & Vandierendonck, 1999, 2001; De Rammelaere & Vandierendonck, 2001; Deschuyteneer & Vandierendonck, 2005a, 2005b; Deschuyteneer, Vandierendonck, & Muyllaert, 2006; Hecht, 2002; Lemaire, Abdi, & Fayol, 1996; Seitz & Schumann-Hengsteler, 2000, 2002). The role of phonological working memory resources in simple arithmetic is less clear. In some studies, an effect of phonological load on simple-arithmetic problem solving was observed (see, e.g., Lee & Kang, 2002; Lemaire et al., 1996; Seitz & Schumann-Hengsteler, 2002), whereas, in other studies, it was not (see, e.g., De Rammelaere et al., 1999, 2001; Seitz & Schumann-Hengsteler, 2000). Investigations of the role of the visuospatial "sketch pad" in simple arithmetic are scarce (but see Lee & Kang, 2002; Seitz & Schumann-Hengsteler, 2000), and the findings in the few studies conducted are equivocal.

A drawback of all the studies mentioned above, however, is that none of them showed any distinction between retrieval and nonretrieval trials. Yet it has been shown that adults use several strategies to solve even the simplest arithmetic problems (see, e.g., Hecht, 1999; LeFevre, Bisanz, et al., 1996; LeFevre, Sadesky, & Bisanz, 1996). For instance, although direct memory retrieval (i.e., *knowing* that 3 × 4 = 12) is the most frequently used strategy, nonretrieval strategies (or *procedural* strategies) are used as well. Examples of such procedural strategies are transformation (e.g., 9 × 6 = (10 × 6) − 6 = 60 − 6 = 54) and counting (e.g., 4 × 7 = 7 . . . 14 . . . 21 . . . 28). The studies mentioned above notwithstanding, it is impossible to discern the specific simple-arithmetic strategies in which executive and phonological working memory resources are needed.

Investigations of the roles of executive and phonological working memory across different simple-arithmetic strategies have only very recently been conducted, beginning with Hecht (2002). In his study, simple addition equa-

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tions (e.g., 4 + 3 = 8) had to be verified under no load, phonological load, and executive load. After each trial, participants had to report which strategy they had used. Results showed that all strategies (i.e., retrieval, transformation, and counting) were slowed under executive working memory loads, whereas only the counting strategy was slowed under phonological working memory loads. On the basis of the results of his regression analyses, however, Hecht concluded that retrieval does not rely on the central executive, whereas the counting strategy relies on both executive and phonological working memory resources.

Seyler, Kirk, and Ashcraft (2003) also studied the role of working memory in simple-arithmetic strategies. In their first experiment, simple subtraction problems had to be solved while a two-, four-, or six-letter string had to be remembered. Results showed that solving subtraction problems with minuends of 11 or greater (e.g., 11 − 5) relied more heavily on working memory than did solving problems with minuends smaller than 11 (e.g., 8 − 5). In an experiment using strategy reports, Seyler et al. (2003) showed that subtraction problems with minuends of 11 or greater were more frequently solved with procedural strategies than were problems with minuends smaller than 11. It was concluded that working memory is more involved when simple subtraction problems are solved via procedural strategies.

A drawback of both previous studies is that neither Hecht (2002) nor Seyler et al. (2003) controlled for strategy selection effects; participants were always free to choose any strategy they wanted. Consequently, nonretrieval strategies would have been employed more frequently on large problems, whereas retrieval would have been employed more frequently on small problems. Such strategy selection effects may have influenced strategy efficiency data and, therefore, all resulting conclusions. In order to exclude such biasing effects of strategy selection on strategy efficiency, the choice/no-choice method (Siegler & Lemaire, 1997) should be used. Using the choice/no-choice method in combination with selective working memory loads provides unbiased data about the role of working memory in strategy selection and strategy efficiency.

The combination of the choice/no-choice method and selective working memory loads was first used by Imbo and Vandierendonck (in press). They investigated the role of executive and phonological working memory resources in simple-arithmetic strategies. In their study, simple addition and subtraction problems had to be solved under no-load, passive phonological load, active phonological load, or central executive load conditions. Results showed that retrieval of addition and subtraction facts relied on executive working memory resources. Solving addition or subtraction problems by means of a nonretrieval strategy, on the other hand, required both executive and active phonological working memory resources. The passive phonological store was involved only when counting was used to solve subtraction problems. Obviously, the role of executive and phonological working memory resources was significantly larger in nonretrieval strategies (i.e., transformation and counting) than in direct memory retrieval.

To summarize, the three studies described above showed that the role of working memory differs across strategies. Whether the central executive is needed in retrieval remains a debated topic. Hecht (2002) does not believe that this working memory component is needed in retrieval, whereas Imbo and Vandierendonck (in press) present evidence that retrieval requires executive working memory resources. Nevertheless, all three studies seem to agree that phonological working memory resources are needed when nonretrieval strategies are used to solve simple addition and subtraction problems.

Whereas our knowledge about the role of working memory in addition and subtraction strategies is limited, we know practically nothing about the role of working memory in multiplication and division strategies. For instance, we know that solving simple multiplication and division problems requires working memory resources (De Rammelaere & Vandierendonck, 2001; Deschuyteneer & Vandierendonck, 2005b; Deschuyteneer et al., 2006; Lee & Kang, 2002; Seitz & Schumann-Hengsteler, 2000, 2002), but up until now, no study has investigated the role of working memory across the different multiplication and division strategies.

Multiplication and division certainly cannot be said to be the counterparts of addition and subtraction, and studying the role of working memory in multiplication and division strategies is far more than just an extension of previous research. Indeed, there exist many differences across operations, especially between addition and subtraction on the one hand and multiplication and division on the other. Differences across the execution of arithmetic operations begin to develop during childhood and continue to exist in adulthood. Addition and subtraction problem-solving procedures are taught before those for multiplication and division. Furthermore, the acquisition of addition and subtraction skills and strategies is mainly based on counting procedures, whereas the acquisition of multiplication and division skills and strategies is based on the memorization of problem–answer pairs. In adults, the highest percentages of retrieval use are observed in multiplication (98%), whereas the lowest percentages of retrieval use are observed in division (69%), with those for addition and subtraction lying in between (88% and 72%, respectively; Campbell & Xue, 2001). Also, adults' strategy efficiencies differ greatly across operations, with multiplication RTs (930 msec) being much faster than division RTs (1,086 msec; Campbell & Xue, 2001).

These results seem to suggest that accessing long-term memory and selecting the correct response are very difficult for division, but rather easy for multiplication. As getting access to long-term memory and selecting the correct response are processes requiring executive working memory resources, it is quite clear that an executive load must affect division efficiency. It is less certain, however, whether an executive load will affect the overlearned retrieval of multiplication facts. It may also be expected that phonological working memory loads will affect nonretrieval strategy efficiencies, but not retrieval strategy efficiencies. Indeed, when nonretrieval strategies are used, intermediate values have to be kept in working memory temporarily, a function accomplished by the phonological working memory component (Ashcraft, 1995). Effects of phonological working memory loads on nonretrieval strategies have been observed in addition and subtraction, but several authors (see, e.g., Campbell, 1994; Dehaene, 1997) suppose that multiplication is more heavily based on auditory–verbal number codes than are other operations. Effects of phonological working memory loads may, therefore, be more readily apparent in the present study.

In order to investigate the role of executive and phonological working memory resources[1] in multiplication and division strategies, the present study included two frequently used and approved methods: the selective-interference paradigm and the choice/no-choice method. The selective-interference paradigm is the methodological approach most frequently chosen for studying the role of different working memory resources in mental arithmetic. It entails using both a single-task condition, in which the primary task (mental arithmetic) is executed without any working memory load, and a dual-task condition, in which the primary task is combined with a secondary task that serves to load a specific working memory component. If both the primary and the secondary tasks demand the same working memory resources, performance decrements should be observed in either task. In the present study, three secondary tasks were used to load three specific working memory components—more specifically, the passive phonological component (the phonological store), the active phonological component (the subvocal rehearsal process), and the central executive.

The choice/no-choice method (designed by Siegler & Lemaire, 1997) is used to collect data on strategy selection (Which strategies are chosen?) and strategy efficiency (Are strategies executed efficiently?) independently. In this method, each participant is tested under two types of conditions: a choice condition, in which participants are free to choose any strategy they want, and no-choice conditions, in which participants are required to solve all the problems using one particular strategy. There are as many no-choice conditions as there are strategies available in the choice condition. Data obtained under no-choice conditions provide information about strategy efficiency, whereas data gathered under the choice condition provide information about strategy selection.

In addition to investigating the role of working memory in multiplication and division strategies, the present study was also designed to test whether simple-arithmetic strategies are influenced by factors other than those imposed by the experimenter. For each participant, several individual-difference measures were obtained—namely, arithmetic skill, math experience, gender, calculator use, math anxiety, and associative strength. Effects of arithmetic skill have already been reported (see, e.g., Campbell & Xue, 2001; Gilles, Masse, & Lemaire, 2001; Kirk & Ashcraft, 2001; LeFevre & Bisanz, 1986; LeFevre, Bisanz, et al., 1996; LeFevre, Sadesky, & Bisanz, 1996). Generally, strategy use is more efficient (i.e., faster) in high-skill participants than in low-skill participants. Effects of math experience, in contrast, have been reported only rarely. However, Roussel, Fayol, and Barrouillet (2002) observed that experienced participants (primary school teachers) performed slower on arithmetic tasks than did inexperienced participants (undergraduate psychology students). In contrast, experienced and inexperienced participants did not differ in their strategy choices. In one of our own studies, arithmetic experience (based on the participants' high school curricula) was found to predict both strategy selection and strategy efficiency for multiplication problems, but not for addition problems (Imbo, Vandierendonck, & Rosseel, in press). Gender effects have been investigated in children, rather than in adults. Several studies with children showed more frequent and more efficient retrieval use in boys than in girls (see, e.g., Carr & Jessup, 1997; Carr, Jessup, & Fuller, 1999; Royer, Tronsky, Chan, Jackson, & Marchant, 1999). Whether these differences exist in adulthood is a debated topic. Because some recent studies (see, e.g., Geary, Saults, Liu, & Hoard, 2000; Imbo et al., in press) showed significant gender differences in adults' arithmetic processing, with males outperforming females, gender was included in the present study. Only two studies investigated the possible effects of calculator use, one showing no effects (Campbell & Xue, 2001), and the other showing effects of calculator use on strategy efficiency (Imbo et al., in press). Participants who reported highly frequent calculator use were remarkably slower in both retrieval efficiency and procedural efficiency. The present study purposely addressed this issue and included a short questionnaire about calculator use. For math anxiety, it was expected that high-anxious participants would perform worse on the simple-arithmetic tasks than would the low-anxious participants. Effects of math anxiety have previously been shown in complex-arithmetic tasks (see, e.g., Ashcraft & Kirk, 2001), but not yet in simple-arithmetic tasks. The associative strength variable, finally, is an estimate of how strong the participants' problem–answer associations are in long-term memory and is operationalized as the participants' percentage of retrieval use in choice conditions. It was hypothesized that participants with stronger problem–answer associations would be faster at retrieving arithmetic facts from long-term memory.

EXPERIMENT 1
Multiplication

Method

Participants. Sixty participants took part in the present experiment (15 men and 45 women). Their mean age was 21 years and 0 months. Half of them were first-year psychology students at Ghent University who participated for course requirements and credits. The other half were paid €10 for their participation.

Procedure. Each participant was tested individually in a quiet room for approximately 1 h. The experiment started with short questions about the age of the participant and his/her math experience (i.e., the number of mathematics lessons per week during the last year of secondary school), calculator use (on a rating scale from 1 *never* to 5 *always*), and math anxiety (on a rating scale from 1 *low* to 5 *high*).[2] All participants solved the simple-arithmetic problems under four conditions: the choice condition (in order to exclude influence of no-choice conditions on the choice condition), and then three no-choice conditions, the order of which was randomized across participants. In the choice condition, 6 practice problems and

42 experimental problems were presented. After the choice condition, participants needed no more practice; the no-choice conditions, therefore, comprised the 42 experimental problems only. Each condition was further divided into two blocks: one in which no working memory component was loaded and another in which one working memory component was loaded. The working memory load differed across participants: For 20 participants, the central executive was loaded; for 20 other participants, the active phonological rehearsal process was loaded; and for the remaining 20 participants, the passive phonological store was loaded. For half of the participants, each condition started with the no-load block and was followed by the working memory load block; the order was reversed for the other half of the participants.

Simple-arithmetic task. The multiplication problems presented in the simple-arithmetic task consisted of 2 one-digit numbers (e.g., 6×7). Problems involving 0, 1, or 2 as an operand (e.g., 5×0, 1×4, 2×3) and tie problems (e.g., 3×3) were excluded. Commuted pairs (e.g., 9×4 and 4×9) were considered to be two different problems. These selection criteria resulted in 42 multiplication problems ranging from 3×4 to 9×8. *Small problems* were defined as problems with a correct product smaller than 25, whereas *large problems* were defined as problems with a correct product larger than 25 (Campbell, 1997; Campbell & Xue, 2001). A trial started with the presentation of a fixation point for 500 msec. Then the multiplication problem was presented horizontally in the center of the screen, with the operation sign at the fixation point. The problem remained on the screen until the participant responded. Timing began when the stimulus appeared and ended when the response triggered the sound-activated relay. To enable this sound-activated relay, we had participants wear a microphone that was activated when they spoke their answer. This microphone was connected to a software clock accurate to within 1 msec. On each trial, feedback was presented to the participants: a green “Correct” when their answer was correct and a red “Incorrect” when it was not.

Immediately after solving each problem, participants in the choice condition were presented four strategies on the screen (see, e.g., Campbell & Gunter, 2002; Campbell & Xue, 2001; Kirk & Ashcraft, 2001; LeFevre, Sadesky, & Bisanz, 1996; Seyler et al., 2003): retrieval, counting, transformation, and other. These four choices had been extensively explained by the experimenter:

1. Retrieval: You solve the problem by remembering or knowing the answer directly from memory. It means that you know the answer without any additional processing. For example, you know that $5 \times 6 = 30$ because 30 “pops into your head.”

2. Counting: You solve the problem by counting a certain number of times to get the answer. You recite the tables of multiplication. For example, $4 \times 7 = 7 \ldots 14 \ldots 21 \ldots 28$, or $5 \times 3 = 5 \ldots 10 \ldots 15$.

3. Transformation: You solve the problem by referring to related operations or by deriving the answer from known facts. You change the presented problem to take advantage of a known arithmetical fact. For example, $9 \times 8 = (10 \times 8) - 8 = 80 - 8 = 72$, or $6 \times 7 = (6 \times 6) + 6 = 36 + 6 = 42$.

4. Other: You solve the problem by a strategy unlisted here, or you do not know what strategy you used to solve the problem. For example, guessing.

After each problem, participants were asked to verbally report which of these strategies they had used. The experimenter also emphasized that the presented strategies were not meant to encourage use of a particular strategy. If the participant felt like using only one of the presented strategies, he/she was completely free to do so. When the participant acknowledged generally using a mix of strategies, he/she was free to do this as well.

In the no-choice conditions, participants were asked to use one particular strategy to solve all problems. In the no-choice/retrieval condition, they were required to retrieve the answer. More specifically, participants were asked to pronounce the answer that first popped into their heads. In the no-choice/transformation condition, participants were required to transform the problem by employing an intermediate step. The experimenter proposed several intermediate steps, and all participants recognized using at least a few of them. Examples were as follows: (1) going via tens, for example, $9 \times 6 = (10 \times 6) - 6 = 60 - 6 = 54$, and $5 \times 7 = (10 \times 7) \div 2 = 70 \div 2 = 35$; (2) using the double, for example, $4 \times 6 = 2 \times 2 \times 6 = 2 \times 12$; and (3) using ties, for example, $7 \times 8 = (7 \times 7) + 7 = 49 \times 7 = 56$. However, if participants normally used any transformation step not proposed by the experimenter, they were free to do so. In the no-choice/counting condition, participants had to say (subvocally) the tables of multiplication until they reached the correct total (e.g., $4 \times 7 = 7 \ldots 14 \ldots 21 \ldots 28$). After having solved the problem, participants also had to answer *yes* or *no* to indicate whether they had succeeded in using the required strategy. This enabled us to exclude noncompliant trials from our analyses.

In the choice and no-choice conditions, the experimenter recorded the answer of the participant, the strategy information, and the validity of the trial online. All invalid trials, such as failures of the voice-activated relay, were discarded and returned at the end of the block, which minimized data loss due to unwanted failures.

Executive secondary task. A continuous choice reaction time task (CRT task) was used to load the executive working memory component (Szmalec, Vandierendonck, & Kemps, 2005). Stimuli for this task consisted of low tones (262 Hz) and high tones (524 Hz) that were sequentially presented with intervals of 900 or 1,500 msec. Participants had to press the 4 on the numerical keyboard when they heard a high tone and the 1 when they heard a low tone. The duration of each tone was 200 msec. The tones were presented continuously during the simple-arithmetic task. The CRT task was also performed alone (i.e., without the concurrent solving of arithmetic problems). In this single-task condition, the multiplication problems were presented with their correct answers, which the participants had to read off the screen. This ensured that the procedure and vocalization in the single-task condition remained very similar to the procedure and vocalization in the dual-task condition. Differences in secondary-task performance could thus only be due to the mental arithmetic process itself.

Active phonological secondary task. In this task, letter strings of three consonants (e.g., “T K X”) were read aloud by the experimenter. Known letter strings (e.g., BMW, LSD) were not used. The participant had to retain these letters and repeat them after three simple-arithmetic problems. Following the participant’s response, the experimenter presented a new three-letter string. This task was also tested individually (i.e., without the concurrent solving of arithmetic problems), using the same methodology as in the CRT single-task condition.

Passive phonological secondary task. In this task, irrelevant speech was presented to the participants. This speech consisted of dialogues in Swedish, taken from a compact disc used in courses of language instruction. The Swedish dialogues were presented at an agreeable loudness (around 70 dB) through headphones. Because both Swedish and Dutch (the participants’ native language) are Germanic languages, phonemes strongly match between the two languages. None of the participants had any knowledge of Swedish.

French Kit. After the simple-arithmetic experiment, all participants completed the French Kit (French, Ekstrom, & Price, 1963), a paper-and-pencil test of complex arithmetic. The test consisted of two subtests: one page with complex addition problems and one page with complex subtraction and multiplication problems. Participants were given 2 min per page and were instructed to solve the problems as quickly and accurately as possible. The correct answers on both subtests were summed to yield a total score of arithmetic skill.

Results

Failures of the sound-activated relay spoiled 6.9% of the trials. All these invalid trials were presented again at the end of the block, so most of them were recovered from data loss. This reduced the percentage of trials that were spoiled due to failures of the sound-activated relay to 1.8%. All incorrect trials (4.4%), all choice trials on which participants reported having used an *other* strategy (0.1%), and all no-choice trials on which participants failed to use the required strategy (8.8%) were deleted. All data were analyzed on the basis of the multivariate general linear model, and all reported results were considered to be significant if $p < .05$, unless stated otherwise.

To test whether the three subject groups—loaded by the passive phonological task, the active phonological task, or the executive task—differed from each other, ANOVAs were conducted on the scores from the French Kit[3] (arithmetic skill), the scores from the calculator-use questionnaire, the number of arithmetic lessons in the last year of secondary school (math experience), and the scores from the math anxiety questionnaire. Results showed that the three groups did not differ in any of these variables (all $F\text{s} < 1.2$, all $p\text{s} > .30$).

Strategy efficiency. Only the RTs uncontaminated by strategy choices (i.e., no-choice RTs) were considered, since only these RTs provided clear data concerning strategy efficiency. A 3 × 2 × 3 × 2 ANOVA was conducted on correct RTs with working memory component (passive phonological, active phonological, executive) as a between-subjects factor and load (no load vs. load), strategy (retrieval, transformation, counting), and size (small vs. large) as within-subjects factors (see Table 1).

The main effects of load, size, and strategy were significant [$F(1,57) = 10.24$, $MS_e = 1,326,374$, $F(1,57) = 198.87$, $MS_e = 1,598,084$, and $F(2,56) = 110.27$, $MS_e = 5,221,560$, respectively]. RTs were longer under load (3,061 msec) than under no load (2,786 msec) and longer for large problems (3,588 msec) than for small problems (2,259 msec). RTs were also longer for counting (4,759 msec) than for transformation (2,992 msec) [$F(1,57) = 138.10$, $MS_e = 3,378,924$] and longer for transformation than for retrieval (1,020 msec) [$F(1,57) = 82.98$, $MS_e = 4,514,306$]. The main effect of strategy was modified by a strategy × load interaction and a strategy × size interaction. The strategy × load interaction [$F(2,56) = 5.15$, $MS_e = 683,977$] indicated that the load effect (i.e., load RTs − no-load RTs) was larger for counting than for retrieval [$F(1,57) = 10.04$, $MS_e = 750,311$] and larger for counting than for transformation [$F(1,57) = 7.01$, $MS_e = 807,632$]. Load effects did not differ between retrieval and transformation [$F(1,57) < 1$]. The strategy × size interaction [$F(2,56) = 69.61$, $MS_e = 1,705,536$] indicated that the problem-size effect (i.e., RTs on large problems − RTs on small problems) was larger in counting than in retrieval [$F(1,59) = 141.63$, $MS_e = 2,275,821$] and larger in counting than in transformation [$F(1,59) = 132.01$, $MS_e = 262,806$], but as large in retrieval as in transformation [$F(1,59) = 2.13$, $MS_e = 212,582$, $p = .15$].

The working memory component × load interaction did not reach significance [$F(2,57) = 1.91$, $MS_e = 1,326,374$, $p = .16$]. However, since differential load effects were predicted for the different working memory components, planned comparisons were conducted. These analyses showed that the effect of load (i.e., load RTs − no-load RTs) was significant for the executive component [$F(1,57) = 11.59$, $MS_e = 1,326,374$], but did not reach significance for the active phonological component [$F(1,57) = 1.87$, $p = .18$] or the passive phonological component [$F(1,57) < 1$]. This interpretation was verified by separate ANOVAs that tested the effects of the different working memory loads for each single strategy. Retrieval RTs were affected by an executive load [$F(1,57) = 35.69$, $MS_e = 28,055$], but not by an active phonological load [$F(1,57) = 2.38$, $p = .13$] or a passive phonological load [$F(1,57) < 1$]. Transformation RTs tended to be affected by an executive load [$F(1,57) = 2.88$, $MS_e = 1,054,430$, $p = .09$], but not by an active phonological load [$F(1,57) < 1$] or a passive phonological load [$F(1,57) < 1$]. Counting RTs, finally, were affected by an executive load [$F(1,57) = 10.16$, $MS_e = 1,611,840$] and tended to be affected by an active phonological load [$F(1,57) = 2.75$, $MS_e = 1,611,840$, $p = .10$], but were not affected by a passive phonological load [$F(1,59) = 1.82$,

Table 1
Mean Correct Reaction Times (in Milliseconds) of Experiment 1 (Multiplication) and Experiment 2 (Division) As a Function of Load, Working Memory Component (Passive Phonological, Active Phonological, Executive), Strategy, and Size

		PL Passive				PL Active				Executive			
		No Load		Load		No Load		Load		No Load		Load	
Strategy	Size	*M*	*SE*	*M*	*SE*	*M*	*SE*	*M*	*SE*	*M*	*SE*	*M*	*SE*
Experiment 1: Multiplication													
Retrieval	Small	854	52	843	58	922	52	977	58	736	52	957	58
	Large	1,129	80	1,089	78	1,259	80	1,319	78	964	80	1,191	78
Transformation	Small	2,874	357	2,954	380	3,280	357	3,240	380	2,379	357	2,761	380
	Large	3,235	304	3,126	334	3,110	304	3,312	334	2,616	304	3,013	334
Counting	Small	2,881	269	3,162	292	2,980	269	3,342	292	2,556	269	2,964	292
	Large	6,261	661	6,704	761	6,284	661	6,863	761	5,874	661	7,275	761
Experiment 2: Division													
Retrieval	Small	745	59	725	75	917	59	908	75	906	59	1,210	75
	Large	893	78	860	96	1,159	78	1,131	96	1,057	77	1,402	96
Via multiplication	Small	1,593	195	1,696	193	1,590	193	1,671	193	1,410	195	1,764	193
	Large	1,996	281	2,246	321	1,972	281	2,107	321	1,930	281	2,342	321

$p = .18$]. High variance on the counting RTs may have prevented this effect from reaching significance.

To consolidate the results described above and investigate the influence of individual differences, correlations[4] were calculated between strategy efficiency (i.e., retrieval RTs, transformation RTs, and counting RTs), strategy selection, working memory load (i.e., executive, active phonological, and passive phonological), problem size, and individual-difference variables (i.e., math anxiety, arithmetic skill, calculator use, gender, and math experience).

When looking at the correlation measures presented in Table 2, we see that strategies were executed more slowly when problem size was larger and when the central executive was loaded. This confirms the ANOVA results. Moreover, the efficiency of the different strategies correlated with several individual-difference variables. The efficiency of all three strategies was higher in high-skill participants than in low-skill participants. Participants with stronger problem–answer associations were more efficient in retrieval, but not in transformation or counting. Retrieval efficiency was higher in infrequent calculator users than in frequent calculator users and higher in males than in females.

Strategy selection. In order to investigate effects on strategy selection, a 3 × 2 × 2 ANOVA was conducted on percentages of use of each strategy (in the choice condition), with working memory component (passive phonological, active phonological, executive) as a between-subjects factor and load (no load vs. load) and size (small vs. large) as within-subjects factors (see Table 3).

For retrieval, the main effect of size was significant [$F(1,57) = 71.47$, $MS_e = 96$], indicating more frequent retrieval use on small problems (89%) than on large problems (72%). The main effects of load and working memory component did not reach significance, nor did any interaction (highest $F = 2.31$). For transformation, the main effect of size was significant [$F(1,57) = 50.22$, $MS_e = 11{,}395$], indicating more frequent transformation use on large problems (16%) than on small problems (3%). None of the other effects reached significance (highest $F = 1.79$). Finally, counting tended to be used more often on large problems (11%) than on small problems (9%), but this effect did not reach significance [$F(1,57) = 3.13$, $MS_e = 403$, $p = .08$]. None of the other effects reached significance (highest $F = 1.18$).

In Table 2, the correlations between retrieval frequency, working memory load, problem size, and individual differences are presented. Percentage of retrieval use correlated with problem size, but did not correlate with any of the working memory loads. This confirms the ANOVA results. Percentage of retrieval use correlated with all individual-difference variables, however. More specifically, retrieval was more frequently used by high-skill participants than by low-skill participants, by infrequent calculator users than by frequent calculator users, by more experienced participants than by less experienced participants, by low-anxious participants than by high-anxious participants, and by males than by females.

Secondary task performance. An ANOVA was conducted on CRT accuracy, CRT speed, and letter task accuracy (see Table 4), with condition as a within-subjects variable (single, choice, no-choice/retrieval, no-choice/transformation, and no-choice/counting). CRT speed tended to differ across conditions [$F(4,16) = 2.56$, $MS_e = 3{,}862$, $p = .08$]. Participants reacted faster to the tones in the CRT-only condition (626 msec) than in the other conditions (660 msec), but this difference did not reach significance [$F(1,19) = 2.21$, $MS_e = 8{,}516$, $p = .15$]. CRT accuracy differed across conditions as well [$F(4,16) = 6.51$, $MS_e = 67$]. More specifically, CRT accuracy was significantly higher in the CRT-only condition (87%) than in the other conditions (80%) [$F(1,19) = 4.17$, $MS_e = 167$]. When few executive working memory resources were available, performance was impaired not only on the primary task, but also on the secondary task. CRT accuracy was higher in the no-choice/retrieval condition than in either the choice condition [$F(1,19) = 7.31$, $MS_e = 32$] or the other no-choice conditions [$F(1,19) = 7.04$, $MS_e = 40$]. Note that the slowest CRT performance was observed in the no-choice/transformation condition, where the effect of an executive load failed to reach significance ($p = .09$; see above). A trade-off between efficient transformation use and efficient CRT performance, then, may account for the insignificant effect of executive load on transformation RTs. Performance on the active phonological task (i.e., the letter task) differed across conditions as well [$F(4,16) = 12.56$, $MS_e = 166$]. Accuracy was significantly higher in the single-task condition (84%) than in the dual-task conditions (68%) [$F(1,19) = 19.91$, $MS_e = 210$].

Table 2
Correlation Table for Experiment 1 (Multiplication)

	Transform RT	Count RT	Retrieval Use %	Problem Size	Arith. Skill	Calc. Use	Math Exper.	Math Anx.	Gender	Phon. Passive	Phon. Active	Exec.
Retrieval RT	.424*	.393*	−.370*	.311*	−.415*	.294*	−.006	.009	−.210*	−.021	.045	.193*
Transform RT		.509*	−.113	.539*	−.208*	.093	.002	−.051	−.047	.037	.046	.096
Count RT			−.002	.063	−.284*	.109	.006	−.112	.012	−.012	.045	.080
Retrieval use %				−.349*	.190*	−.205*	.256*	−.202*	.270*	.016	.007	.048
Arithmetic skill						−.440*	.014	.012	.410*	–	–	–
Calculator use							.127	.096	−.332*	–	–	–
Math experience								−.455*	.159	–	–	–
Math anxiety									−.186	–	–	–
Gender										–	–	–

Note—Associative strength is operationalized by the participants' percentage of retrieval use. RT, reaction time; Phon., phonological; Exec., executive. *$p < .0038$ (the Bonferroni-corrected alpha level of .05 when correlating 13 variables); $df = 238$.

Table 3Mean Percentages of Strategy Use in Experiment 1 (Multiplication) and Experiment 2 (Division) As a Function of Load, Working Memory Component (Passive Phonological, Active Phonological, Executive), and Size

		PL Passive				PL Active				Executive			
		No Load		Load		No Load		Load		No Load		Load	
Strategy	Size	*M*	*SE*	*M*	*SE*	*M*	*SE*	*M*	*SE*	*M*	*SE*	*M*	*SE*
Experiment 1: Multiplication													
Retrieval	Small	88	4	90	4	87	4	86	4	88	4	91	4
	Large	70	6	71	6	68	6	70	6	76	6	79	6
Transformation	Small	2	2	2	4	4	2	3	1	2	2	2	1
	Large	17	5	17	4	20	5	16	4	15	5	13	4
Counting	Small	9	3	8	4	10	3	11	4	9	3	7	4
	Large	13	3	12	3	12	3	14	3	10	3	8	3
Experiment 2: Division													
Retrieval	Small	82	5	88	5	80	5	81	5	86	5	86	5
	Large	68	6	69	5	68	6	72	5	74	6	74	5
Via multiplication	Small	18	5	12	5	20	5	19	5	14	5	14	5
	Large	32	6	31	5	32	6	28	5	26	6	26	5

Summary

Results concerning strategy efficiency showed that the roles of the different working memory resources differed across strategies. Executive working memory resources were needed in all strategies, whereas phonological working memory resources were especially needed in the counting strategy. Working memory load did not have any effect on strategy selection. Both strategy efficiency and strategy selection correlated significantly with several individual-difference variables. The interpretation of the possible roles of these individual differences will be addressed in the General Discussion of the present study.

EXPERIMENT 2: Division

Method

Participants. Sixty participants (10 men and 50 women) took part in the present experiment. Their mean age was 21 years and 4 months. Half of them were first-year psychology students at Ghent University, who participated for course requirements and credits. The other half were paid €10 for their participation. None of them had participated in Experiment 1.

Stimuli and Procedure. The 43 division problems were the reverses of the multiplication problems used in Experiment 1. The procedure was identical to the one used in Experiment 1, with one exception. It has been shown that only two strategies are frequently used to solve simple division problems (Campbell & Xue, 2001; LeFevre & Morris, 1999; Robinson, Arbuthnott, & Gibbons, 2002): direct memory retrieval and solving the division problem via the related multiplication problem (e.g., solving 48 ÷ 8 via ? × 8 = 48). Therefore, the choices in the choice condition of this experiment were restricted to three:

1. Retrieval: You solve the problem by remembering or knowing the answer directly from memory. It means that you know the answer without any additional processing. For example, you know that 30 ÷ 6 = 5 because 5 "pops into your head."
2. Via multiplication: You solve the division problem by using the related multiplication problem. For example, when you have to solve 42 ÷ 6, you think about how many times 6 goes into 42, i.e., 6 × ? = 42. You might also check your answer by doing the multiplication 6 × 7 = ?.
3. Other: You solve the problem by a strategy unlisted here, or you do not know what strategy you used to solve the problem. For example, guessing.

Accordingly, there were only two no-choice conditions: no-choice/retrieval, in which participants were asked to retrieve the answer, and no-choice/via-multiplication, in which participants were asked to solve the division problem via the related multiplication problem.

Table 4
Performance on the Secondary Tasks in Experiment 1 (Multiplication) and Experiment 2 (Division)

	CRT Accuracy		CRT Speed		Letter-Task Accuracy	
Condition	*M*	*SE*	*M*	*SE*	*M*	*SE*
Experiment 1: Multiplication						
Single	87	5	626	26	84	3
Choice	79	3	656	17	56	3
Retrieval	84	3	646	20	75	4
Transformation	79	4	672	18	68	5
Counting	79	3	666	16	74	4
Experiment 2: Division						
Single	88	2	647	23	90	2
Choice	73	3	661	8	62	5
Retrieval	75	3	664	15	78	4
Via multiplication	75	3	646	13	77	4

Note—CRT Accuracy, percent correct choice reaction time; CRT Speed, speed (in milliseconds) choice reaction time.

Results

Failures of the sound-activated relay spoiled 5.6% of the trials. All these invalid trials were presented again at the end of the block, so most of them were recovered from data loss. This reduced the percentage of trials lost due to failures of the sound-activated relay to 1.5%. All incorrect trials (10.0%), choice trials on which participants reported having used an *other* strategy (0.7%), and no-choice trials on which participants failed to use the required strategy (6.0%) were deleted. The low percentage of *other* strategy use confirmed that the two strategies allowed in the choice condition (i.e., direct memory retrieval and the via-multiplication strategy) covered the choice pattern generally used by participants when solving simple division problems. All data were analyzed on the basis of the multivariate general linear model, and all reported results were considered to be significant if $p < .05$, unless stated otherwise.

To test whether the three participant groups (i.e., loaded by either the passive phonological task, the active phonological task, or the executive task) differed from each other, we conducted four ANOVAs. Results showed no group differences in arithmetic skill, calculator use, math experience, or math anxiety (all Fs < 1.1, all ps > .30).

Strategy efficiency. A 3 × 2 × 2 × 2 ANOVA was conducted on correct no-choice RTs with working memory component (passive phonological, active phonological, executive) as a between-subjects factor and load (no load vs. load), strategy (retrieval vs. via multiplication), and size (small vs. large) as within-subjects factors (see Table 1).

The main effects of load, strategy, and problem size were significant. RTs were longer under load (1,505 msec) than under no load (1,304 msec) [$F(1,57) = 29.08$, $MS_e = 102{,}768$]; retrieving division facts (993 msec) was faster than solving them via multiplication (1,860 msec) [$F(1,57) = 52.84$, $MS_e = 1{,}400{,}216$]; and small problems (1,261 msec) were solved faster than were large problems (1,591 msec) [$F(1,57) = 59.60$, $MS_e = 219{,}528$].

Strategy also interacted with problem size and load. The strategy × size interaction indicated a larger problem-size effect (i.e., RTs on large problems − RTs on small problems) when division problems were solved via multiplication than when they were retrieved from memory [$F(1,57) = 16.69$, $MS_e = 157{,}848$]. The strategy × load interaction showed larger effects of working memory load (i.e., load RTs − no-load RTs) when division problems were solved via multiplication than when they were retrieved from memory [$F(1,57) = 5.05$, $MS_e = 99{,}248$].

There was also a significant interaction between working memory component and load [$F(2,57) = 11.30$, $MS_e = 102{,}769$], which showed that load effects were significant for the executive component [$F(1,57) = 48.72$, $MS_e = 102{,}769$], but not for the active phonological component [$F(1,57) < 1$] or the passive phonological component [$F(1,57) = 2.19$, $p = .14$]. This interpretation was verified by separate ANOVAs that tested the effects of the different working memory loads for each strategy. Retrieval RTs were affected by executive loads [$F(1,57) = 75.27$, $MS_e = 27{,}985$], but not by active phonological or passive phonological loads (each $F < 1$). Via-multiplication RTs were affected by executive loads [$F(1,57) = 16.87$, $MS_e = 174{,}031$], but not by active phonological loads [$F(1,57) = 1.33$, $p = .25$]. However, via-multiplication RTs tended to be affected by passive phonological loads [$F(1,57) = 3.59$, $MS_e = 174{,}032$, $p = .06$].

To consolidate the results described above, and to investigate the influence of individual differences, correlations were calculated between strategy efficiency (i.e., retrieval RTs and via-multiplication RTs), strategy selection, working memory load (i.e., executive, active phonological, and passive phonological), problem size, and individual-difference variables (i.e., math anxiety, arithmetic skill, calculator use, gender, and math experience).

Correlation measures are presented in Table 5 (see note 4). Strategy efficiencies were smaller when problem size was larger and when the central executive was loaded. This confirms the ANOVA results. Strategy efficiencies correlated with several individual-difference variables as well. More specifically, retrieval and via-multiplication efficiencies were higher in high-skill participants than in low-skill participants, and higher in low-anxious participants than in high-anxious participants. Associative strength correlated significantly with the efficiency of the via-multiplication strategy, but not with the efficiency of the retrieval strategy. Finally, the efficiency of the via-multiplication strategy was higher in more experienced participants than in less experienced participants.

Strategy selection. In order to investigate effects on strategy selection, a 3 × 2 × 2 ANOVA was conducted on percentages of use of each strategy in the choice condition, with working memory component (passive phonological, active phonological, executive) as a between-subjects factor and load (no load vs. load) and size (small vs. large) as within-subjects factors (see Table 3).

For retrieval, the main effect of size was significant [$F(1,57) = 49.36$, $MS_e = 10{,}431$], indicating more frequent retrieval use on small problems (84%) than on large problems (71%). The main effects of load and working memory component did not reach significance, nor did any interaction (highest $F = 1.11$). The via-multiplication

Table 5
Correlation Table for Experiment 2 (Division)

	Multiplication RT	Retrieval Use (%)	Problem Size	Arith. Skill	Calc. Use	Math Exper.	Math Anx.	Gender	Phon. Passive	Phon. Active	Exec.
Retrieval RT	.494*	−.149	.233*	−.264*	.019	−.047	.195*	−.130	−.020	−.014	.240*
Multiplication RT		−.206*	.210*	−.328*	.083	−.230*	.233*	−.105	.045	.027	.097
Retrieval use (%)			−.274*	.003	−.063	.150	−.006	.062	.041	.031	.000
Arithmetic skill					−.241*	.299*	−.321*	.002	–	–	–
Calculator use						.208*	.241*	−.030	–	–	–
Math experience							−.207*	.040	–	–	–
Math anxiety								−.128*	–	–	–
Gender									–	–	–

Note—Associative strength is operationalized by the participants' percentage of retrieval use. RT, reaction time; Phon., phonological; Exec., executive. *$p < .0042$ (the Bonferroni-corrected alpha level of .05 when correlating 12 variables); $df = 238$.

strategy, in contrast, was used more frequently on large problems (29%) than on small problems (16%) [$F(1,57) = 49.36$, $MS_e = 10{,}431$]. None of the other effects reached significance (highest $F = 1.11$)

In Table 5, the correlations between retrieval frequency, working memory load, problem size, and individual differences are presented. Percentage of retrieval use correlated with problem size, but did not correlate with any of the working memory loads. This confirms the ANOVA results. None of the individual-difference variables correlated significantly with strategy selection.

Secondary task performance. An ANOVA was conducted on CRT accuracy, CRT speed, and letter-task accuracy (Table 4), with condition as a within-subjects variable (single, choice, no-choice/retrieval, no-choice/via-multiplication). CRT accuracy differed across conditions [$F(3,17) = 11.80$, $MS_e = 56$]. More specifically, CRT accuracy was higher in the CRT-only condition (88%) than in the other conditions (75%) [$F(1,19) = 33.86$, $MS_e = 78$]. CRT speed did not differ across conditions [$F(3,17) = 1.06$, $p = .39$]. Performance on the active phonological task (the letter task) differed across conditions [$F(3,17) = 15.06$, $MS_e = 180$]. Accuracy was higher in the single-task condition (90%) than in the dual-task condition (72%) [$F(1,19) = 13.26$, $MS_e = 350$].

Summary

Concerning strategy efficiency, it was shown that, as in Experiment 1, the roles of the different working memory resources differed across strategies. The retrieval strategy was affected by an executive load only, whereas the via-multiplication strategy was affected by an executive load and by a passive phonological load. Strategy efficiency further correlated significantly with several individual-difference variables, the interpretation of which will be addressed in the General Discussion of the present study. Also, as in Experiment 1, strategy selection was not influenced by working memory load.

GENERAL DISCUSSION

In the present study, the choice/no-choice method and the selective-interference paradigm were combined in order to investigate the role of working memory in simple-arithmetic strategy selection and strategy efficiency. Results showed that the executive working memory component was involved in all strategies—retrieval, transformation, and counting in Experiment 1, and retrieval and via-multiplication in Experiment 2. Phonological working memory components played a much smaller role and tended to be needed in some nonretrieval strategies, such as the counting strategy in the multiplication experiment and the via-multiplication strategy in the division experiment).

The Role of Executive Working Memory Resources

Executive working memory resources were needed in direct retrieval of multiplication and division facts. Getting access to information stored in long-term memory is indeed one of the main executive (or attentional) functions (see, e.g., Baddeley, 1996; Baddeley & Logie, 1999; Cowan, 1995; Engle, Kane, & Tuholski, 1999; Ericsson & Kintsch, 1995). Consequently, executive (or attentional) working memory resources have long been hypothesized to play a significant role in retrieving arithmetic facts from long-term memory (see, e.g., Ashcraft, 1992, 1995; Ashcraft, Donley, Halas, & Vakali, 1992; Barrouillet, Bernardin, & Camos, 2004; Geary & Widaman, 1992; Kaufmann, 2002; Kaufmann, Lochy, Drexler, & Semenza, 2003; Lemaire et al., 1996; Seitz & Schumann-Hengsteler, 2000, 2002; Zbrodoff & Logan, 1986), and the present study succeeded in showing this through the use of a rigorous method—solving simple-arithmetic problems in a no-choice/retrieval condition under an executive working memory load.

We suppose that executive working memory resources are needed to select the correct response. Indeed, the presentation of a simple multiplication or division problem does automatically activate several candidate answers in long-term memory (see, e.g., Campbell, 1997; Galfano, Rusconi, & Umiltà, 2003; Rusconi, Galfano, Rebonato, & Umiltà, 2006; Rusconi, Galfano, Speriani, & Umiltà, 2004; Thibodeau, LeFevre, & Bisanz, 1996). After this automatic activation of several associated responses, a deliberate choice of the correct response has to be executed in order to complete the retrieval.

Executive working memory resources also played a role when nonretrieval strategies were used to solve multiplication or division problems. Of course, executing nonretrieval strategies also requires retrieval of known responses, which relies on executive resources. Moreover, executing nonretrieval strategies requires other demanding processes as well, such as performing calculations (see, e.g., Ashcraft, 1995; Imbo, Vandierendonck, & De Rammelaere, 2007; Imbo, Vandierendonck, & Vergauwe, 2007; Logie, Gilhooly, & Wynn, 1994), manipulating interim results (Fürst & Hitch, 2000), and monitoring counting sequences (see, e.g., Ashcraft, 1995; Case, 1985; Hecht, 2002; Logie & Baddeley, 1987).

The central executive did not play a role in strategy selection: Percentages of strategy use did not change under an executive working memory load. This is in agreement with previous studies (see, e.g., Hecht, 2002; Imbo & Vandierendonck, in press) and suggests that selecting simple-arithmetic strategies does not rely on executive working memory resources. The absence of load effects on the strategy selection process is in agreement with the adaptive strategy choice model of Siegler and Shipley (1995). In this model, strategy selection is based solely on problem–answer association strengths (i.e., the answer that is most strongly associated with the presented problem is retrieved) and not on metacognitive processes, such as executive (or attentional) processes.

The Role of Phonological Working Memory Resources

Phonological working memory resources tended to be needed in nonretrieval strategies. More specifically, an active phonological load tended to affect the counting strategy in Experiment 1 ($p = .10$), and a passive phonological load tended to affect the via-multiplication strategy in Experiment 2 ($p = .06$). These results are in agreement with previous studies (Hecht, 2002; Imbo & Vandierendonck, in press; Seyler et al., 2003) that also showed a significant role for the phonological loop in nonretrieval strategies.

The main function of the active phonological rehearsal process is storing intermediate and partial results (Ashcraft, 1995; Hitch, 1978; Logie et al., 1994), a function that is needed in nonretrieval strategies only. Using the counting strategy to solve multiplication facts (e.g., $4 \times 7 = 7 \ldots 14 \ldots 21 \ldots 28$) indubitably requires the storing of intermediate results, and thus relies on active phonological resources. The passive phonological store comes into play when more than one number needs to be maintained at any one time (Logie & Baddeley, 1987). This may explain our finding that passive phonological resources were needed when the via-multiplication strategy was used to solve division problems. In order to transform a division problem into a multiplication problem (e.g., transforming $56 \div 8$ into $8 \times ? = 56$), participants had to maintain the dividend and the divisor while they subvocally recited the multiplication tables.

The present study also sheds further light on the equivocal results observed in previous studies that investigated the role of the phonological loop in simple arithmetic. Whereas some studies did show an effect of phonological load (see, e.g., Lee & Kang, 2002; Lemaire et al., 1996; Seitz & Schumann-Hengsteler, 2002), others did not (see, e.g., De Rammelaere et al., 1999, 2001; Seitz & Schumann-Hengsteler, 2000). Present results suggest that strategy choices may have played a role. Studies in which participants relied more heavily on nonretrieval strategies may have shown larger effects of phonological working memory loads than have studies in which participants relied mainly on direct memory retrieval.

The Impact of Individual Differences

In addition to investigating the role of working memory in people's arithmetic strategy use, we also explored whether individual differences might have influenced strategy efficiency or strategy selection processes. Some of the possible roles of these individual-difference variables are discussed below.

Arithmetic skill correlated significantly with all strategy efficiencies. More specifically, high-skill participants were more efficient in executing both retrieval and nonretrieval strategies to solve multiplication and division problems. This observation is not very surprising, however, as both our primary task (solving simple arithmetic problems) and the French Kit are speeded performance tests. Hence, correlations between arithmetic skill and strategy efficiency have been observed previously (see, e.g., Campbell & Xue, 2001; Imbo et al., in press; Kirk & Ashcraft, 2001; LeFevre & Bisanz, 1986). Arithmetic skill correlated with strategy selection only in the multiplication experiment: High-skill participants used retrieval more frequently than did low-skill participants, an observation that is in agreement with previous studies (see, e.g., Imbo et al., in press; LeFevre, Bisanz, et al., 1996; LeFevre, Sadesky, & Bisanz, 1996).

Associative strength (i.e., percentages of retrieval use) correlated with retrieval efficiency in Experiment 1, but not in Experiment 2 (in which the correlation was quite high and in the correct direction, but not significant). Indeed, it has been asserted that problems with higher associative strengths are retrieved more efficiently from long-term memory (see, e.g., Ashcraft et al., 1992; Hecht, 2002). The correlation between associative strength and via-multiplication strategy efficiency in Experiment 2 may reflect the fact that fast retrieval of multiplication facts is a critical component of this strategy.

Concerning math anxiety, the results of Experiment 1 indicated effects on strategy selection. Retrieval use was significantly less frequent in high-anxious participants than in low-anxious participants. Anxious participants may set higher confidence criteria and will retrieve an answer only when they are sure of its correctness. No effects of math anxiety on strategy efficiency were found in Experiment 1, probably because solving simple multiplication problems is rather easy. Indeed, math anxiety affects arithmetic performance only when the task is resource-demanding (Ashcraft, 1995; Faust, Ashcraft, & Fleck, 1996). This also explains why math anxiety affected strategy efficiency in Experiment 2, in which division problems had to be solved, and both retrieval and nonretrieval strategy use were less efficient in high-anxious participants than in low-anxious participants. Math-anxious participants are often preoccupied with worries and intrusive thoughts when performing arithmetic tasks. Because such thoughts constitute a load on working memory resources, high-anxious participants have fewer working memory resources left for solving arithmetic tasks efficiently (Ashcraft & Kirk, 2001; Faust et al., 1996). It is reasonable to believe that solving division problems is more resource demanding than solving multiplication problems, which explains why math anxiety affected strategy efficiency in Experiment 2, but not in Experiment 1.

The frequency of calculator use correlated with strategy selection and strategy efficiency in Experiment 1 (multiplication), but not in Experiment 2 (division). More frequent calculator use was related to less efficient and less frequent retrieval use. Effects of calculator use on strategy efficiency had been observed earlier (Imbo et al., in press), but no previous study has shown a reliable effect of calculator use on simple-arithmetic strategy selection.

Math experience correlated with strategy selection and strategy efficiency. More experienced participants used the retrieval strategy more frequently (Experiment 1) and were more efficient in the execution of the via-multiplication strategy (Experiment 2). Comparable effects have been observed previously (see, e.g., Imbo et al., in press) and indicate that daily arithmetic practice has a substantial effect on strategy selection and strategy efficiency.

Gender, finally, correlated with strategy selection and strategy efficiency in Experiment 1, but not in Experiment 2. When solving multiplication problems, men more frequently used retrieval than did women, an effect observed earlier (see, e.g., Carr & Jessup, 1997; Carr et al., 1999; Fennema, Carpenter, Jacobs, Franke, & Levi, 1998; Geary et al., 2000). We also observed more efficient retrieval use in men than in women, which confirms the hypothesis that gender differences in mental arithmetic are due to the fact that retrieval use is faster in men than in women (Royer et al., 1999). However, gender might correlate with many other individual-difference variables as well, such as calculator use, math experience, math anxiety, and arithmetic skill. Hence, further research is needed to disentangle gender effects from other confounding variables.

On the basis of these exploratory correlations, it may be concluded that individual differences influence people's strategy efficiency and strategy selection processes. However, the effects were not always significant and differed across operations (multiplication vs. division) and across strategic performance measures (efficiency vs. selection). This was especially the case for the individual-difference variables that were based on single questions (e.g., calculator use and math anxiety). We acknowledge that the reliability of such measures can be questioned. Hence, future studies, in which individual differences are tested more thoroughly, are needed to confirm or disconfirm the exploratory results found here. For example, one might choose to use the full Mathematics Anxiety Rating Scale (MARS; Richardson & Suinn, 1972) in order to test participants' math anxiety. Further research might also investigate the impact of individual differences in a more experimental way—for example, by training participants, by manipulating their anxiety levels, or by augmenting or reducing their calculator use.

Conclusion

In the present study, we used a combination of two frequently used and widely approved methods: the selective-interference paradigm and the choice/no-choice method. The selective-interference paradigm enabled us to investigate the role of three different working memory components; the choice/no-choice method enabled us to study strategy selection and strategy efficiency independently. Another novelty of the present study is that multiplication and division strategies were investigated. These operations differ greatly from addition and subtraction, from childhood through adulthood. Moreover, the role of working memory in multiplication and division strategies has, until now, never been investigated. A final novelty of the present study is that several individual-difference variables were included.

Concerning strategy efficiency, results showed that executive working memory resources were involved in both retrieval and nonretrieval strategies. Active and passive phonological working memory resources played a much smaller role and tended to be involved in nonretrieval strategies only. Strategy selection, on the other hand, was not affected by executive or phonological working memory loads. It was further shown that individual differences had a large impact as well. Arithmetic skill, calculator use, math experience, gender, and math anxiety influenced strategy efficiency and strategy selection. Individual differences should not, therefore, be ignored when the cognitive systems underlying simple-arithmetic performance are investigated. Indeed, many effects caused by individual differences can be explained by cognitive variables. Effects of math anxiety, for example, can be explained by working memory limits (Ashcraft & Kirk, 2001; Faust et al., 1996), and effects of math experience can be explained by differential problem–answer strengths in long-term memory (Imbo et al., in press). Arithmetic models and theories could be challenged to incorporate these individual differences and their respective cognitive processes.

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NOTES

1. Given the poorer elaboration of the role of the visuospatial sketchpad in simple arithmetic (on theoretical, methodological, and empirical level), this working memory component was not included in the present study.

2. The correlation between rating math anxiety on a scale from 1 to 5 and rating math anxiety with the Mathematics Anxiety Rating Scale (MARS; Richardson & Suinn, 1972) ranges from .45 to .85 (Mark Ashcraft, personal communication).

3. Both subtests of the French Kit correlated significantly with each other ($p < .01$); $r = .675$ in Experiment 1 and $r = .531$ in Experiment 2, indicating high reliability. Correlations are not 100% because both subtests test other operations (addition vs. multiplication–subtraction).

4. Gender was coded as a dummy variable: Girls were coded as −1 and boys were coded as 1. Each working memory load was coded as a dummy variable as well. This variable was −1 for no-load conditions and 1 for load conditions.

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